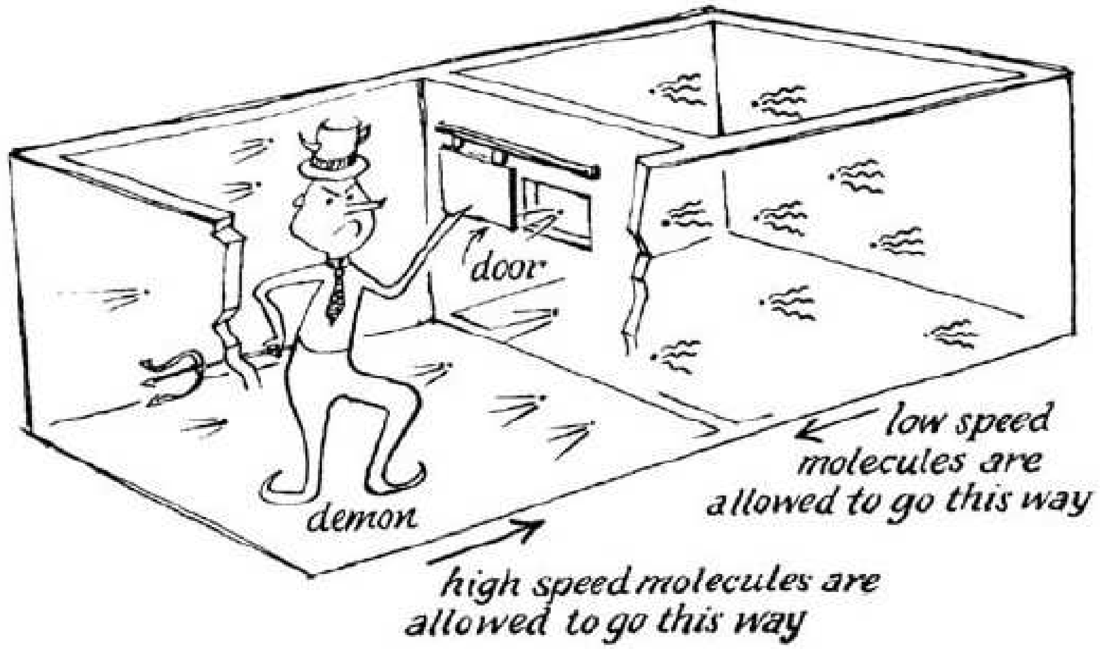
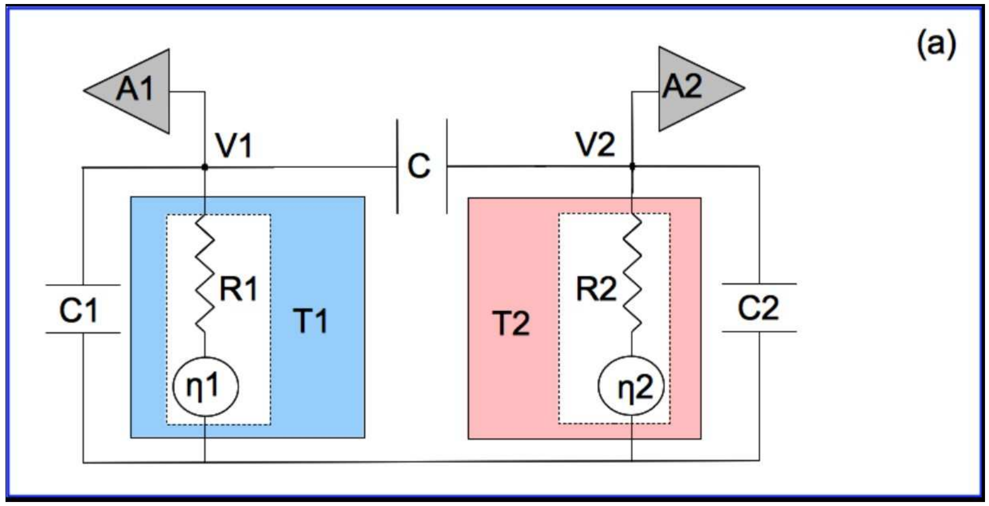
Espoo 19-21September 2018,

Jukka Pekola 60th Birthday

QT60 – Workshop on thermodynamics, thermoelectrics and transport in quantum devices





Heat transport in electric circuits

Landauer's bound and Maxwell's demon
Thermodynamics-information connections

Stochastic and out-equilibrium thermodynamics

HAPPY BIRTHDAY JUKKA !!!

Espoo September 2018,
Jukka Pekola 60th Birthday

Heat flux and entropy produced by thermal fluctuations

Sergio Ciliberto
Laboratoire de Physique
Ecole Normale Supérieure de Lyon and CNRS, France

The role of the coupling in the energy transfer between two stochastic systems coupled to different thermal baths

- **General aspects of Stochastic Thermodynamics**
- **Two electric circuits (conservative coupling)**
- **Two Brownian particles (dissipative coupling)**
- **Transient statistical phenomena**

Bérut et al, JSTAT,P054002(2016)

Collaborators

Antoine Bérut

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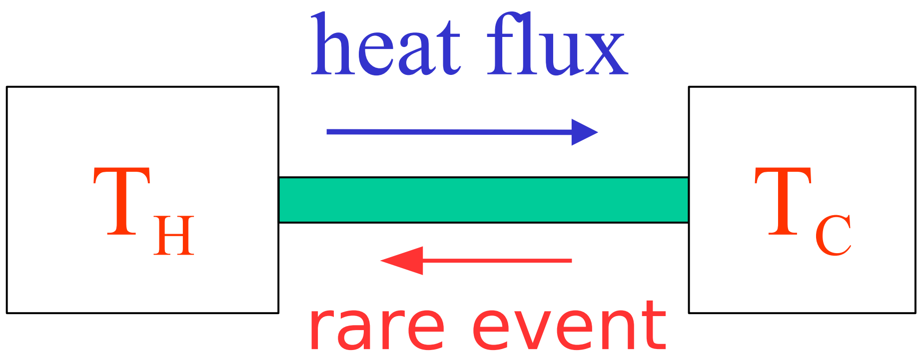
Antoine Naert

Alberto Imparato
University of Aarhus Denmark

On the heat flux between two reservoirs at different temperature

A) In the stationary case for the heat flux between two reservoirs at different temperatures

$$\ln \frac{P(Q_\tau)}{P(-Q_\tau)} = \left(\frac{1}{T_C} - \frac{1}{T_H}\right) \frac{Q_\tau}{k_B}$$



Fluctuation Theorem (FT)

Only few experiments

Theory :

T. Bodinau, B. Deridda, Phy.Rev. Lett 92, 180601 (2004).
C. Jarzynski and D. K. Wójcik Phys. Rev. Lett. 92, 230602 (2004).
C. Van den Broeck, R. Kawai and P. Meurs, Phys. Rev. Lett 93, 090601 (2004).
P Visco, J. Stat. Mech., page P06006, (2006).
K. Saito and A. Dhar Phys. Rev. Lett. 99, 180601 (2007).
D. Andrieux, P. Gaspard, T. Monnai, S. Tasaki, New J. Phys. 11, 043014 (2009).
Evans D. , Searles D. J. Williams S. R., J. Chem. Phys. 132, 024501 2010.
M. Campisi, P. Talkner, P. Hanggi, Rev. Mod. Phys. 83, 771 (2011)
A. Crisanti, A. Puglisi, and D. Villamaina, Phys. Rev. E 85, 061127 (2012)

ENS DE LYON

The Nyquist problem

JULY, 1928 PHYSICAL REVIEW VOLUME 32

THERMAL AGITATION OF ELECTRIC CHARGE IN CONDUCTORS*

By H. Nyquist

Abstract

The electromotive force due to thermal agitation in conductors is calculated by means of principles in thermodynamics and statistical mechanics. The results obtained agree with results obtained experimentally.

Dr. J. B. Johnson[1] has reported the discovery and measurement of an electromotive force in conductors which is related in a simple manner to the temperature of the conductor and which is attributed by him to the thermal agitation of the carriers of electricity in the conductors. The work to be resported in the present paper was undertaken after Johnson's results were available to the writer and consists of a theoretical deduction of the electromotive force in question from thermodynamics and statistical mechanics.[2]

Consider two conductors each of resistance R and of the same uniform temperature T connected in the manner indicated in Fig. 1. The electromotive force due to thermal agitation in conductor I causes a current to be set up in the circuit whose value is obtained by dividing the electromotive force by $2R$. This current causes a heating or absorption of power in conductor II, the absorbed power being equal to the product of R and the square of the current. In other words power is transferred from conductor I to conductor II. In precisely the same manner it can be deduced that power is transferred from conductor II to conductor I. Now since the two conductors are at the same temperature it follows directly from the second law of thermodynamics that the power flowing in one direction is exactly equal to that flowing in the other direction. It will be noted that no assumption has been made as

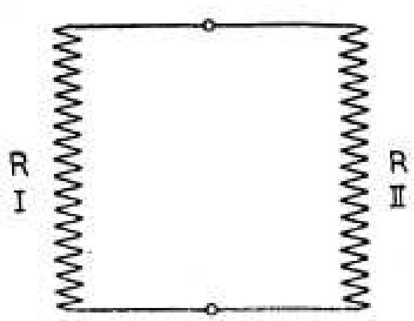
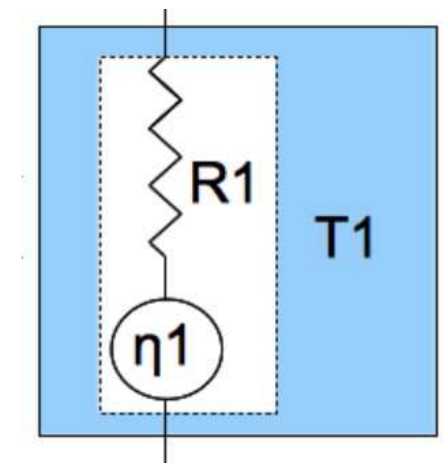


Fig. 1.

Power spectral density of the electric noise

$$|\tilde{\eta}|^2 = 4k_B R T$$



In 1928 well before Fluctuation Dissipation Theorem (FDT), this was the second example, after the Einstein relation for Brownian motion, relating the dissipation of a system to the amplitude of the thermal noise.

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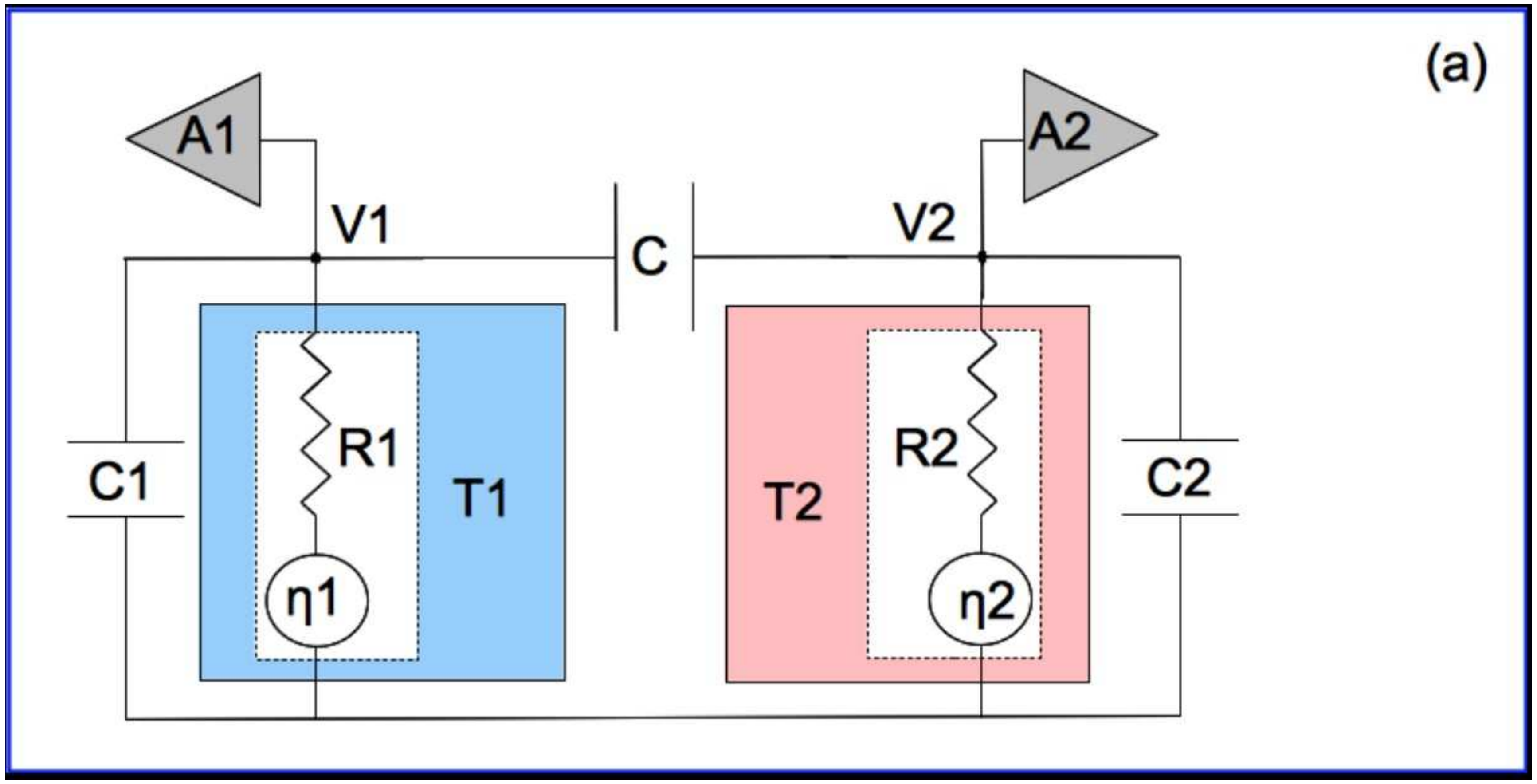
CNRS — dépasser les frontières

What are the consequences of removing the Nyquist equilibrium conditions ?

What are the statistical properties of the energy exchanged between the two conductors kept at different temperature ?

We analyse these questions in an electric circuit within the framework of FT.



