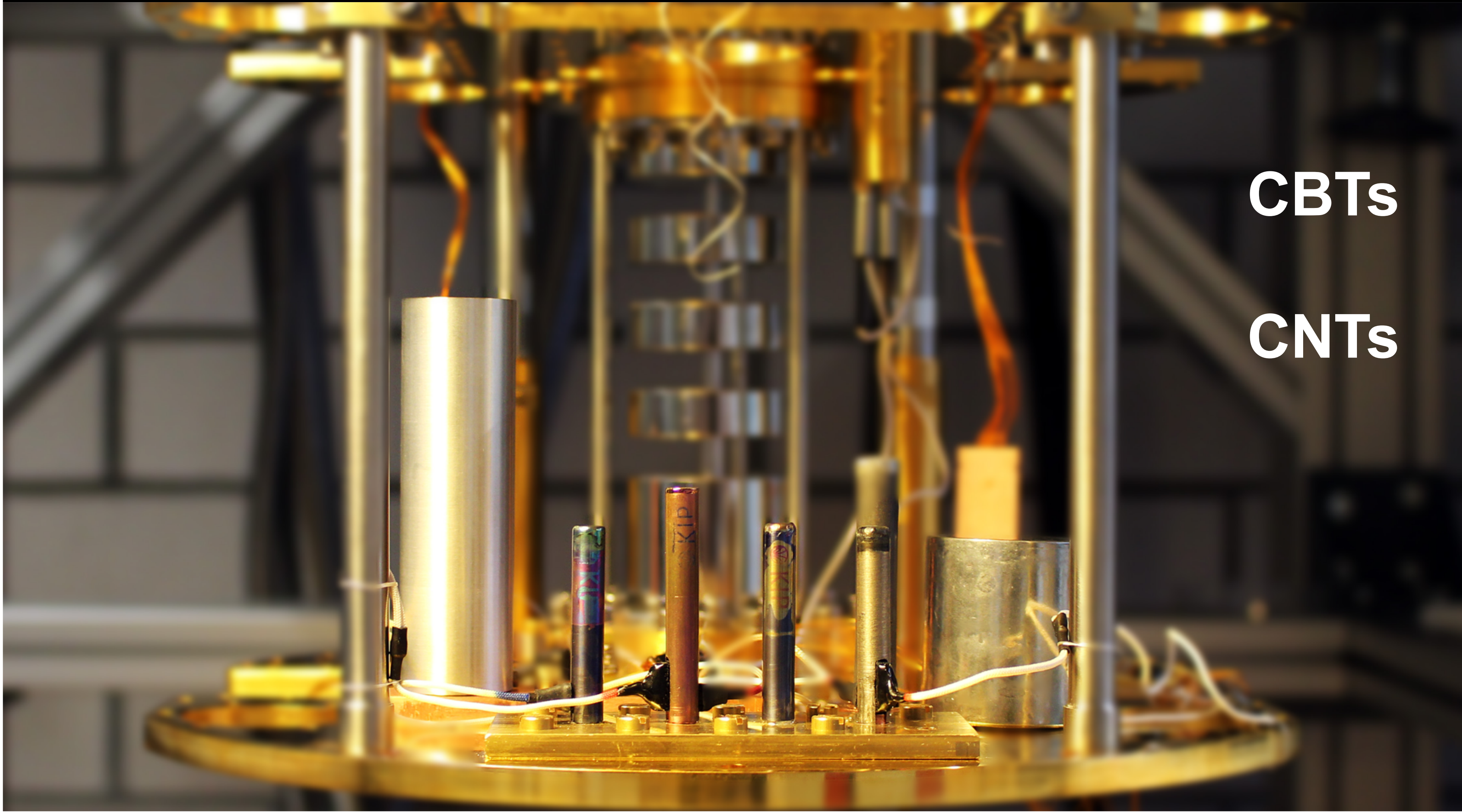
Christian Enss
Heidelberg University

Challenges and Advances of Practical Primary Thermometers for Very Low Temperatures



