# Quantum fluctuations of thermodynamic variables

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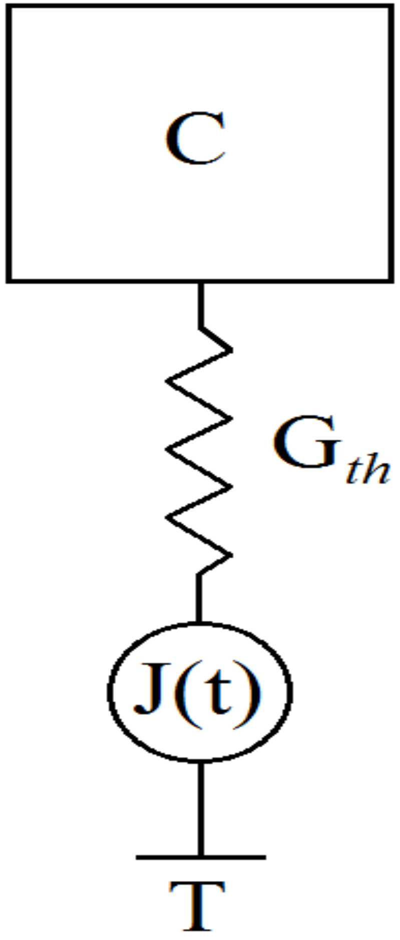
## Outline:

- Introduction: Quantum fluctuations of heat flux; violation of the FDT for thermal conductance.
- Quantum fluctuations of the internal energy.
- Modulation detection of the heat/energy quantum fluctuations.
- Conclusions.

# Classical thermodynamic fluctuations of heat flow/internal energy




"Thermodynamic" equilibrium energy fluctuations due to fluctuations of the level occupancy:

$$\left\langle \tilde{E}^2 \right\rangle = CT^2 .$$

Natural relaxation dynamic of these fluctuations

$$\dot{\tilde{E}} = -\tilde{E}/\tau + J , \qquad \tau = C/G_{th} ,$$

imply definite relation between the thermal conductance and the spectral density of the energy flux:

$$\left\langle \tilde{E}^2 \right\rangle = \frac{1}{2\pi} \int d\omega \frac{S_J}{\omega^2 + 1/\tau^2} = S_J \tau / 2 = CT^2 .$$

$$S_J = 2T^2 G_{th}$$

# Violation of the full (quantum) thermal FDT

1. Microscopically, temperature is not a mechanical quantity

2. There is (was ?) a common view that FDT should work for thermal conductance as well, e.g.

STATISTICAL PHYSICS

Part 2

*Theory of the Condensed State*

*by*

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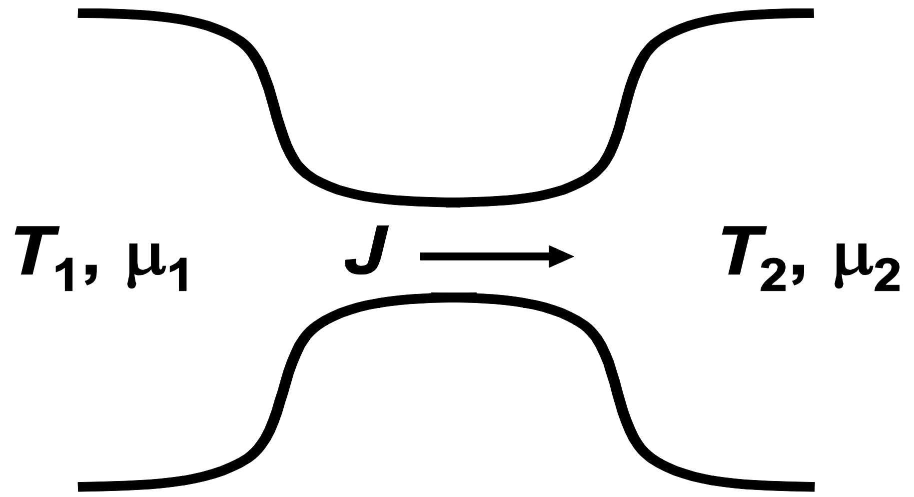
$$S(\omega) = \hbar\omega T \, \mathrm{Re}\, G_{th}(\omega) \coth(\hbar\omega / 2T).$$

3. D.V.A. and J.P. Pekola, PRL **104**, 220601 (2010) - arguments for violation of thermal FDT in quantum regime, partly confirmed later by several authors

Sergi (2011);
Zhan, Denisov, and Hanggi (2013);
Karimi and Pekola (2018).

# Fluctuations of heat flux in mesoscopic transport

Transport through a shot constriction:




$$S(\omega)=\int dt\, e^{-i\omega t}\left\langle \tilde{J}\tilde{J}(t)+\tilde{J}(t)\tilde{J}\right\rangle/2,$$

$$\tilde{J}\equiv J-\langle J\rangle.$$

Phonons

$$J=\frac{\hbar v}{2L}\sum_{k,p}\left(\omega_k\omega_p\right)^{1/2}\operatorname{sgn}(k)(a_k-a_k^+)(a_p^+-a_p).$$

$$S(\omega)=\frac{1}{48\pi\hbar}\sum_{j=1,2}\left[(2\pi T_j)^2+(\hbar\omega)^2\right]\hbar\omega\coth\left(\hbar\omega/2T_j\right).$$