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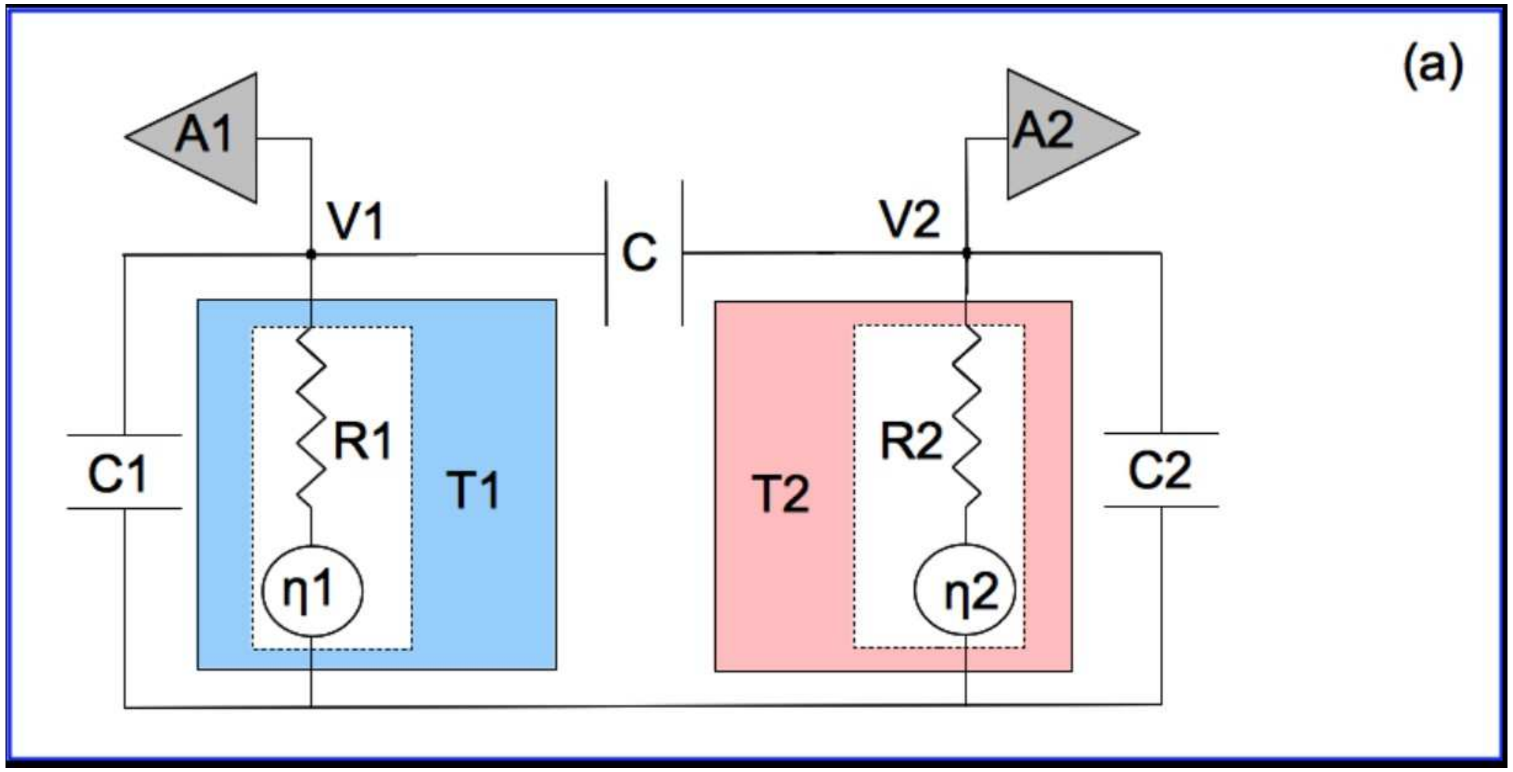
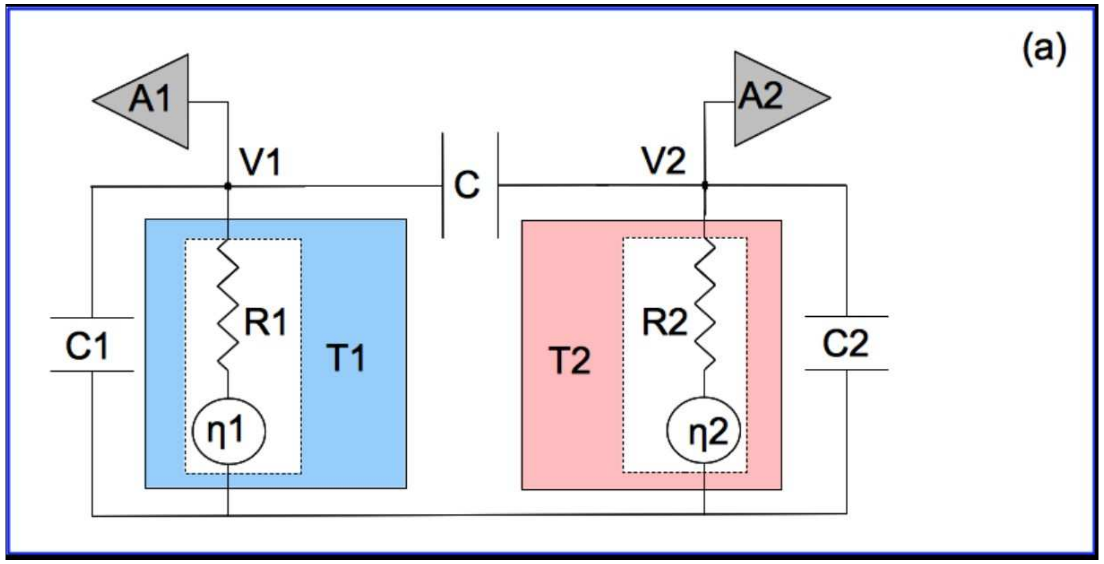


How the variance of V_1 and V_2 are modified because of the heat flux ?

What is the role of correlation between V_1 and V_2 ?

S.Ciliberto, A. Imparato, A. Naert
PRL,110,180601(2013)
JSTAT,P12014 (2013)

Electric Circuit



T_1 is changed by a nitrogen vapor circulation

$T_2 = 296K$ is kept fixed

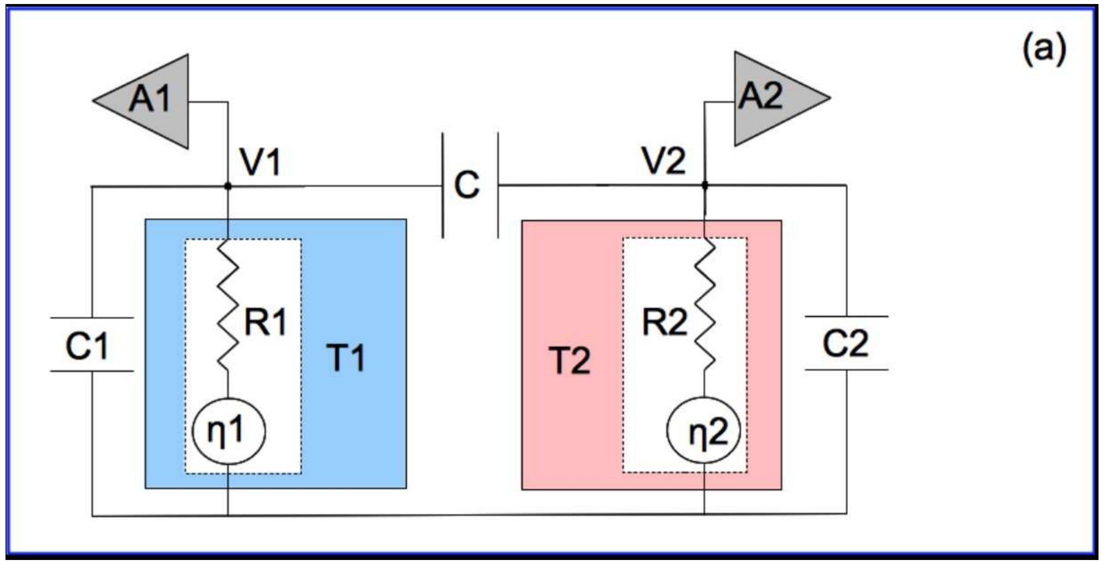
C is the coupling capacitance = 100pF and 1000pF

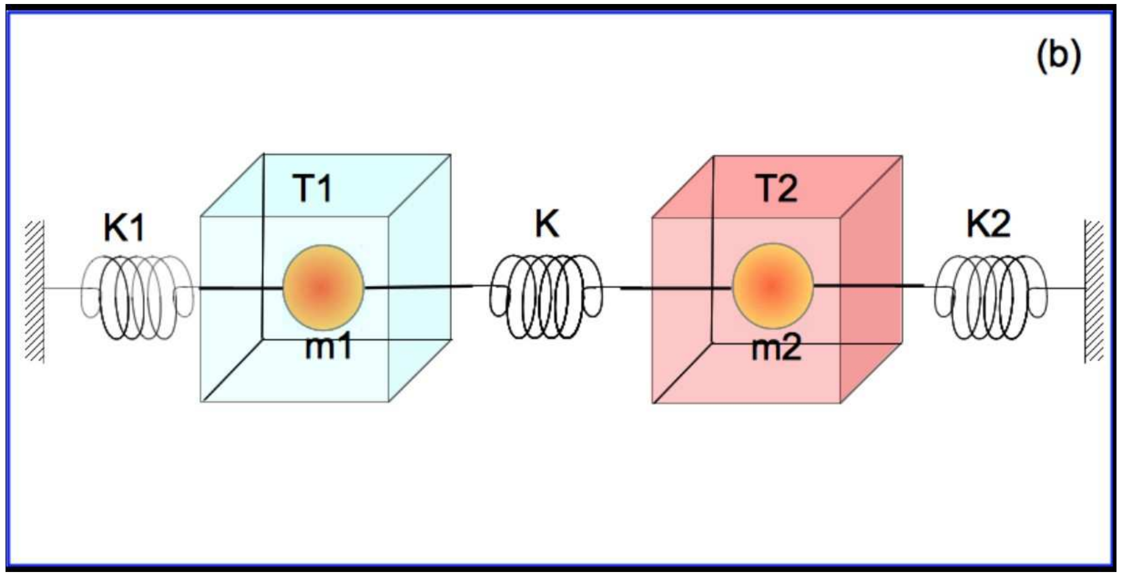
C1 and C2 are the cable and amplifier capacitances $\simeq 500pF$

$R_1 = R_2 = 10M\Omega$

$\tau_o \simeq 0.01s$

Electric Circuit and the mechanical equivalent





$$R_1 \dot{q}_1 = -q_1 \frac{C_2}{X} + (q_2 - q_1)\frac{C}{X} + \eta_1$$

$$R_2 \dot{q}_2 = -q_2 \frac{C_1}{X} + (q_1 - q_2)\frac{C}{X} + \eta_2$$

$$\langle \eta_i(t)\eta_j(t')\rangle = 2\delta_{ij} k_B T_i R_j \delta(t - t')$$

$$X = C_2\, C_1 + C\,(C_1 + C_2)$$

q_m the displacement of the particle m

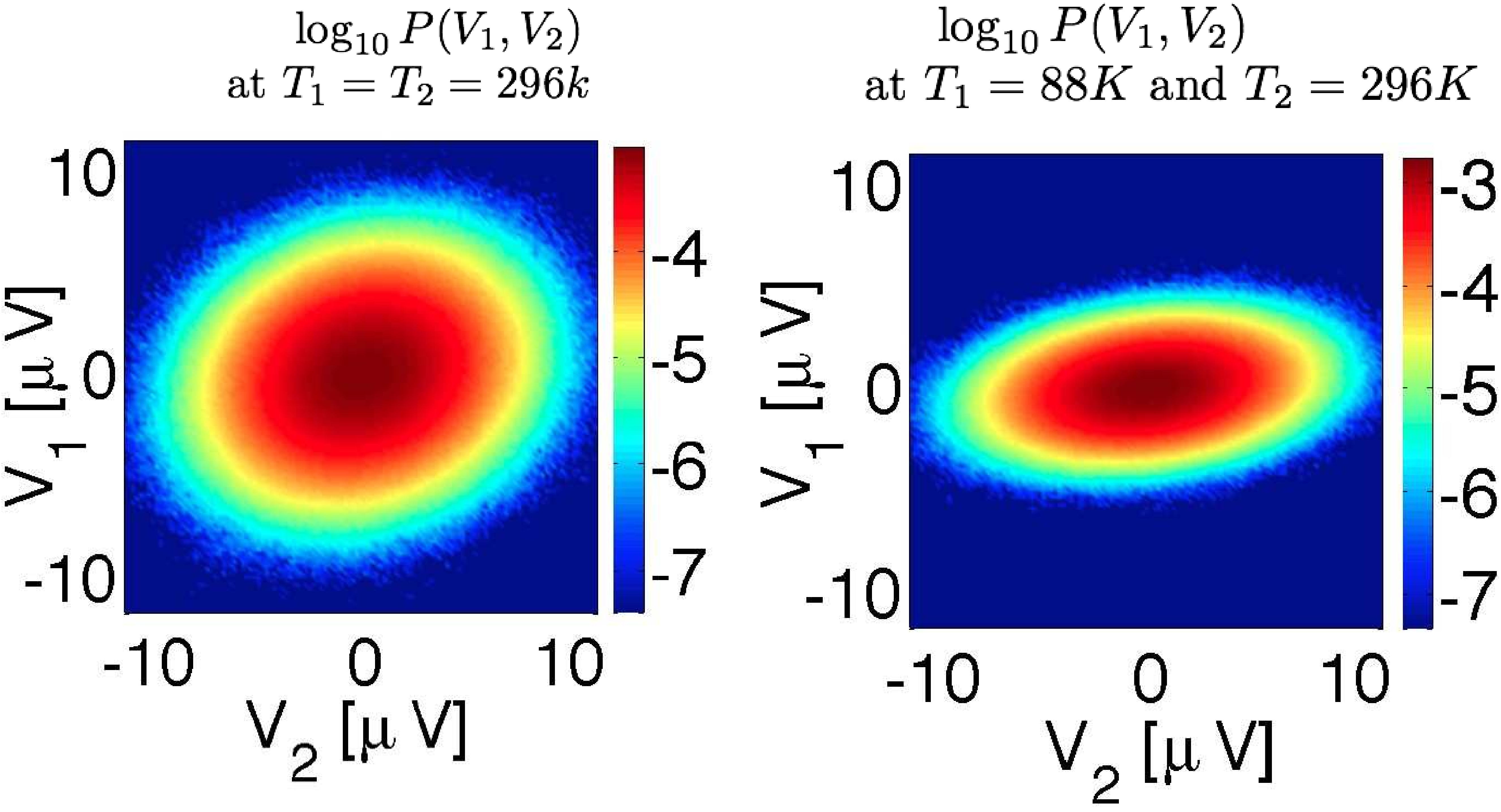
i_m its velocity

$K_m = 1/C_m$ the stiffness of the spring m

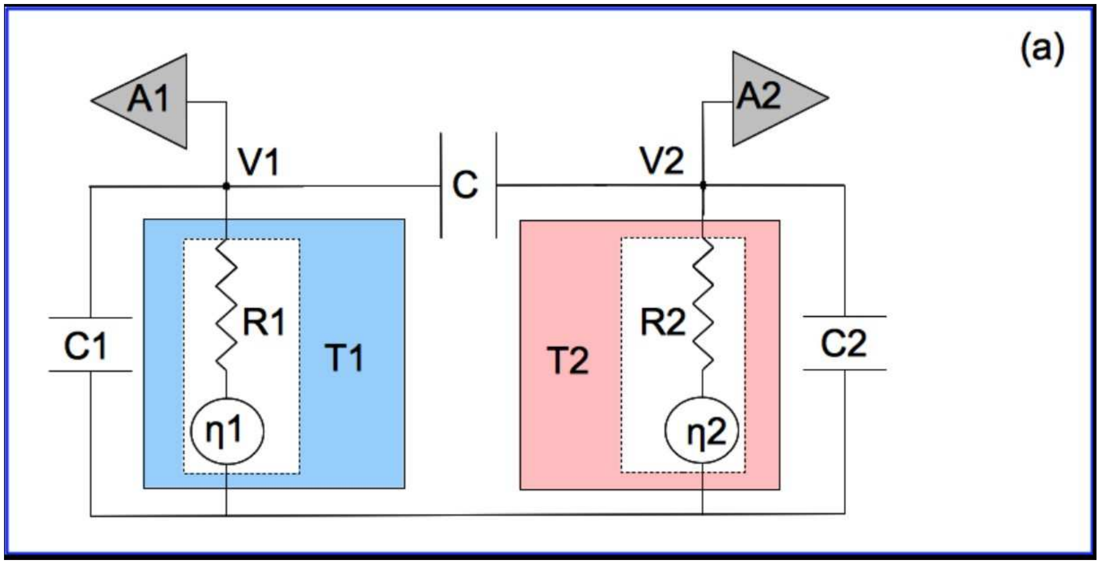
$K = 1/C$ the stiffness of the coupling spring

R_m the viscosity.