CBTs

CNTs


Nano Cryogenics 2008

Temperature is a thermodynamic property of state

It can be defined by a reversible cycle, like a carnot cycle

$$\oint T^{-1}\mathrm{d}Q = 0$$

but this is not very practical

General Considerations

Temperature is by far the most uncertain scale … compare it to time

Primary thermometers: can be used without any prior calibration

Secondary thermometers: must be calibrated against an other thermometer

Thermometry is particular difficult under these conditions:

- for very small systems (nano samples)
- in high magnetic fields
- at ultralow temperatures

Old Wisdom of Low Temperature Physics

If you use one thermometer you have a temperature

If you use two thermometers you have a problem

VOLUME 73, NUMBER 21 PHYSICAL REVIEW LETTERS 21 NOVEMBER 1994

Thermometry by Arrays of Tunnel Junctions

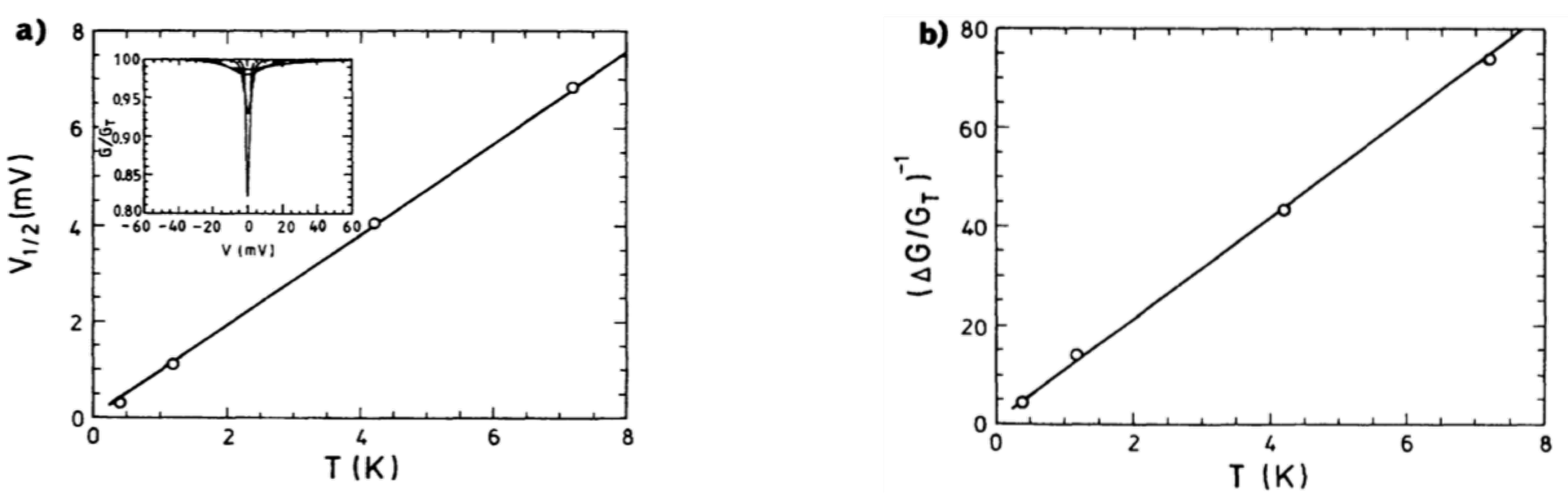
J. P. Pekola, K. P. Hirvi, J. P. Kauppinen, and M. A. Paalanen

Laboratory of Applied Physics, Department of Physics, University of Jyväskylä, P. O. Box 35, 40351 Jyväskylä, Finland

(Received 13 July 1994)