In equilibrium, only the first part corresponds to the thermal FDT with

$$G_{th}=\pi T/6\hbar.$$

# Electrons

Heisenberg equation of motion for the energy density

$$h(x) = \left[\psi^{+}\hat{h}\psi + (\hat{h}\psi^{+})\psi\right]/2, \quad \hat{h} = -(\hbar^2/2m)\partial^2/\partial x^2 + V(x)$$

gives the energy flux operator as

$$J = \frac{v_F}{L}\sum_{k,p}\frac{\varepsilon_k + \varepsilon_p}{2}\left[D(a_k^+ a_p - b_k^+ b_p) + \sqrt{DR}(a_k^+ b_p + b_k^+ a_p)\right],$$

where the energies are measured relative to the chemical potentials $\mu_j$. With this operator one obtains the spectrum of the energy flux noise which in equilibrium reduces to

$$S(\omega) = \left(G/12e^2\right)\left[(2\pi T)^2 + (\hbar\omega)^2\right]\hbar\omega\coth\left(\hbar\omega/2T\right).$$

Both quantitatively and qualitatively, this result is similar to that for phonons.

## Equilibrium quantum fluctuations of energy - model

Use simple tunnel junction as a model:

$$H = H_L + H_R + H_T\ , \qquad H_T = \sum_{k,p} T_{kp} c_k^+ c_p + h.c.$$

The model is to a large extent exactly solvable [Genenko and Ivanchenko (1986)].
At zero temperature,

$$G_{pq}\left(\omega - \varepsilon_p\right) - \sum_{l,r} \frac{T_{lp}^*}{\omega - \varepsilon_l} T_{lr} G_{rp} = \delta_{pq}\ .$$

With the assumption of the random phases of the tunnel matrix elements

$$G_{pq}\left[\omega - \varepsilon_p - \sum_l \frac{\left|T_{lp}\right|^2}{\omega - \varepsilon_l}\right] = \delta_{pq}\ , \qquad \text{i.e.} \qquad G_{pq}(\omega) = \frac{\delta_{pq}}{\omega - \varepsilon_p + i\,sign(\omega)\gamma/2}\ .$$

# Equilibrium quantum fluctuations of energy - result

The fluctuation spectrum of the energy of one electrode can be expressed as

$$S_{EL}(\omega) = \frac{1}{4\pi}\sum_k \varepsilon_k^2 \int d\nu \sum_{\pm} G_k^<(\nu) G_k^>(\nu \pm \omega).$$

Finally,

$$S_{EL}(\omega) = \frac{\hbar^3 G_T}{e^2}\frac{\omega^3/3 + \gamma^2/4}{\omega^2 + \gamma^2}, \qquad \gamma = \frac{G_T}{2e^2\rho_L}.$$

Discussion:

(1) $\gamma = G_{th}/C, \qquad G_{th} = \pi^2 G_T T/3e^2, \qquad C = 2\pi^2 \rho_L T/3.$

(2) $S_{EL}(\omega) = \ldots\omega^3/3 \Leftrightarrow S_J(\omega) = \ldots\omega^3/12 \quad -?$

Resolution of this contradiction: finite energy contained in thermal relaxation

$$J = (\dot{H}_L - \dot{H}_R)/2, \qquad \dot{H}_L \neq -\dot{H}_R, \qquad \dot{H}_L + \dot{H}_R = \dot{H}_T.$$

# Modulation detection of the quantum fluctuations of heat flow

Make the tunnel part of the Hamiltonian time-dependent. Quantum noise is mixed with this modulation of $H_T$ and can be shifted to low frequency, where it behaves classically

$$H_T(t) = f(t) H_T, \qquad f(t) = 1 + a\cos\Omega t.$$

Simple consequences of this modulation, modulation of the average current, and the frequency shift of the current noise:

$$I(t) = G_T V(t) |f(t)|^2, \qquad \overline{S_I(\omega)} = S_I^{(0)}(\omega) + \frac{a^2}{4}\sum_{\pm} S_I^{(0)}(\omega \pm \Omega).$$

The same frequency shift happens for the noise of the heat current $J$. Then

$$S_J(0) = 2T^2 G_{th} + \frac{a^2}{24}\frac{G_T}{e^2}\hbar\Omega\left[(2\pi T)^2 + (\hbar\Omega)^2\right]\coth\left(\hbar\Omega/2T\right).$$

# Quantum fluctuations of energy/temperature

Spectral density of noise at zero frequency can be treated classically

$$\left\langle \delta E^2 \right\rangle = \frac{1}{2\pi}\int d\omega \frac{S_J(0)}{\omega^2+\gamma^2} = \frac{S_J(0)}{\gamma}.$$

If the frequency is sufficiently large, Ω >10 GHz at T≈30 mK,

$$\hbar\Omega >> 2\pi T, \qquad S_J(0) = \frac{a^2}{24}\frac{G_T}{e^2}(\hbar\Omega)^3.$$

The magnitude of the “quantum fluctuations” of energy and temperature then is

$$\left\langle \delta E^2 \right\rangle = \frac{a^2}{8\pi^2} C \frac{(\hbar\Omega)^3}{T}, \qquad \left\langle \delta T^2 \right\rangle = \frac{a^2}{8\pi^2}\frac{(\hbar\Omega)^3}{CT}.$$

# Conclusions

- Relaxation energy contributes directly to the finite frequency energy noise.
- Modulation of the tunneling strength can help detect quantum fluctuations of heat flux/energy.