Joint probability of V_1 and V_2



Electric Circuit and the dissipated energy



Power dissipated in the resistance m=1,2

$$\dot{Q}_m = V_m i_m = V_m[(C_m + C)\dot{V}_m - C\dot{V}_{m'}]$$

$m' = 2$ if $m = 1$, and $m' = 1$ if $m = 2$

Integrating on a time τ

$$Q_{m,\tau} = W_{m,\tau} - \Delta U_{m,\tau}$$

first principle for fluctuations

$$Q_{m,\tau} = \int_t^{t+\tau} i_m V_m \, dt$$

heat flowed in the time τ from reservoir m' to reservoir m

$$W_{m,\tau} = \int_t^{t+\tau} C V_m \frac{dV_{m'}}{dt} dt$$

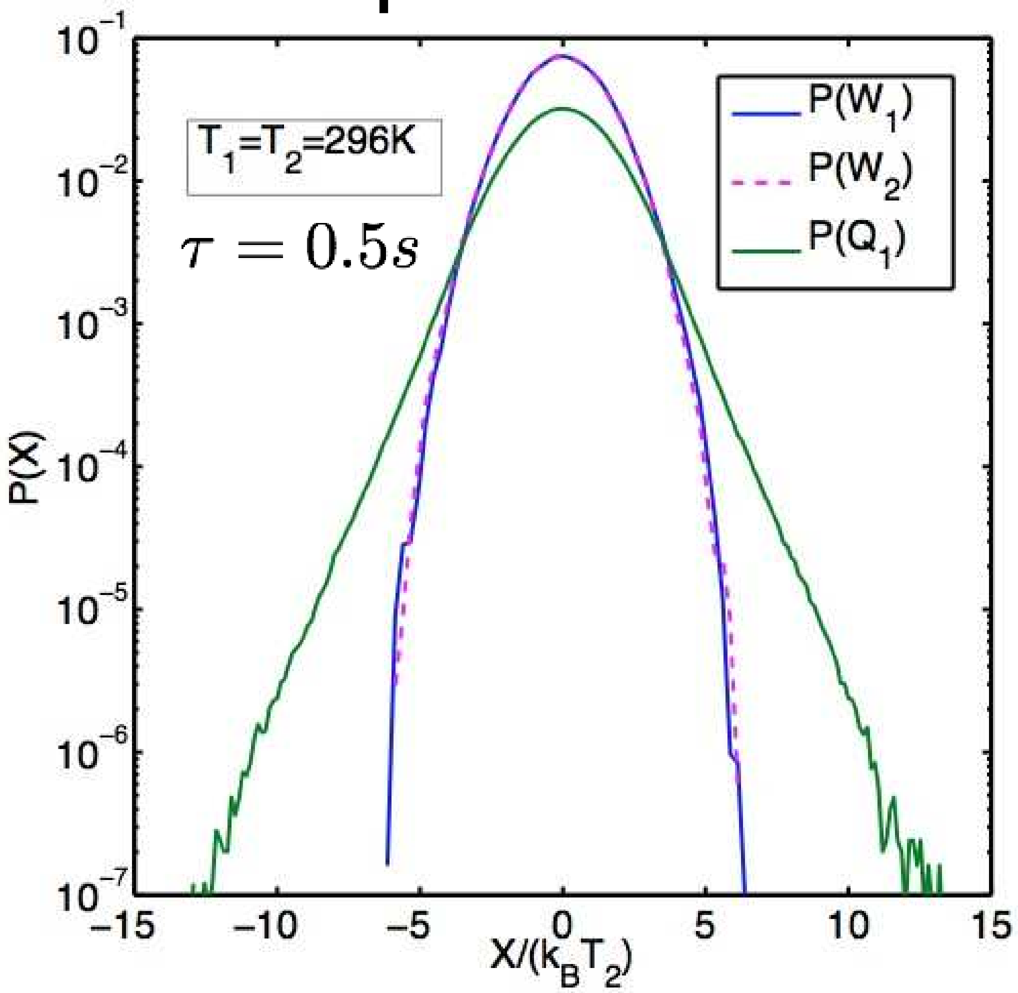
work performed by the circuit m on m' in the time τ

$$\Delta U_{m,\tau} = \frac{(C_m + C)}{2}(V_m(t+\tau)^2 - V_m(t)^2)$$

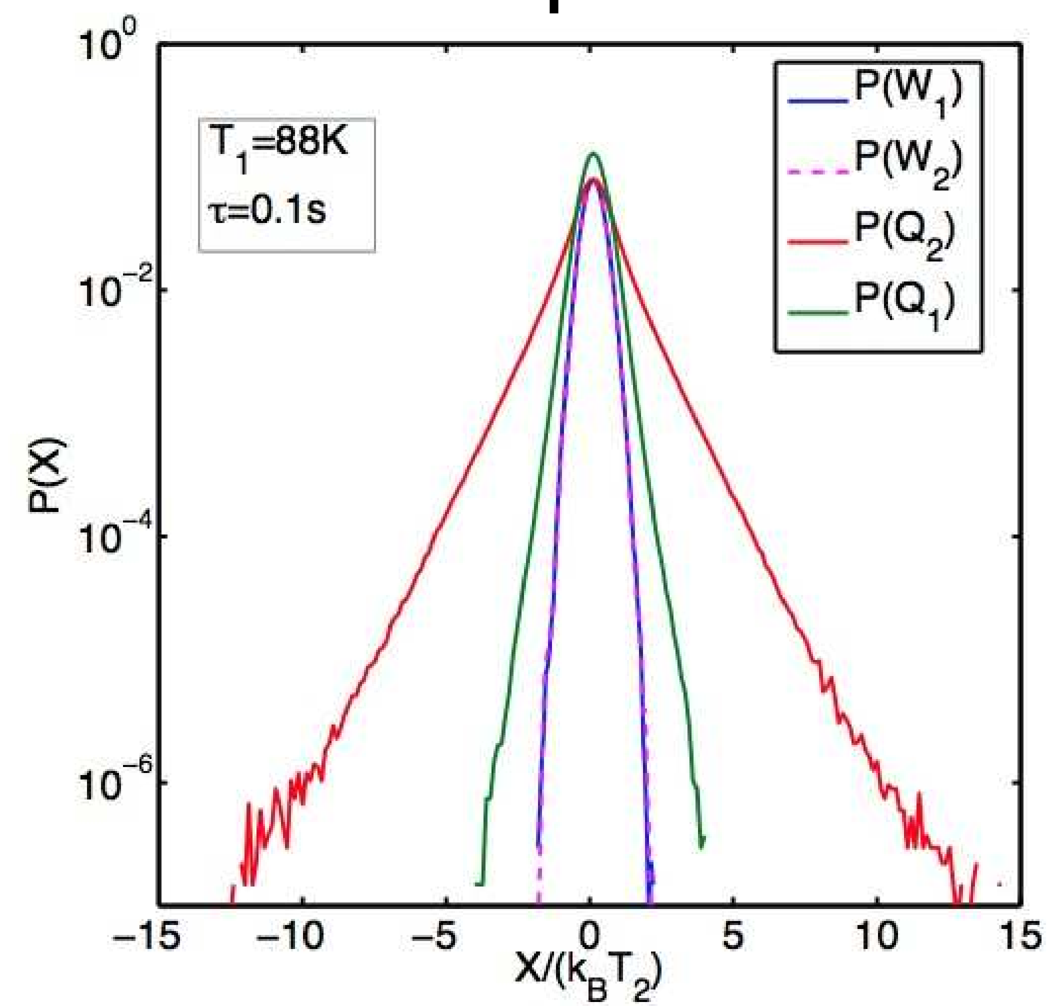
Potential energy change of the circuit m in the time τ.

Statistic of the work and heat

equilibrium



out-equilibrium

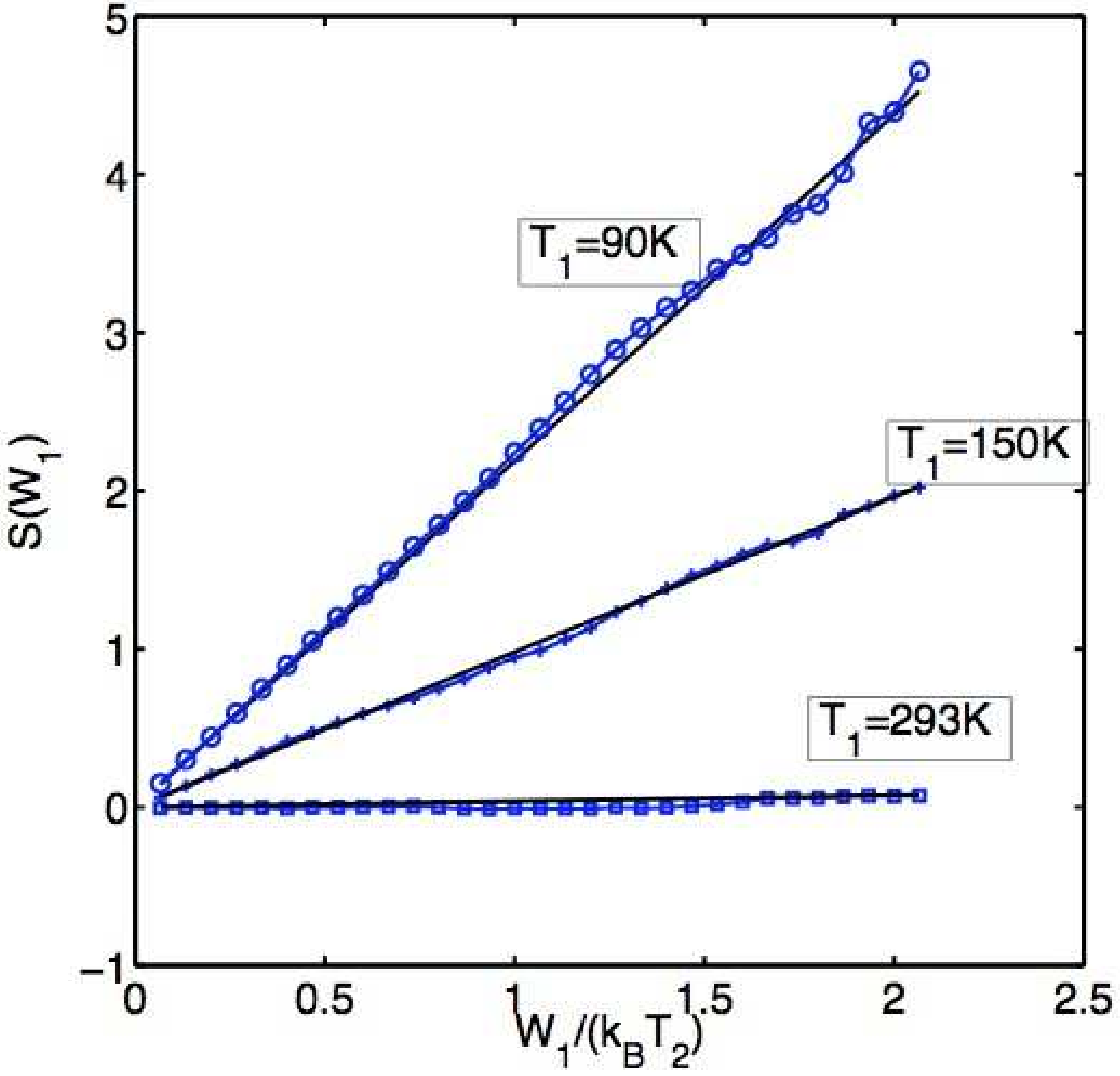


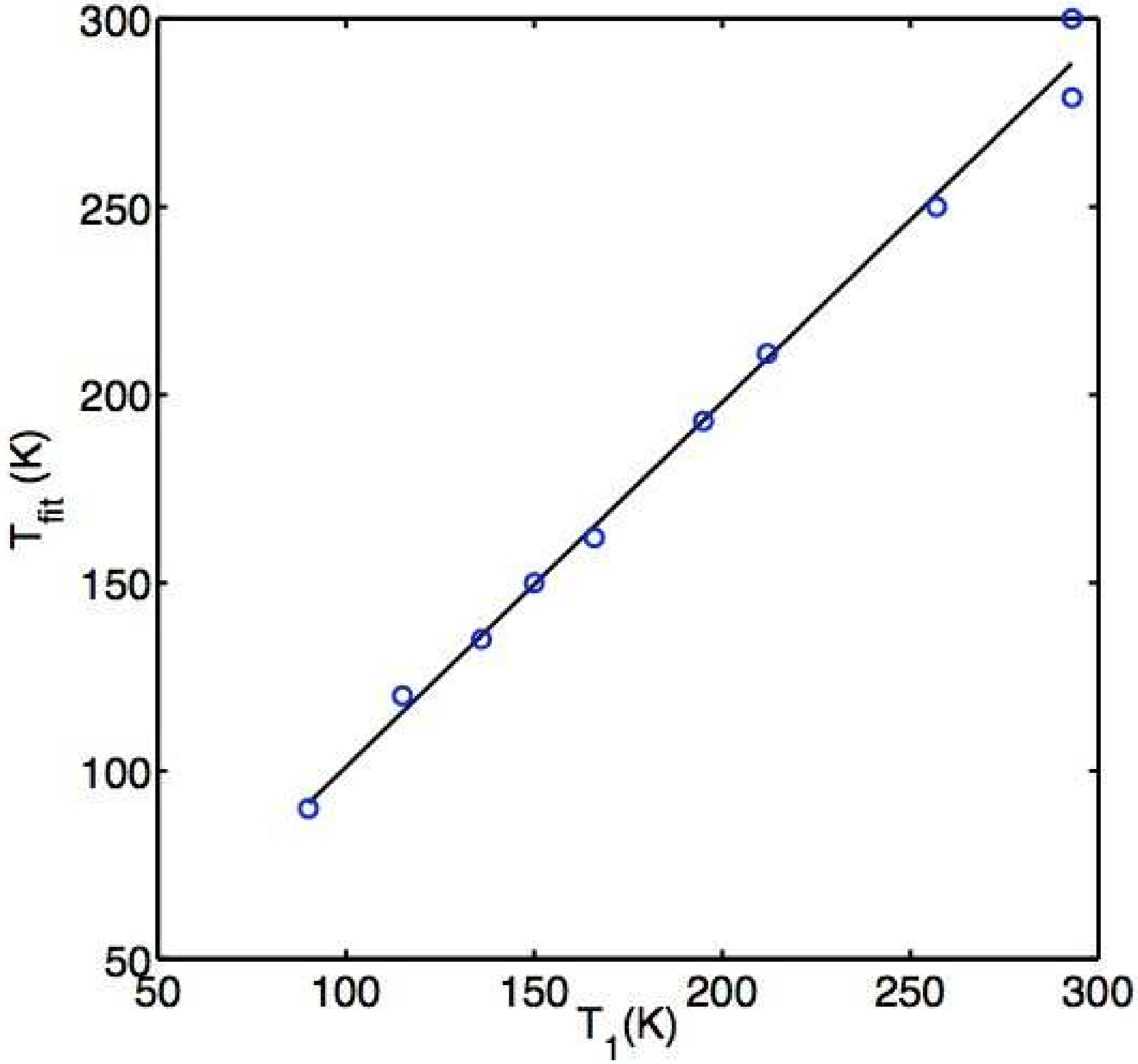
FT for W_τ et Q_τ
for $\tau \rightarrow \infty$

$$S(X_{m,\tau}) = \log \frac{P(X_{m,\tau})}{P(-X_{m,\tau})} = \Delta\beta \frac{X_{m,\tau}}{k_B T_2}$$