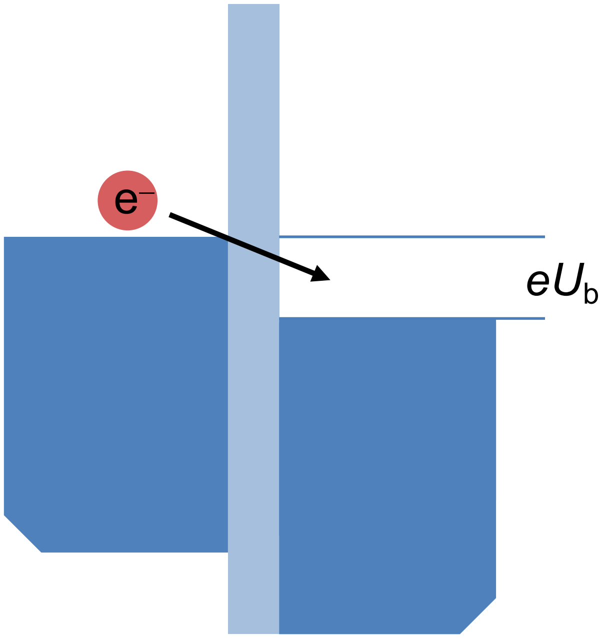
We show that arrays of tunnel junctions between normal metal electrodes exhibit features suitable for primary thermometry in an experimentally adjustable temperature range where thermal and charging effects compete. I-V and dI/dV vs V have been calculated for two junctions including a universal analytic high temperature result. Experimentally the width of the conductance minimum in this regime scales with T and N, the number of junctions, and its value (per junction) agrees with the calculated one to within 3% for large N. The height of this feature is inversely proportional to T.

PACS numbers: 73.40.Gk, 07.20.Dt, 73.40.Rw



J. Pekola *et al.*, PRL **73**, 2903 (1994)

Tunnel Junction

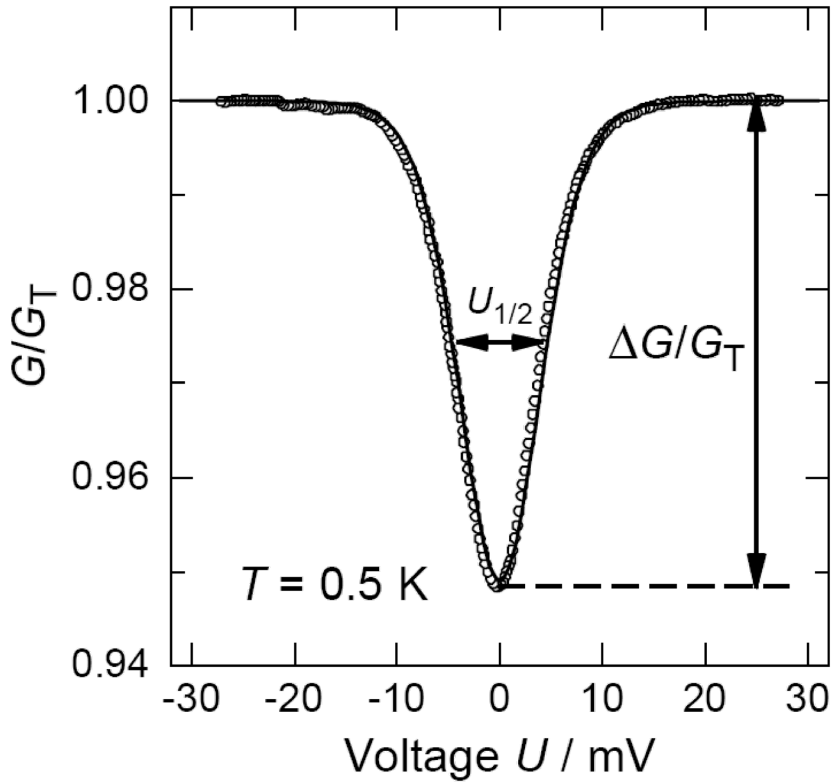


- behaves like ohmic conductor
- tunnel junction has a capacitance

Coulomb Blockade in a nutshell

- electron charges the capacitor, causing a buildup voltage $U = e/C$
- if capacitance is small build up voltage can be large enough to prevent another electron from tunnelling
- electrical current is then suppressed at low bias voltages, and the conduction of the device is no longer constant
- the decrease of the differential conduction around zero bias is called the coulomb blockade

Coulomb Blockade Thermometers (CBT)



charging energy

$$E_{\mathrm{C}} = e^2/2C_{\mathrm{eff}}$$

high temperature limit $E_{\mathrm{C}} \ll k_{\mathrm{B}}T$

differential conduction

$$G = G_{\mathrm{T}}\left[1 - \left(\frac{E_{\mathrm{C}}}{k_{\mathrm{B}}T}\right)\right] g(x)$$

$$g(x) = \frac{x\,\sinh(x) - 4\,\sinh^2(x/2)}{8\,\sinh^4(x/2)}$$

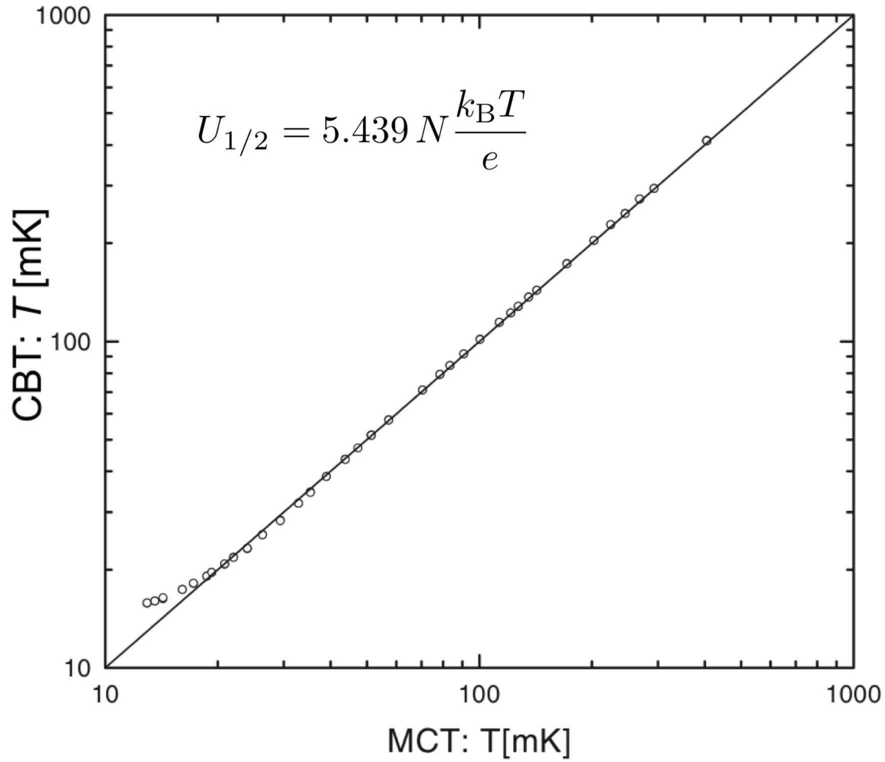
$$\Delta G/G_{\mathrm{T}} = E_{\mathrm{c}}/6k_{\mathrm{B}}T$$