with $\Delta\beta = (T_2/T_1 - 1)$

On the heat flux and entropy produced by thermal fluctuations



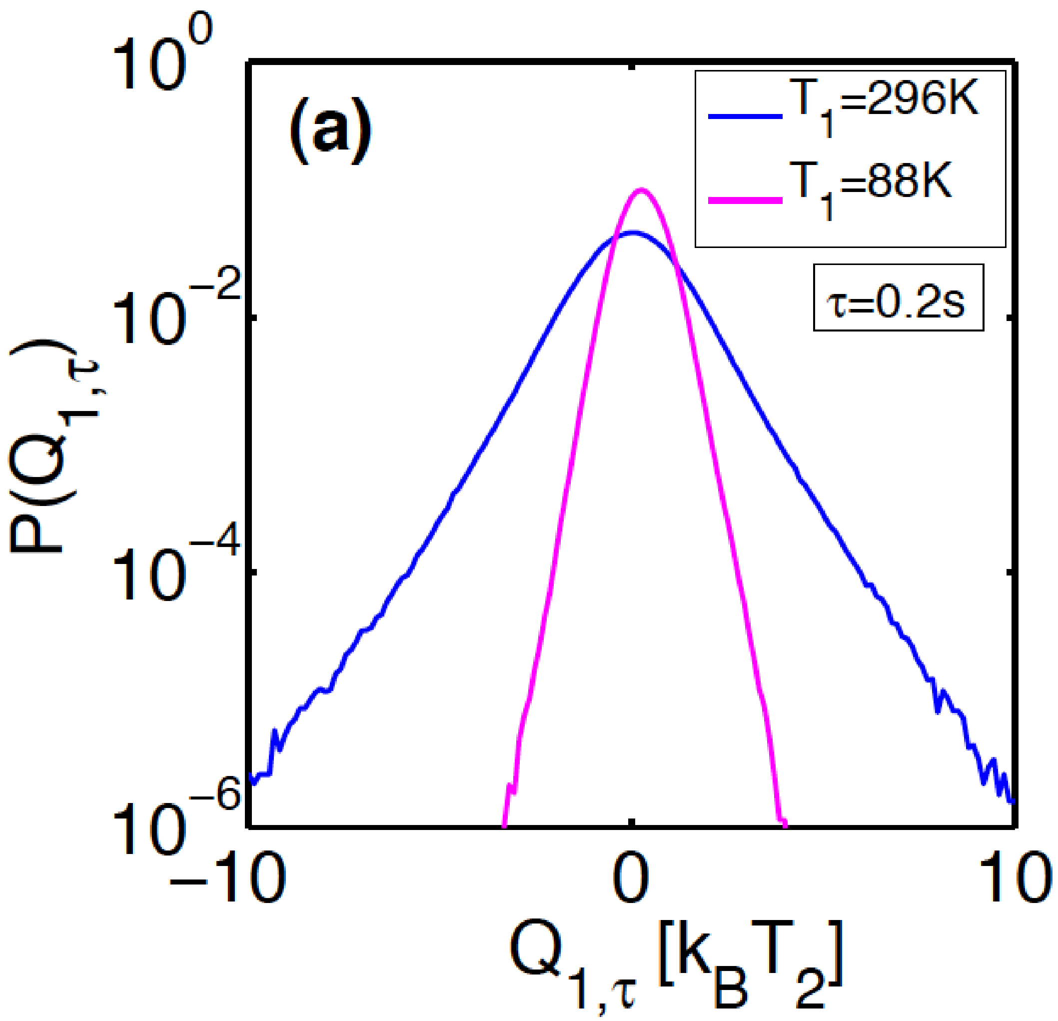


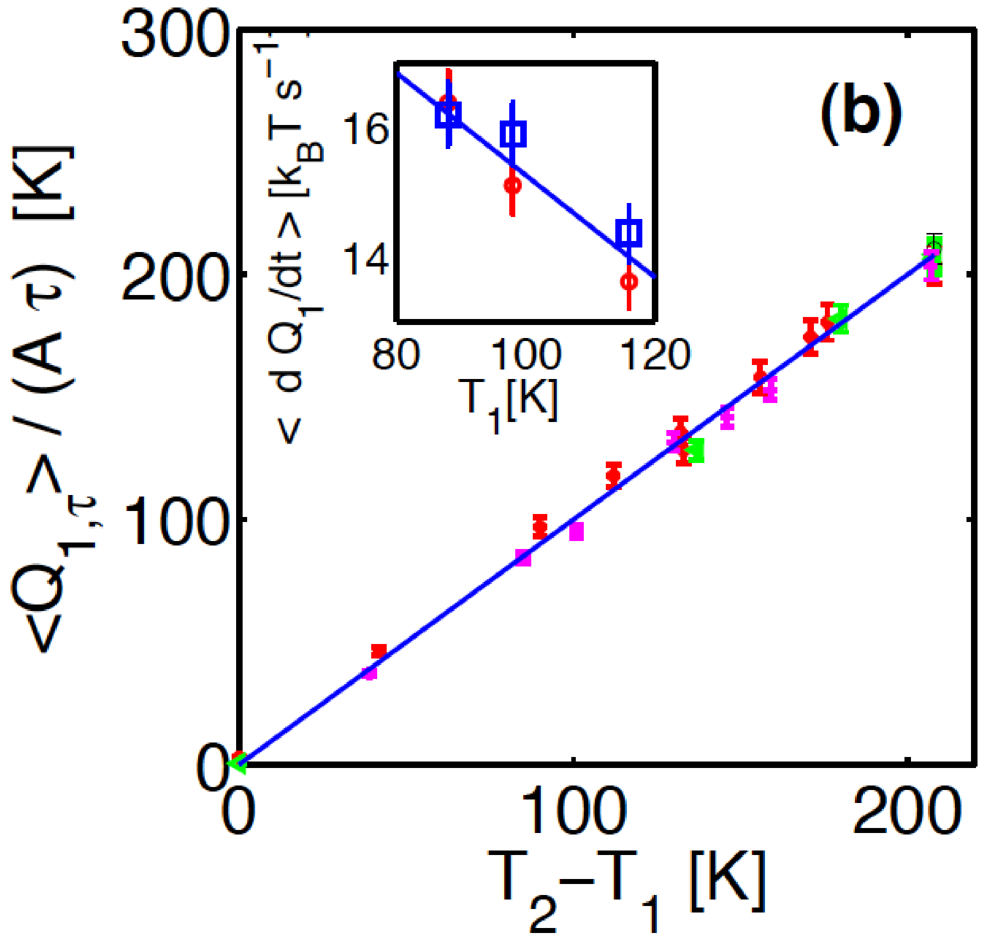
$$S(X_{m,\tau}) = \log \frac{P(X_{m,\tau})}{P(-X_{m,\tau})} = \Delta\beta \frac{X_{m,\tau}}{k_B T_2}$$

with $\Delta\beta = (T_2/T_1 - 1)$

$$T_{fit} = T_2/(\Delta\beta + 1)$$

The heat flux as a function of T_2-T_1





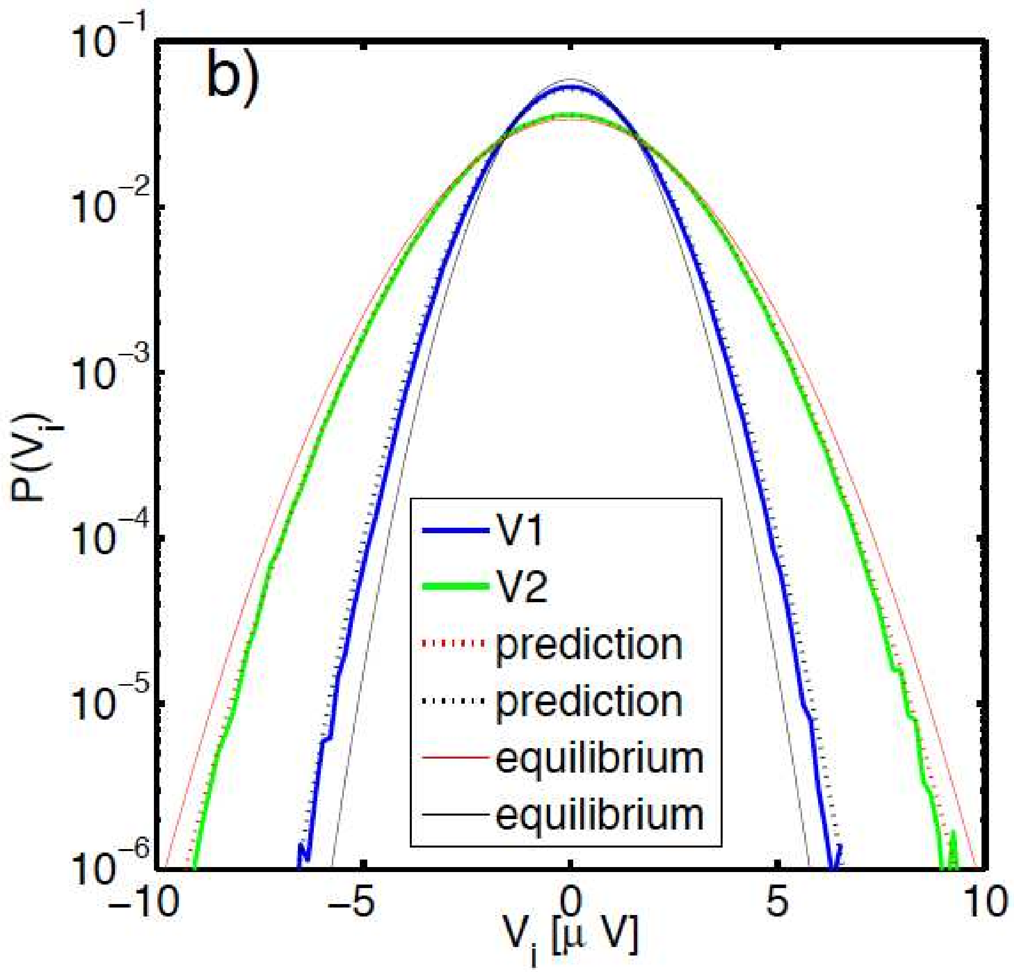
$$\left\langle \dot{Q}_1 \right\rangle = A\,(T_2 - T_1) = \frac{C^2 \Delta T}{XY}$$

$$X = C_2\,C_1 + C\,(C_1 + C_2)$$

$$Y = [(C_1 + C)R_1 + (C_2 + C)R_2] \text{ and } A = C^2/(X\,Y)$$

How the equilbrium variance of V_1 and V_2 is modified

σ_m^2 is
the variance of V_m



$$\sigma_m^2(T_m, T_{m'}) = \sigma_{m,eq}^2(T_m) + < \dot{Q}_m > R_m$$

$$\sigma_{m,\mathrm{eq}}^2(T_m) = k_B T_m (C + C'_m)/X$$

which is an extension to two temperatures of the Harada-Sasa relation

On the entropy produced by thermal fluctuations

$$\Delta S_{r,\tau} = Q_{1,\tau}/T_1 + Q_{2,\tau}/T_2$$

related to the heat exchanged between the reservoirs

Following Seifert, (PRL 95, 040602, 2005)
who developed this concept for a single heat bath,
we introduce a trajectory entropy for the evolving system

$$S_s(t) = -k_B \log P(V_1(t), V_2(t))$$

and the entropy production on the time τ

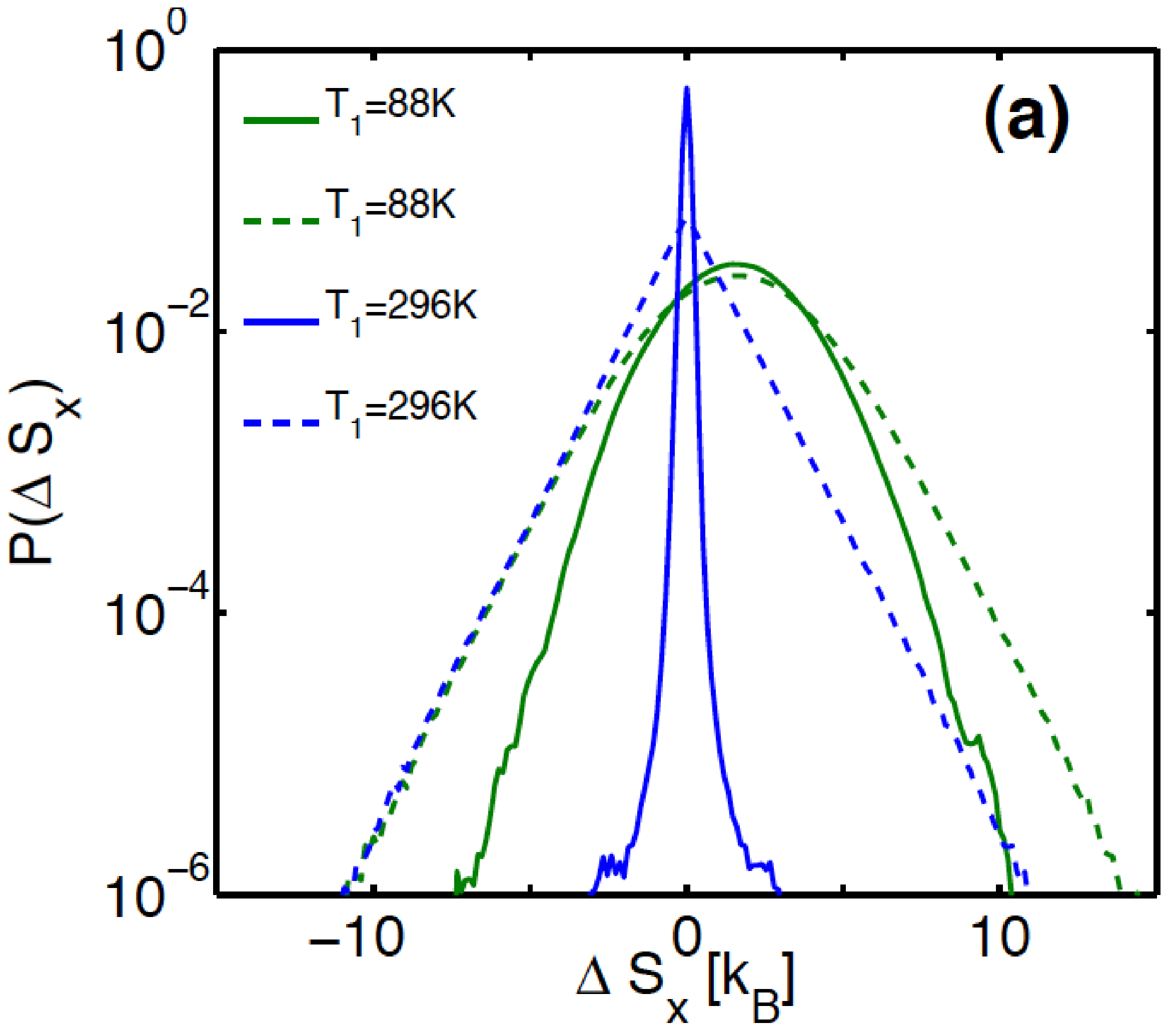
$$\Delta S_{s,\tau} = -k_B \log \left[\frac{P(V_1(t+\tau), V_2(t+\tau))}{P(V_1(t), V_2(t))} \right].$$

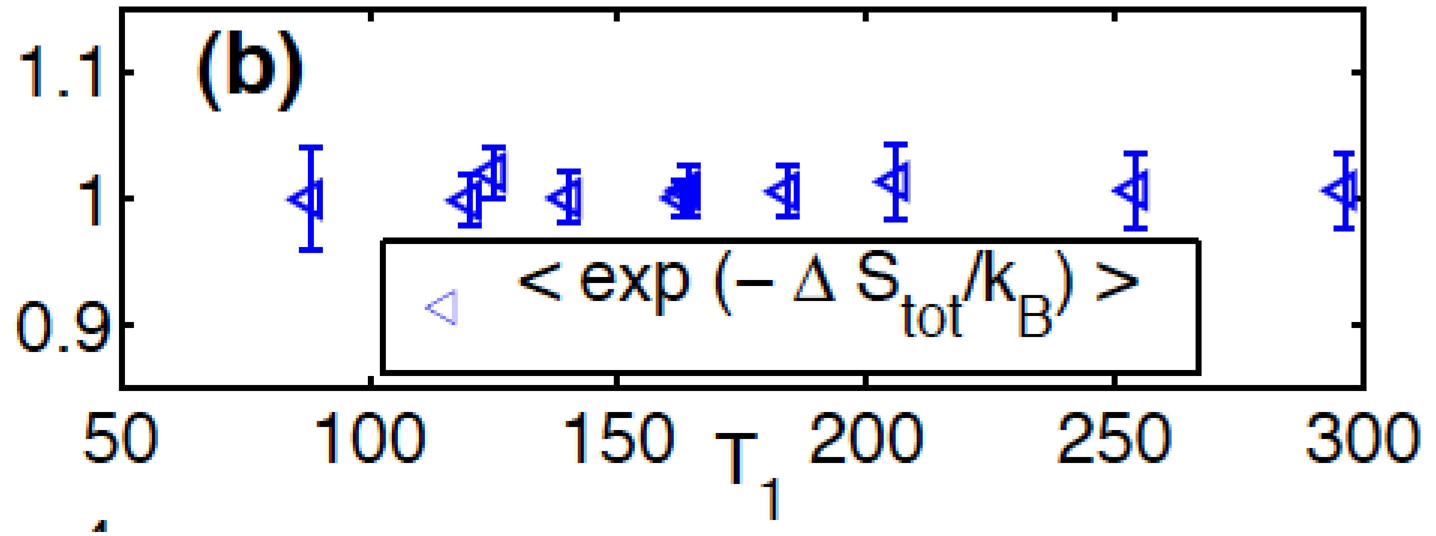
The total entropy is :