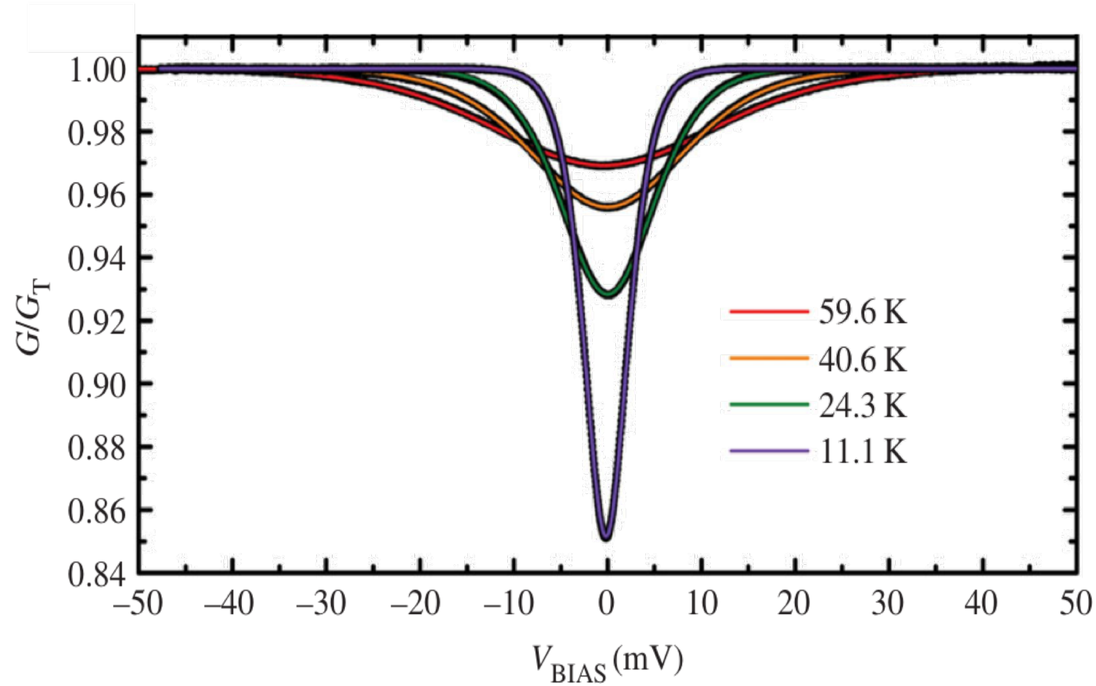
$$U_{1/2} = 5.439\,N\frac{k_{\mathrm{B}}T}{e}$$

$$x = eU/(Nk_{\mathrm{B}}T)$$

J. Pekola *et al.*, PRL **73**, 2903 (1994)

Temperature Dependence



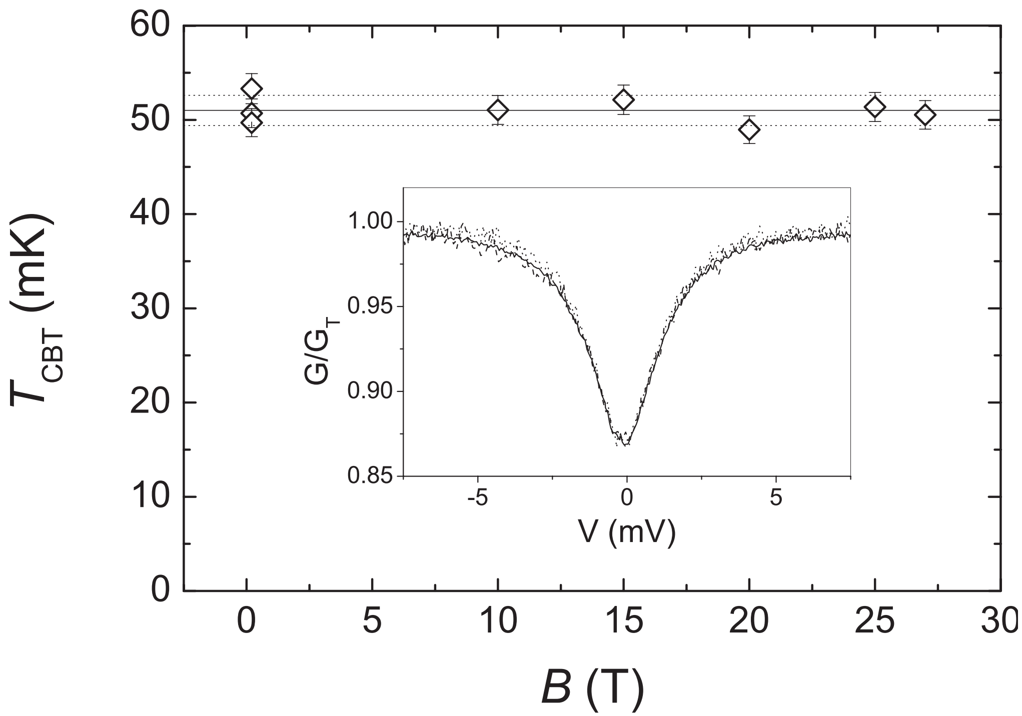


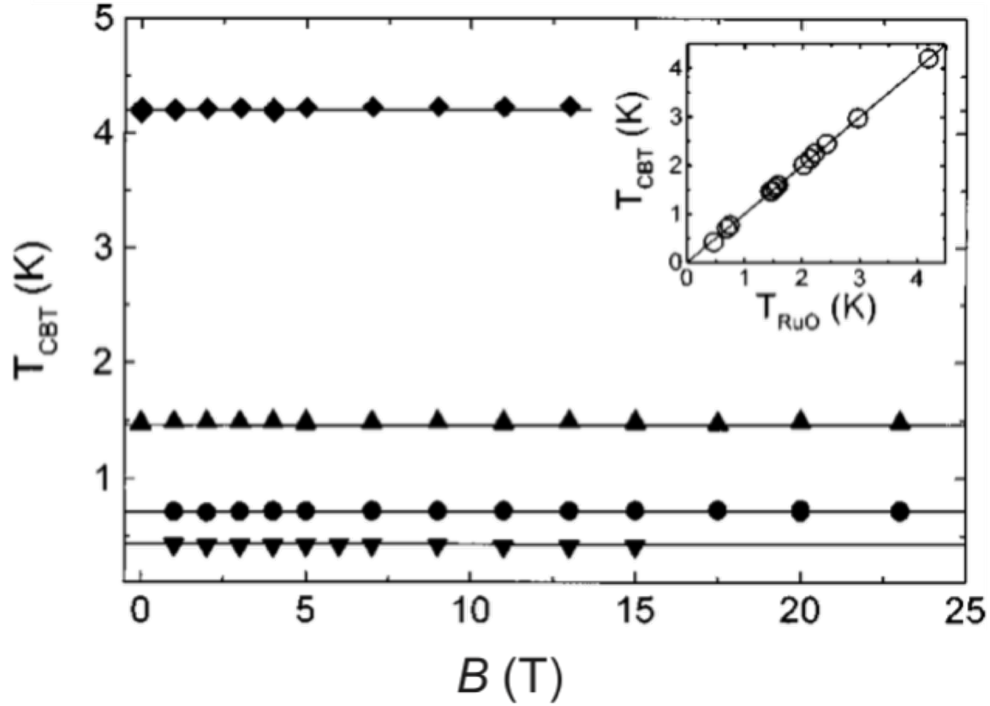
CBT can used over a wide range of temperatures