$$\Delta S_{tot,\tau} = \Delta S_{r,\tau} + \Delta S_{s,\tau}$$

Statistical properties of the total entropy

$$\Delta S_{tot,\tau} = \Delta S_{r,\tau} + \Delta S_{s,\tau}$$





——— $\Delta S_{tot,\tau}$

.... $\Delta S_{r,\tau}$

independently of ΔT and of τ, the following equality always holds

$$\langle \exp(-\Delta S_{tot}/k_B) \rangle = 1$$

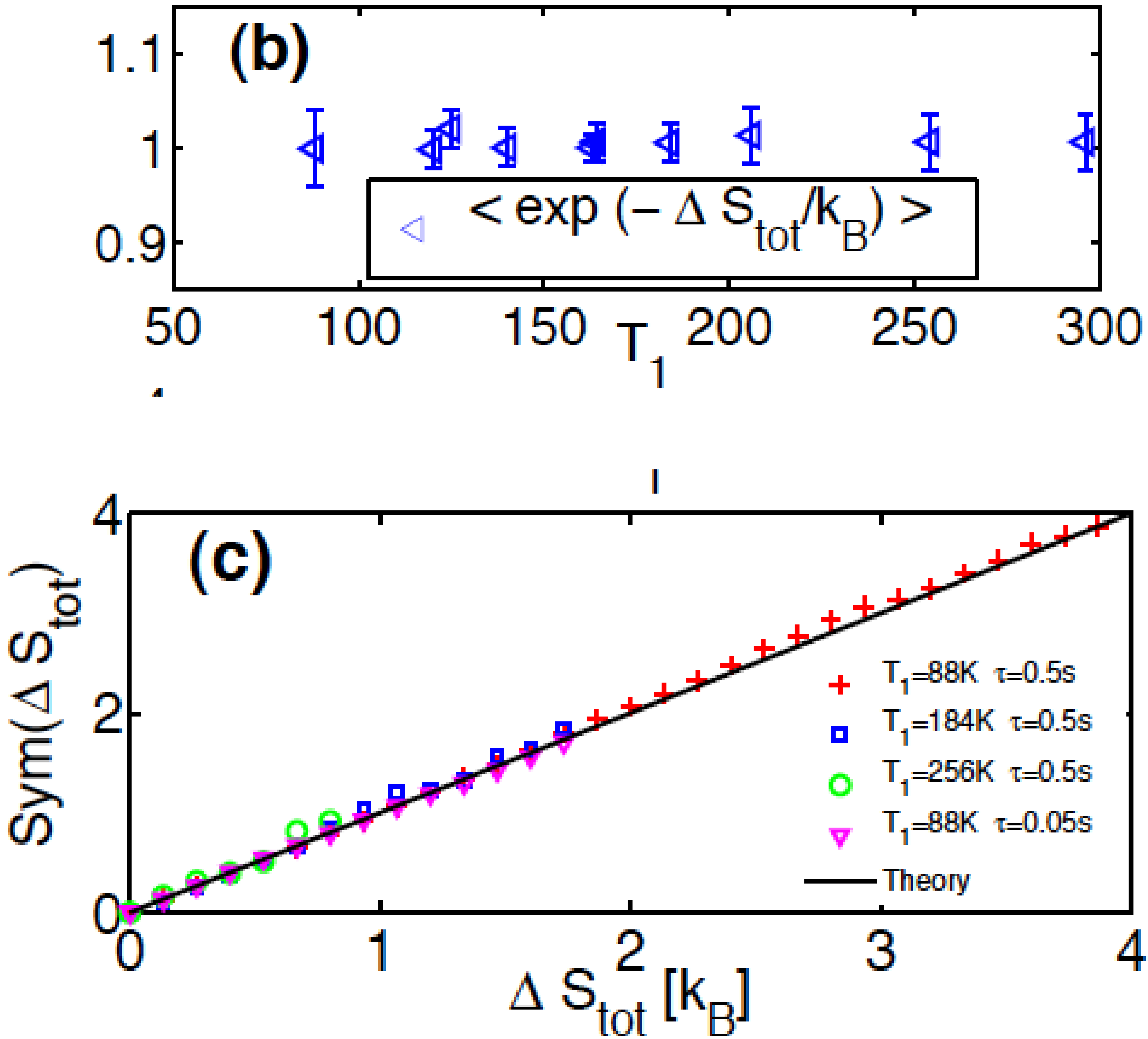
Statistical properties of the total entropy

$$\langle \exp(-\Delta S_{tot}/k_B) \rangle = 1$$

implies that $P(\Delta S_{tot})$ satisfies a FT

$$\log\left[\frac{P(\Delta S_{tot})}{P(-\Delta S_{tot})}\right] = \frac{\Delta S_{tot}}{k_B}$$

$$\forall \tau, \Delta T$$



S.Ciliberto, A. Imparato, A. Naert
PRL,110,180601(2013)
JSTAT,P12014 (2013)

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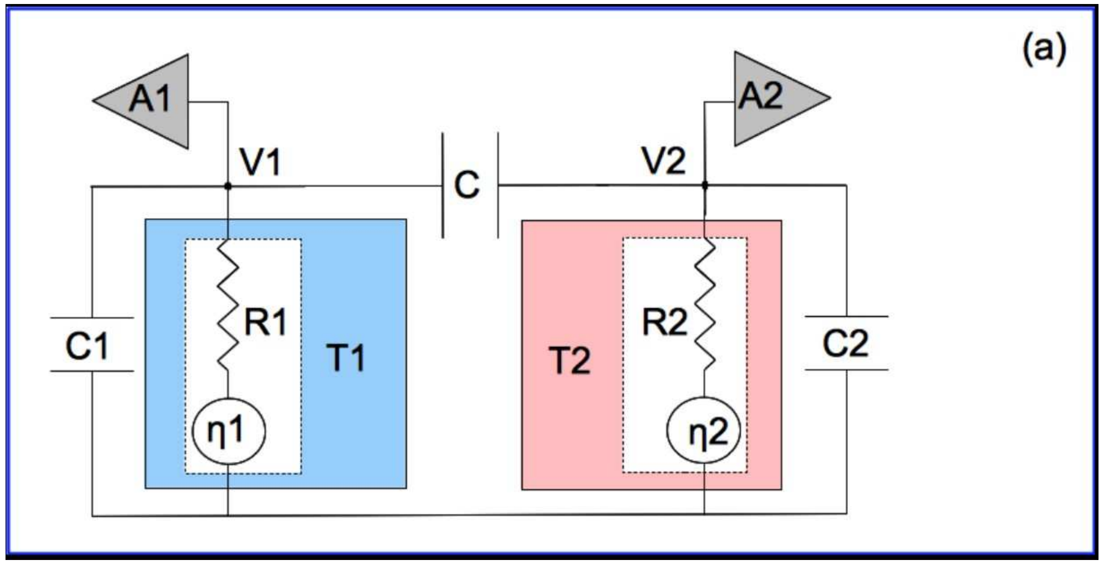
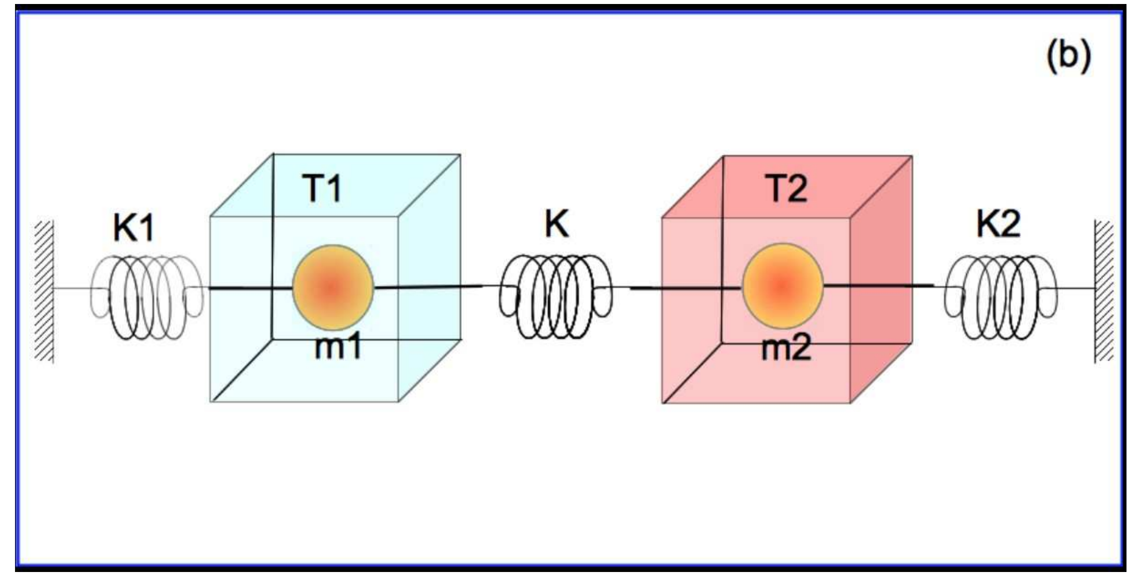
Summary of the experimental and theoretical results ''On the heat flux and entropy produced by thermal fluctuations''

- The mean heat flux $< \dot{Q} > \propto (T_2 - T_1)$
- The pdf of $W_m / < W_m >$ satisfies an asymptotic FT whose prefactor is the entropy production rate $< W_m > (1/T_m - 1/T_{m'})$.
- The out of equilibrium variance :
  $\sigma_m^2(T_m, T_{m'}) = \sigma_{m,eq}^2(T_m) + < \dot{Q}_m > R_m$
  (Extension of Harada-Sasa relation)
- The total entropy ΔS_{tot} satisfies a conservation law which implies the second law and imposes the existence of a FT which is not asymptotic in time.
- ΔS_{tot} is rigorously zero in equilibrium, both in average and fluctuations
- The electrical-mechanical analogy makes these results very general and useful

Electric Circuit and the mechanical equivalent





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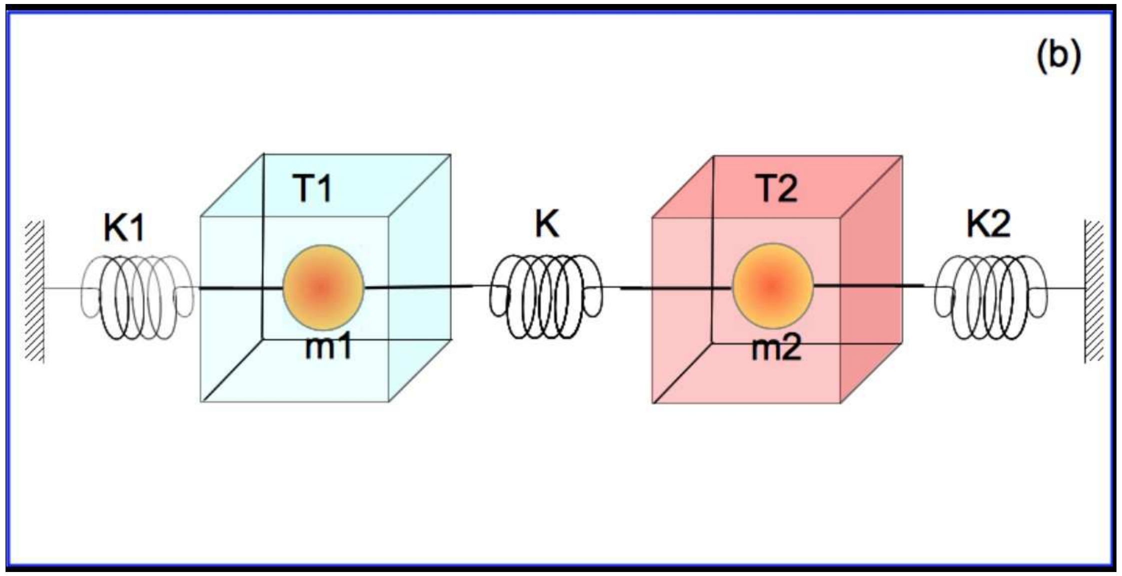
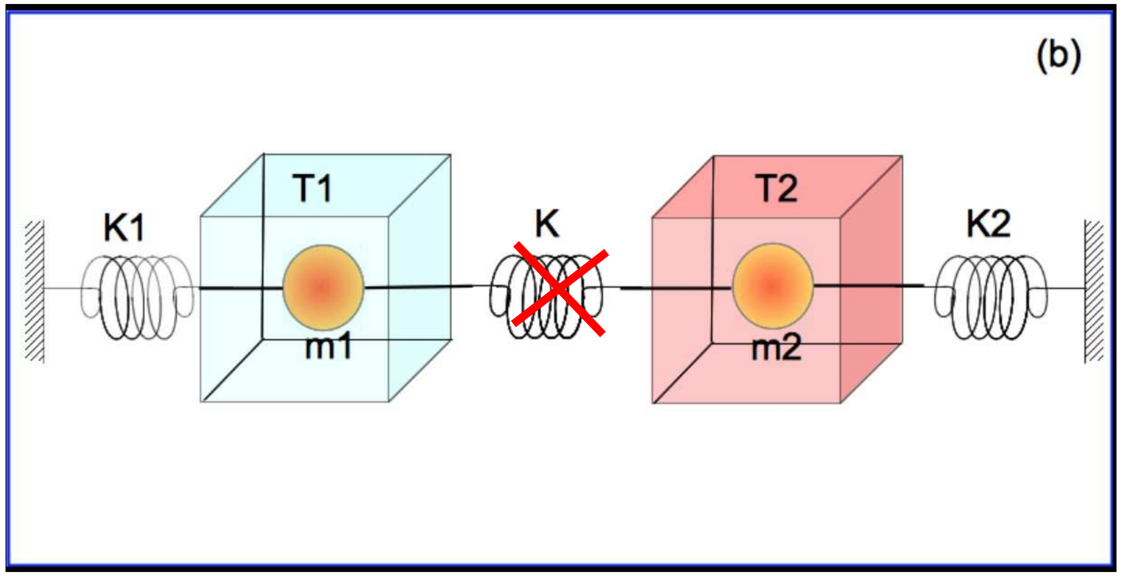
R_m the viscosity.



Two Brownian particles trapped
by two laser beams.



Two Brownian particles trapped
by two laser beams.

Difficulty of having a harmonic coupling
between the particles.

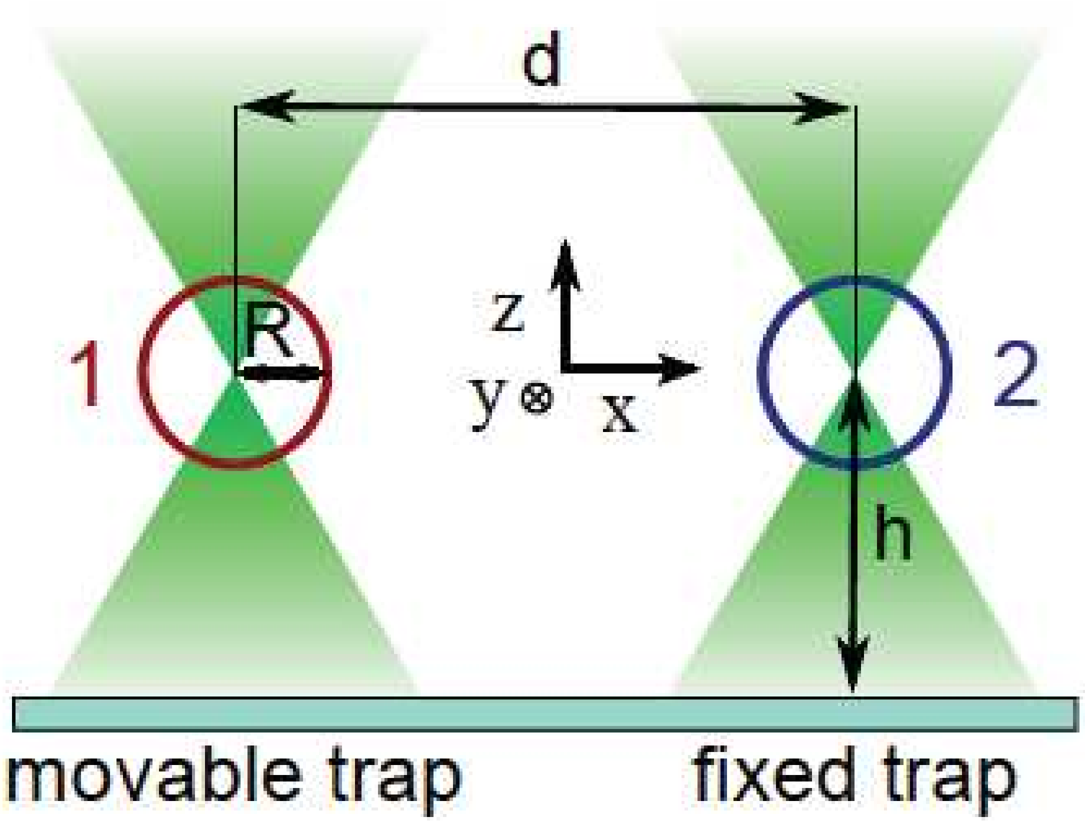
The main source of coupling is hydrodynamic (viscous)