M. Meschke, J.P. Pekola *et al.*, JLTP **134**, 1119 (2004)

M. Meschke, A. Kemppinen and J. P. Pekola
Phil. Trans. Soc. **108**, 191 (1997)

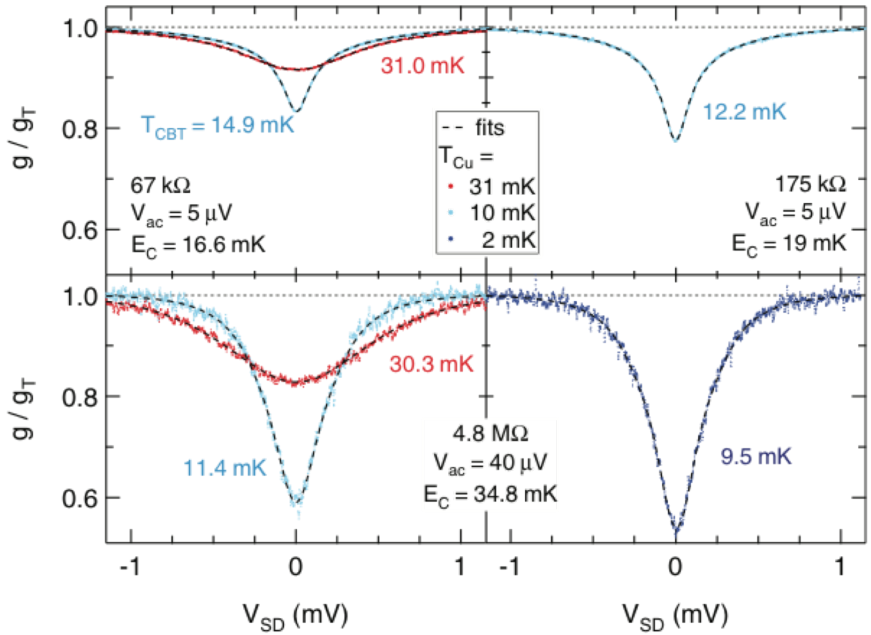
Magnetic Field Independence of CBTs

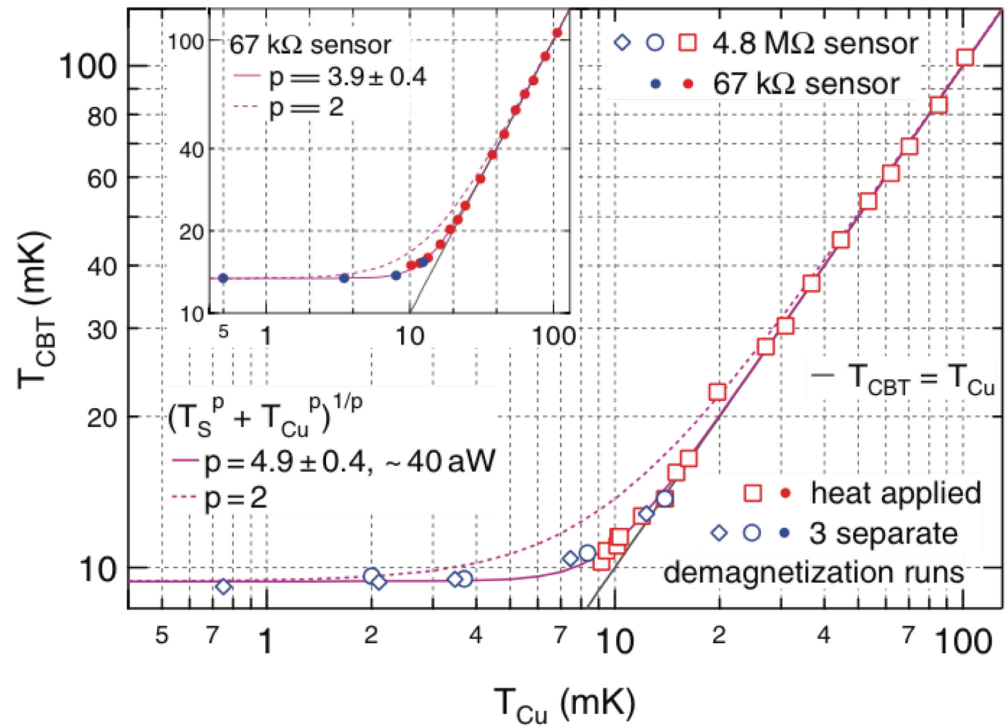




J. Pekola *et al.*, JAP **83**, 5582 (1998)

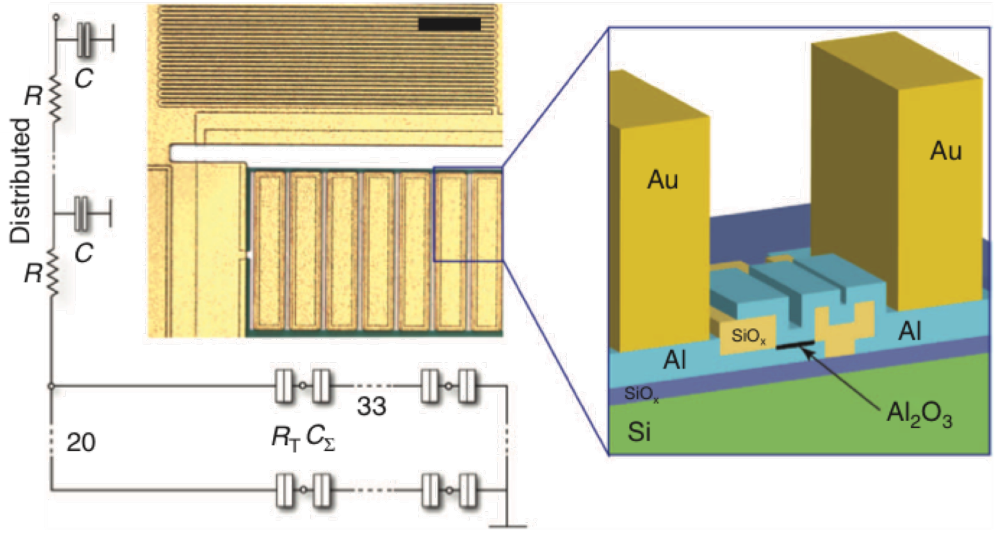
J. Pekola *et al.*, JLTP **128**, 263 (2002)