Difficulty of having two close Brownian particles
at two different temperatures

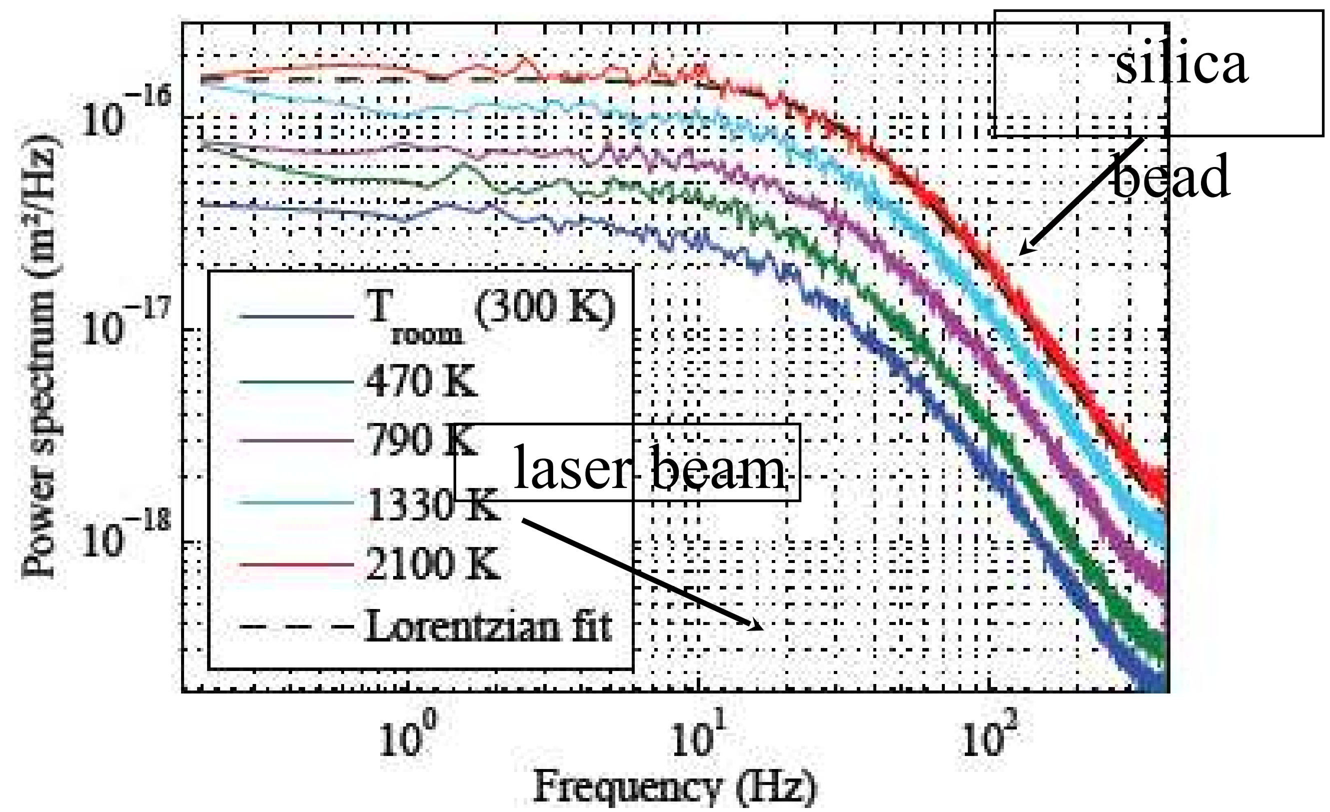
The temperature gradient is done by forcing the motion
of one particle with an external random force

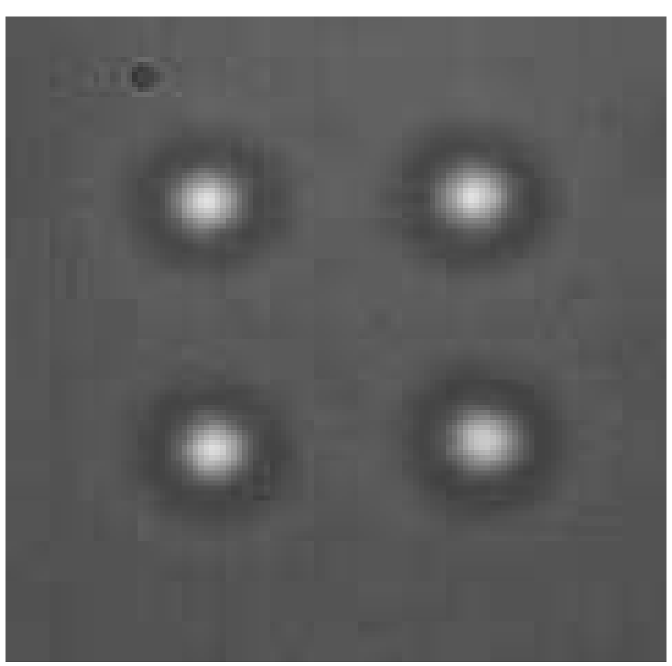
Experimental results

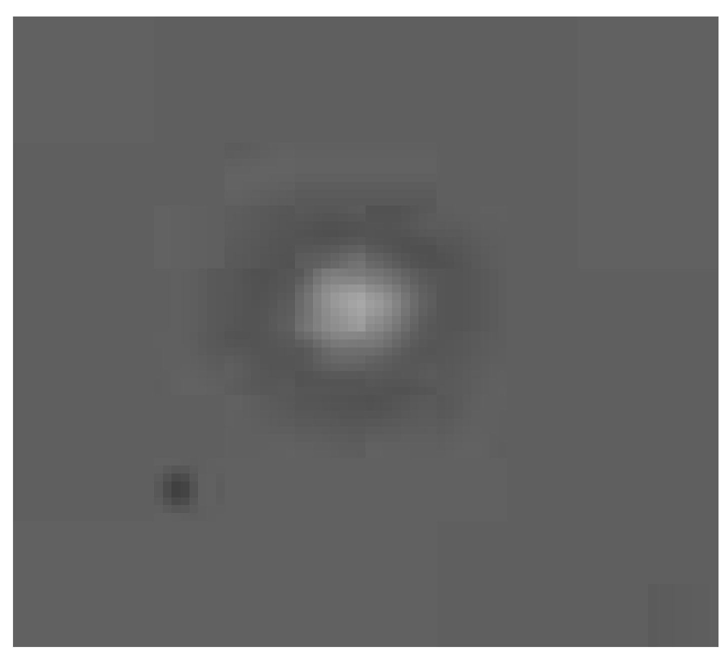
Set up

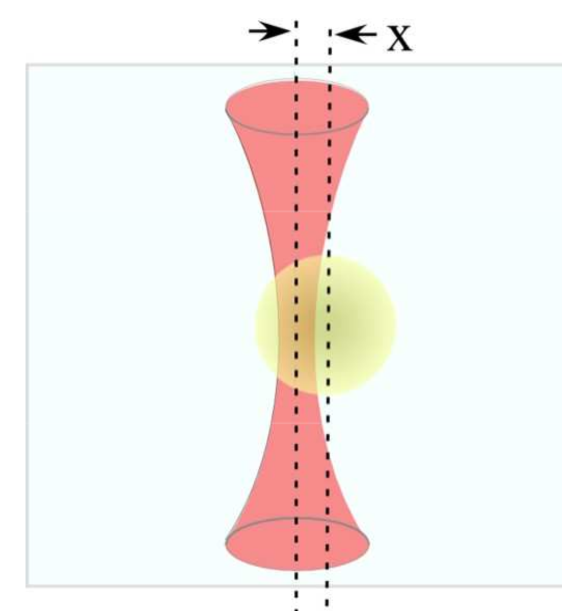


Spectra of excited particle









$$U(x) = \frac{k}{2}\, x^2$$

From the hydrodynamic model one can compute the variances

$$\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_B(T+\Delta T)}{k_1} - \frac{k_2}{k_1}\frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$
$$\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_B \Delta T}{k_1+k_2}$$
$$\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_B T}{k_2} + \frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$

where :

ϵ, T and ΔT are the unknown

ϵ is the coupling coefficient of the particle.
It has to depend on the distance but not
on the random driving amplitude

ΔT is the temperature difference induced by the random driving.

k_1 and k_2 are the stiffness of the optical traps.

The standard hydrodynamic model

It follows that the system of equations is:

$$\begin{cases} \gamma \dot{x}_1 = -k_1 x_1 + \epsilon(-k_2 x_2 + f_2) + f_1 + f^* \\ \gamma \dot{x}_2 = -k_2 x_2 + \epsilon(-k_1 x_1 + f_1 + f^*) + f_2 \end{cases}$$

f_1 and f_2 are the random noise of the heat bath

Out of Equilbrium : forcing on bead 1 $f^* = k_1 x_0(t)$

f^* is a delta correlated noise

Bead 1 has an effective temperature $T^* = T + \Delta T$

The standard hydrodynamic model

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comparison with the electric case

$$\begin{aligned} R_1 \dot{q}_1 &= -q_1 \frac{C_2}{X} + (q_2 - q_1)\frac{C}{X} + \eta_1 \\ R_2 \dot{q}_2 &= -q_2 \frac{C_1}{X} + (q_1 - q_2)\frac{C}{X} + \eta_2 \end{aligned}$$

f_1 and f_2 are the random noise of the heat bath

Out of Equilibrium : forcing on bead 1 $f^* = k_1 x_0(t)$

f^* is a delta correlated noise

Bead 1 has an effective temperature $T^* = T + \Delta T$

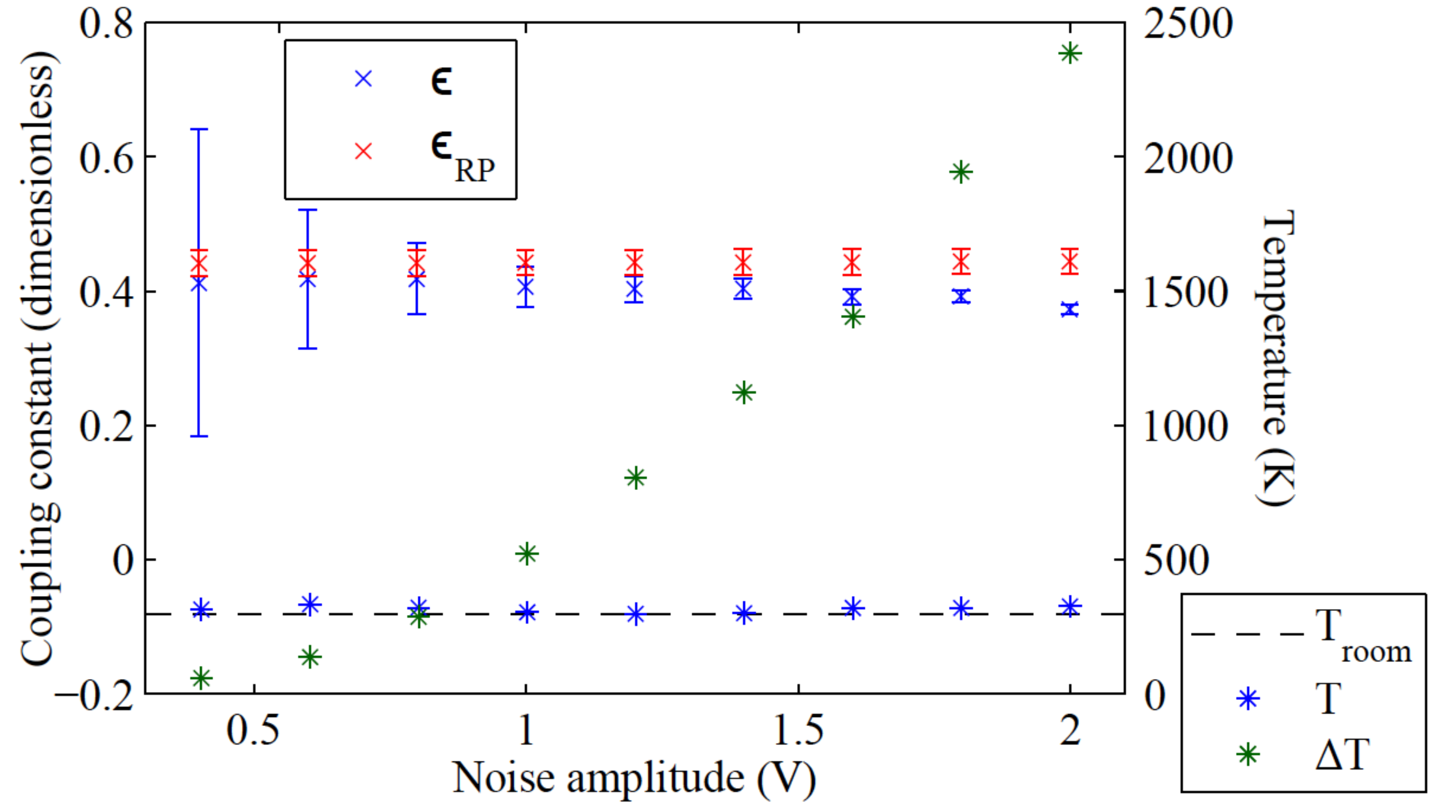
Values of the parameters from the experiment

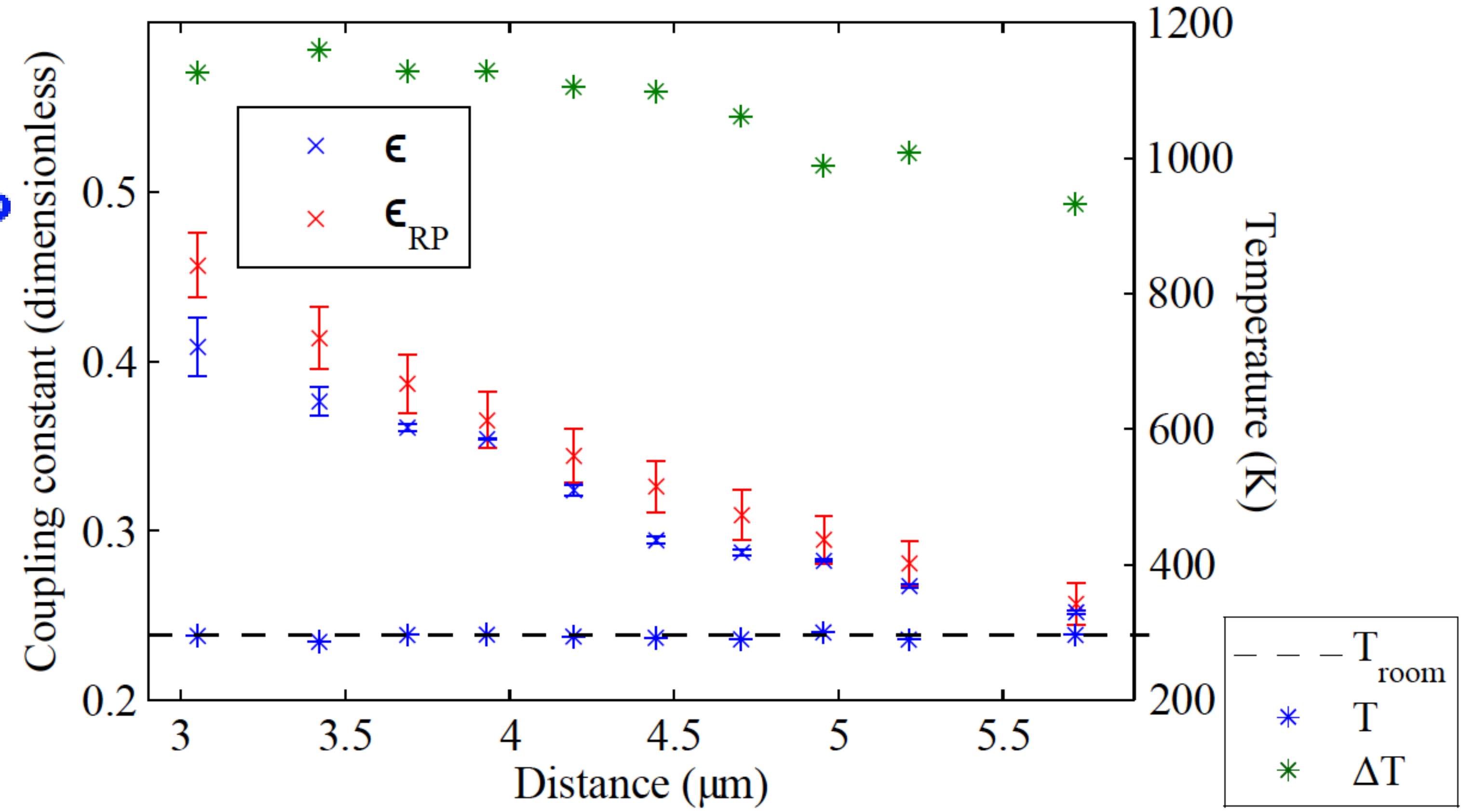
$$\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_\mathrm{B}(T+\Delta T)}{k_1} - \frac{k_2}{k_1}\frac{\epsilon^2 k_\mathrm{B}\Delta T}{k_1+k_2}$$

$$\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_\mathrm{B}\Delta T}{k_1+k_2}$$

$$\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_\mathrm{B} T}{k_2} + \frac{\epsilon^2 k_\mathrm{B}\Delta T}{k_1+k_2}$$

Can we interpret the term proportional to ΔT as the heat flux between the two particles ?





The standard hydrodynamic model

It follows that the system of equations is:

$$\begin{cases} \gamma \dot{x}_1 = -k_1 x_1 + \epsilon(-k_2 x_2 + f_2) + f_1 + f^* \\ \gamma \dot{x}_2 = -k_2 x_2 + \epsilon(-k_1 x_1 + f_1 + f^*) + f_2 \end{cases}$$

heat exchanged by the bead i in the time τ

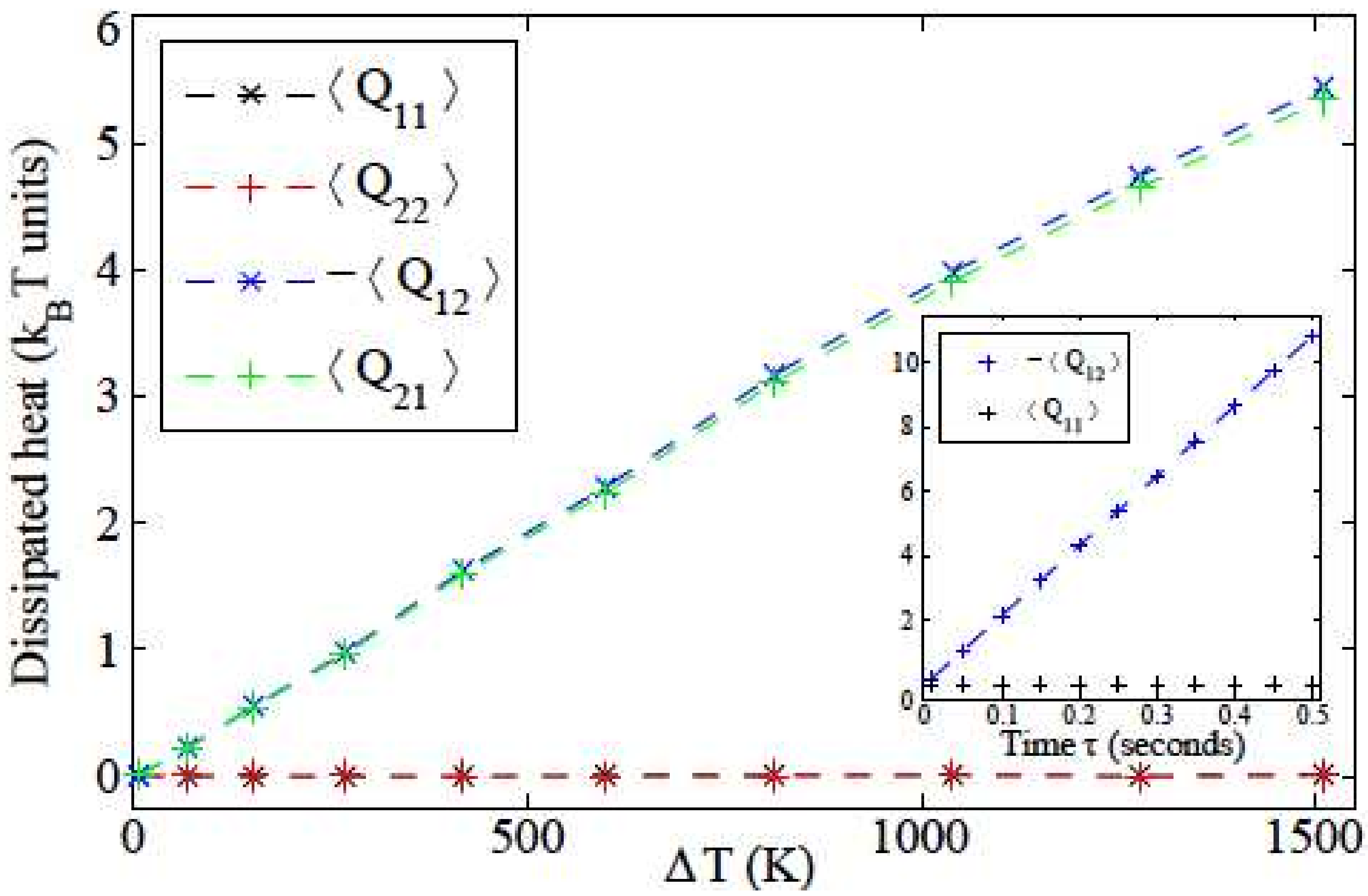
$$Q_i(\tau) = \int_0^{\tau} (\gamma \dot{x}_i - \gamma \xi_i)\, \dot{x}_i \, \mathrm{d}t$$

$$\begin{aligned} \xi_1 &= \frac{1}{\gamma}(f_1 + \epsilon f_2 + f^*) \\ \xi_2 &= \frac{1}{\gamma}(f_2 + \epsilon f_1 + \epsilon f^*) \end{aligned}$$

$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

$$\begin{aligned} q_{ii} &= -\int_0^{\tau} x_i \dot{x}_i \, \mathrm{d}t \\ q_{ij} &= -\int_0^{\tau} x_j \dot{x}_i \, \mathrm{d}t \end{aligned}$$

The heat flux



$$< Q_{i,j} >= \epsilon k_j < q_{i,j} >$$

As for the electric case one obtains that

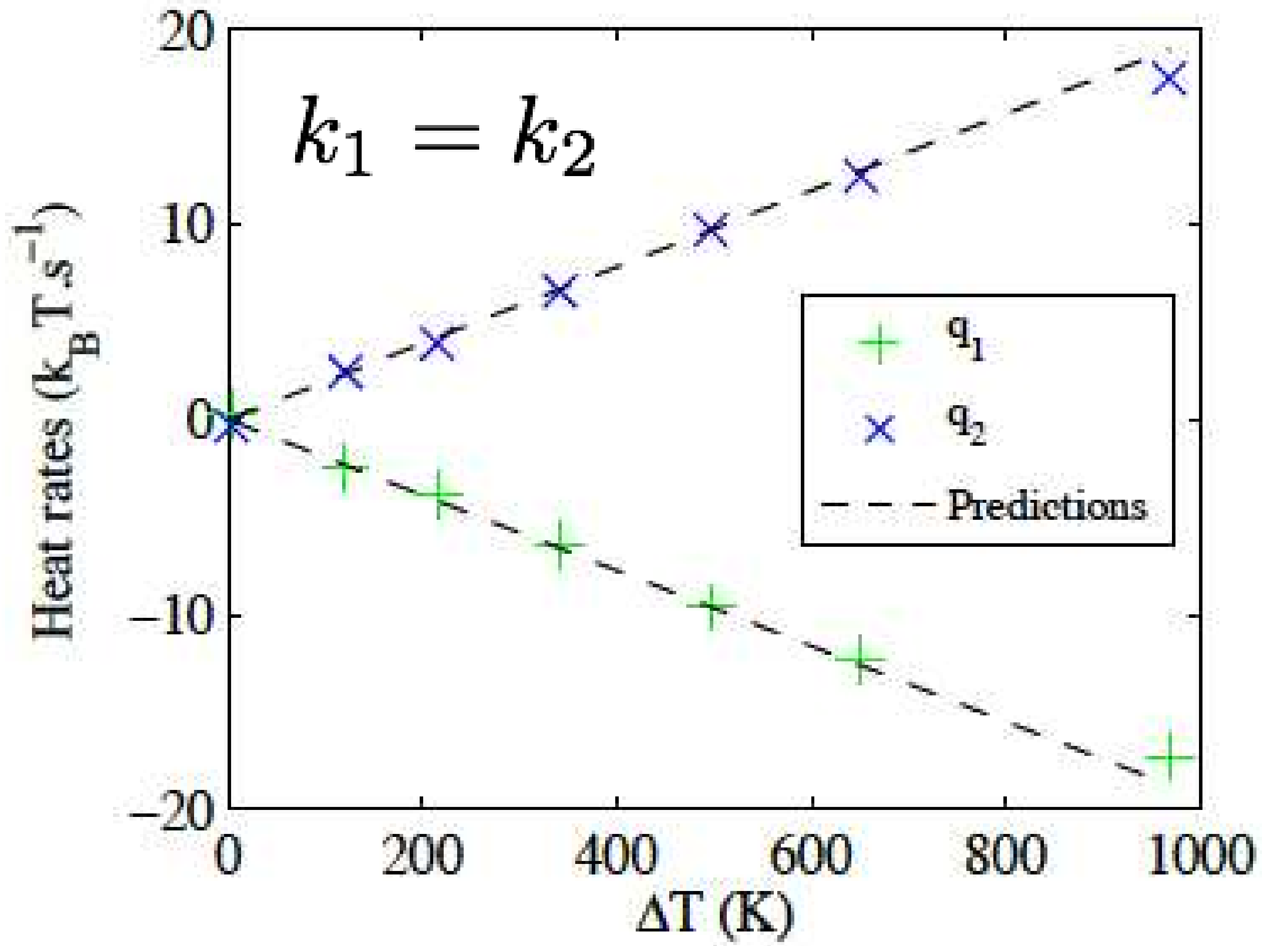
$$\sigma_i^2 - \sigma_{i,equilibrium}^2 \propto < Q_i >$$

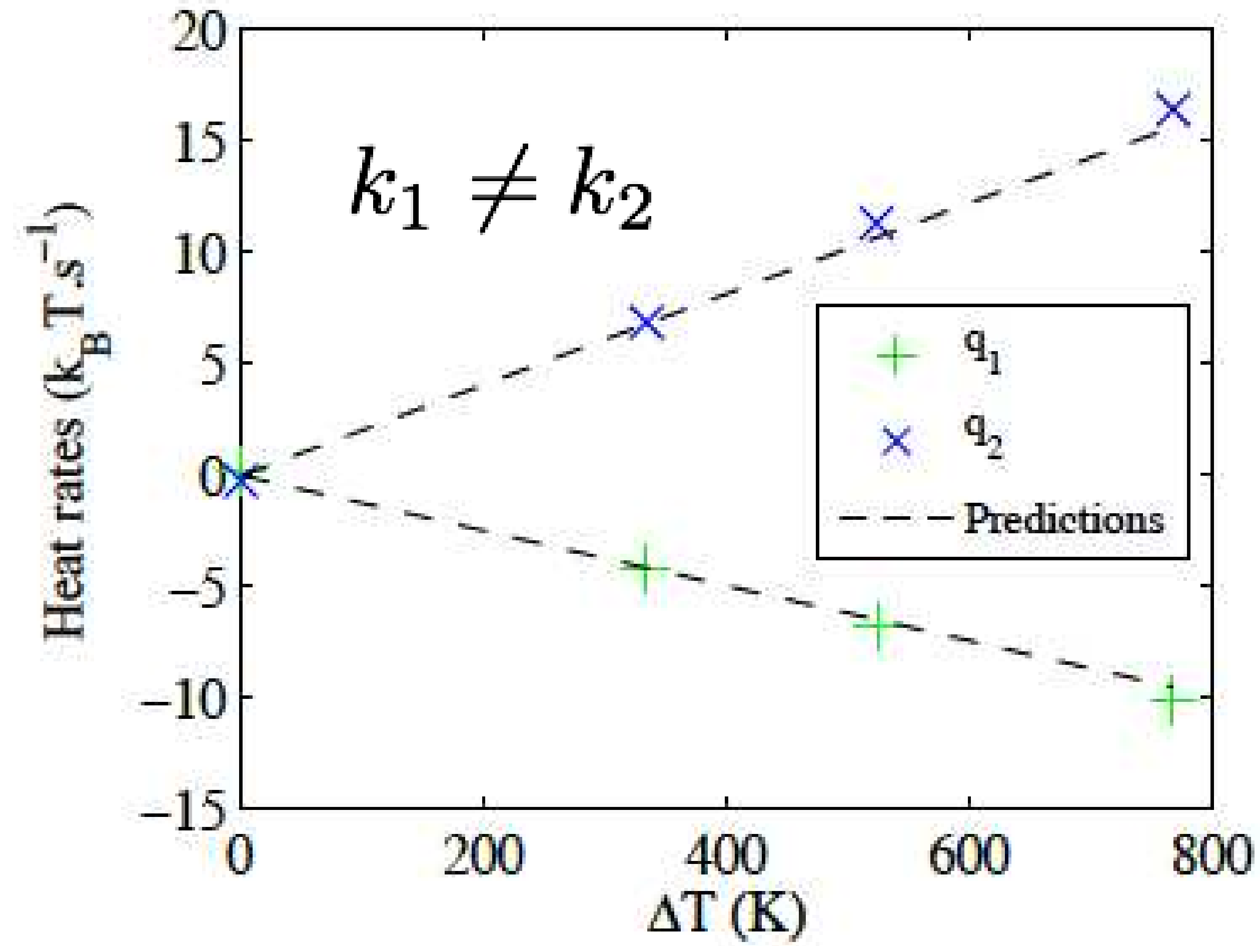
Extension of the Harada-Sasa relation

$$\text{but } < Q_{2,1} >= -\frac{k_1}{k_2} < Q_{1,2} > \quad \text{and}$$

$$< Q_{2,1} > + < Q_{1,2} > \neq 0$$

The heat flux



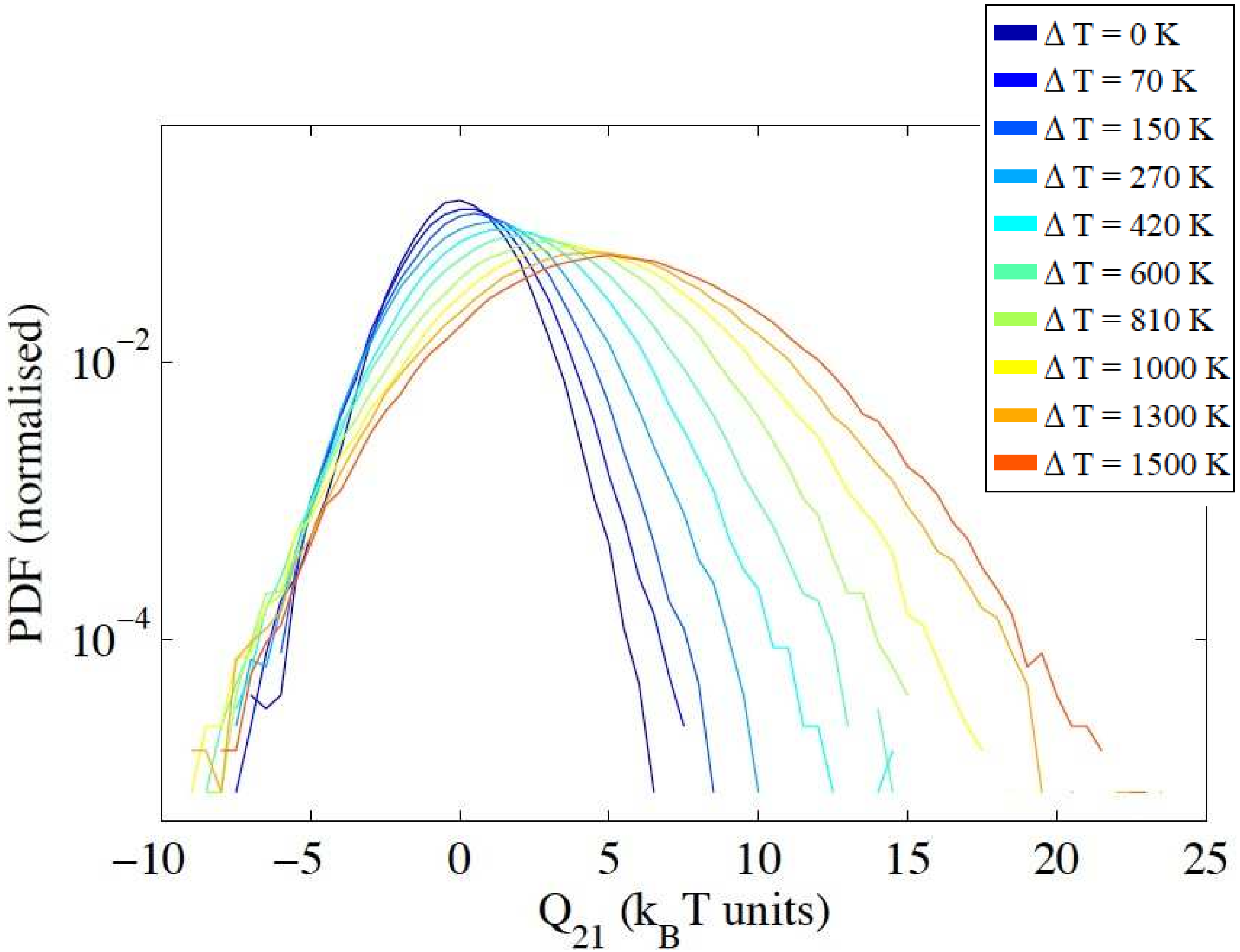


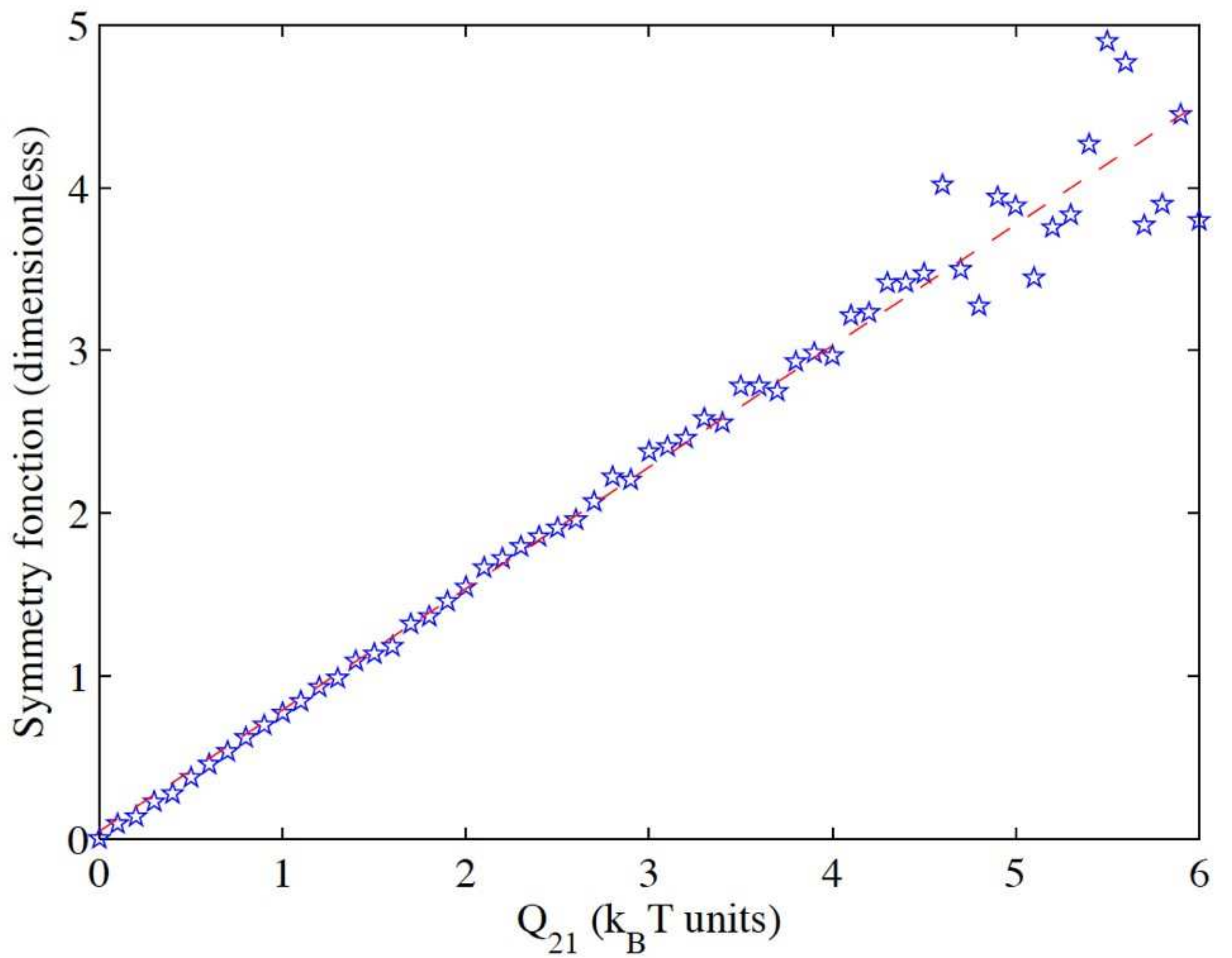
but $< Q_{2,1} >= -\frac{k_1}{k_2} < Q_{1,2} >$ and

$$< Q_{2,1} > + < Q_{1,2} > \neq 0$$

$$Q_{12} - Q_{21} = \text{work of dissipative forces}$$

The Fluctuation Theorem and the effective Temperature





$$S(Q_{2,1}) = \log \frac{P(Q_{2,1})}{P(-Q_{2,1})} = \Delta\beta_{2,1} \frac{Q_{2,1}}{k_B T_2} \qquad \text{with } \Delta\beta_{2,1} = \frac{k_2}{k_1}(1 - T_2/T_1)$$

$$S(Q_{1,2}) = \log \frac{P(Q_{1,2})}{P(-Q_{1,2})} = \Delta\beta_{1,2} \frac{Q_{1,2}}{k_B T_2} \qquad \text{with } \Delta\beta_{1,2} = (1 - T_2/T_1)$$

The transient behavior

What does it occur when the high temperature is switched on ?

This question has been theoretically analyzed in a system with conservative coupling.

B.Cuetara, M. Esposito, and A. Imparato, Phys. Rev. E 89, 052119 (2014).

The main results of this study is that during the transient :

the energy flux from the hot reservoir satisfied the FT for any time

whereas the FT is satisfied only asymptotically for the heat going into the cold reservoir.

We checked this idea in our system which presents a dissipative coupling

The transient behavior

Experimental procedure in the two beads system with $k_1 = k_2$

The two beads are kept at the same temperature $T_1 = T_2$

At $t = 0$ the temperature T_1 is suddenly increased by $\Delta T = 330K$ and it is kept constant for about $1s$.

Q_1 and Q_2 are measured during the transient.

$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

$$q_{ii} = -\int_0^\tau x_i \dot{x}_i \, \mathrm{d}t$$
$$q_{ij} = -\int_0^\tau x_j \dot{x}_i \, \mathrm{d}t$$

The integrals are computed in the interval $0 < t < \tau$

The transient behavior

Experimental procedure in the two beads system with $k_1 = k_2$

The two beads are kept at the same temperature $T_1 = T_2$

At $t = 0$ the temperature T_1 is suddenly increased by $\Delta T = 330K$ and it is kept constant for about $1s$.

Q_1 and Q_2 are measured during the transient.

$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

$$q_{ii} = -\int_0^\tau x_i \dot{x}_i \, \mathrm{d}t$$
$$q_{ij} = -\int_0^\tau x_j \dot{x}_i \, \mathrm{d}t$$

At $t = 1s$ we set $\Delta T = 0$ and we let the system to relax.

This quenching procedure is repeated 4500 times to construct the statistics of Q_1 and Q_2 during the transient.

The transient behavior

$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

$$q_{ii} = -\int_0^\tau x_i \dot{x}_i \, dt$$
$$q_{ij} = -\int_0^\tau x_j \dot{x}_i \, dt$$

Theoretical Prediction in
the case of conservative coupling

B. Cuetara, M. Esposito, and A. Imparato, Phys. Rev. E 89, 052119 (2014).

$$\Sigma(Q_i) = \log \frac{P(Q_i)}{P(-Q_i)} = \Delta\beta_i \frac{Q_i}{k_B T_2}$$

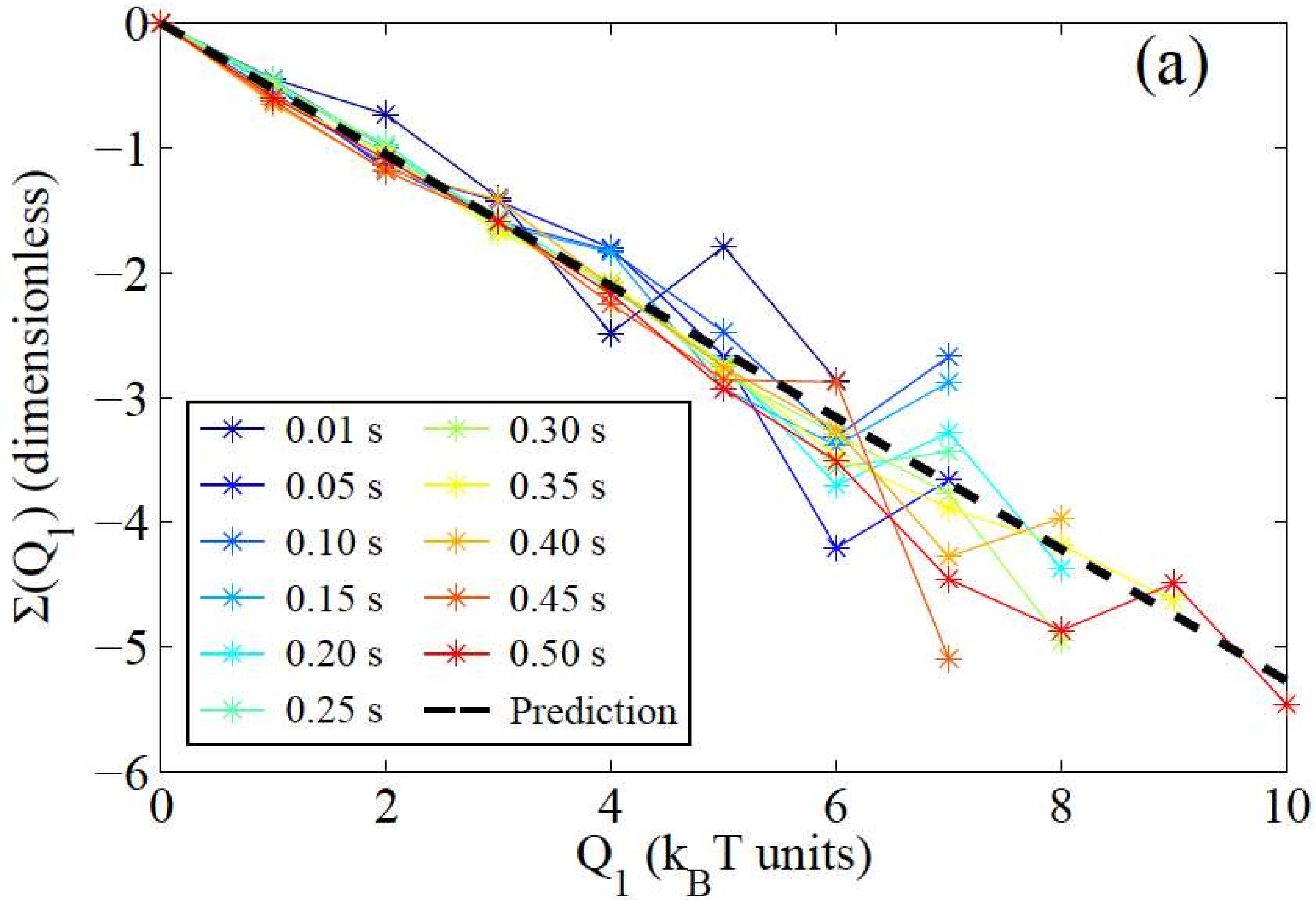
where

$\Delta\beta_1 = (1 - T_2/T_1)$ for any time τ

$\Delta\beta_2 = (1 - T_2/T_1)$ only for $\tau \to \infty$

Q_1 and Q_2 have a different statistical behavior

The transient behavior



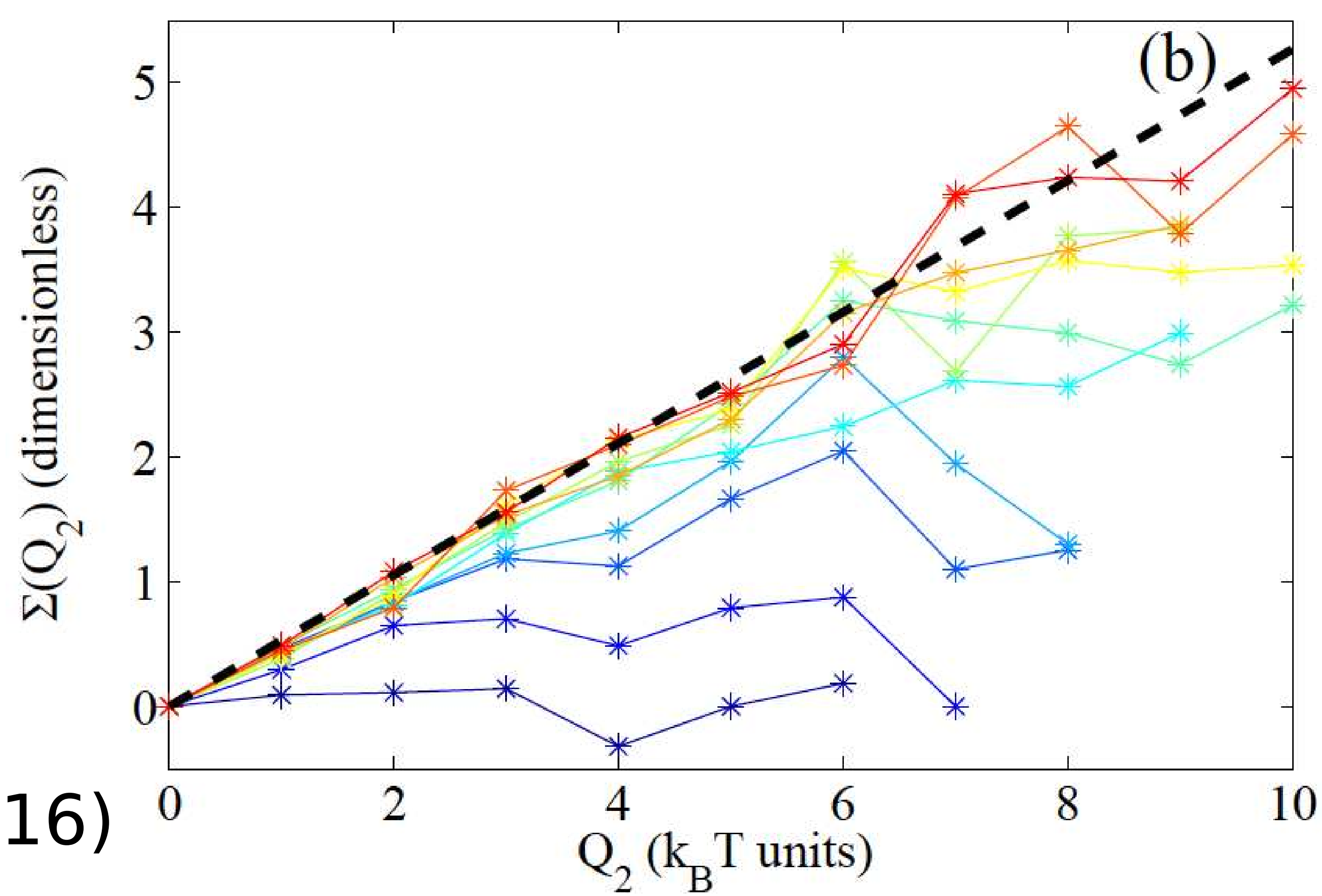
$$\Sigma(Q_i) = \log \frac{P(Q_i)}{P(-Q_i)} = \Delta\beta_i \frac{Q_i}{k_B T_2}$$

where

$\Delta\beta_1 = (1 - T_2/T_1)$ for any time τ

$\Delta\beta_2 = (1 - T_2/T_1)$ only for $\tau \to \infty$

Q_1 and Q_2 have
a different statistical behavior
in the case of viscous coupling



Berut et al, PRL,116, 068301 (2016)

Conclusions on particle interactions

- The differrence between out-equilibrium and equilibrium variance is proportional to the heat flux
- A hydrodynamic model precisely describes the experimental data
- The FT correctly estimates the effective temperature within experimental errors.
- Heat fluxes are affected by the work of dissipative forces
- During the transient the FT for the heat has a different statistical behaviors for the cold and the hot sources.

General conclusions on the role of coupling between systems at two different temperatures

- The differrence between out-equilibrium and equilibrium variance is proportional to the heat flux

- The FT for two heat bath allows us to estimate the entropy production.

- Heat fluxes are affected by the work of dissipative forces in asymmetric systems

- During the transient the FT for the heat has a different statistical behaviors for the cold and the hot sources.

Bérut et al, PRL,116, 068301 (2016)
Bérut et al, PRE, 94,052148 (2016)
Bérut et al, JSTAT,P054002 (2016)