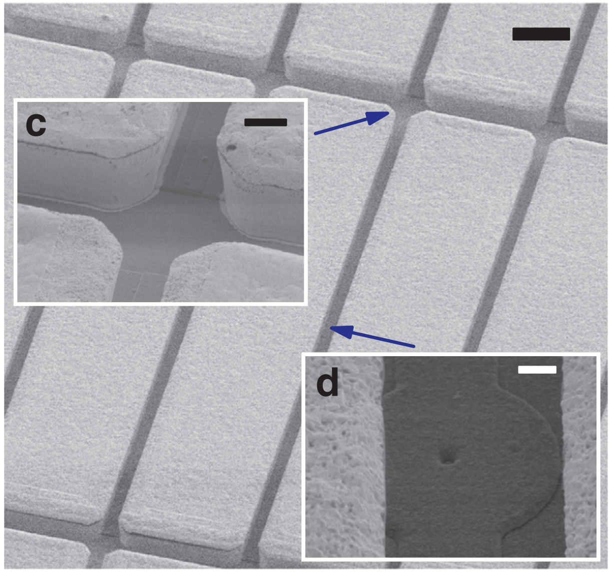


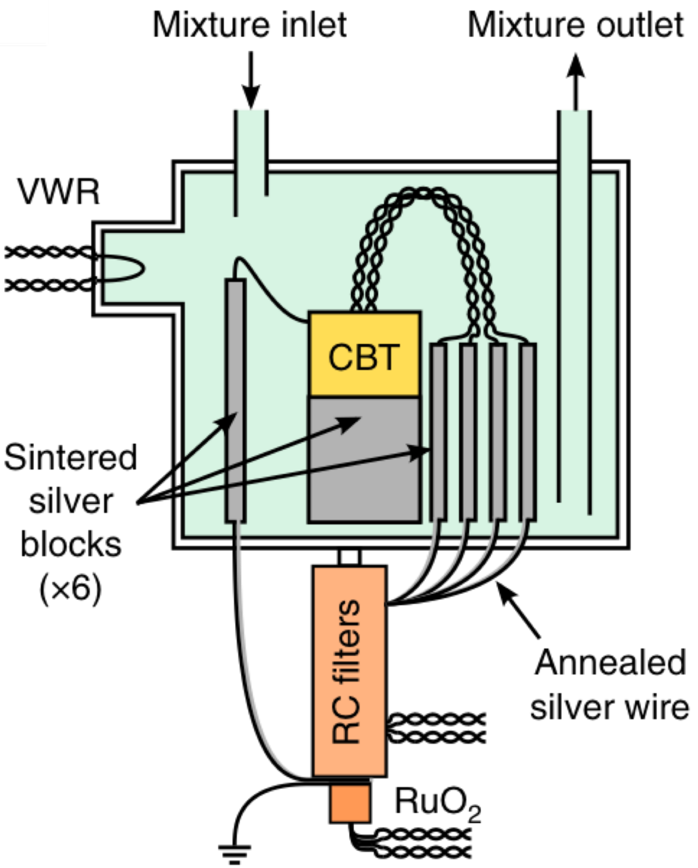


- First CBT measurement below 10 mK

L. Casparis *et al.*, RSI **83**, 083903 (2012)

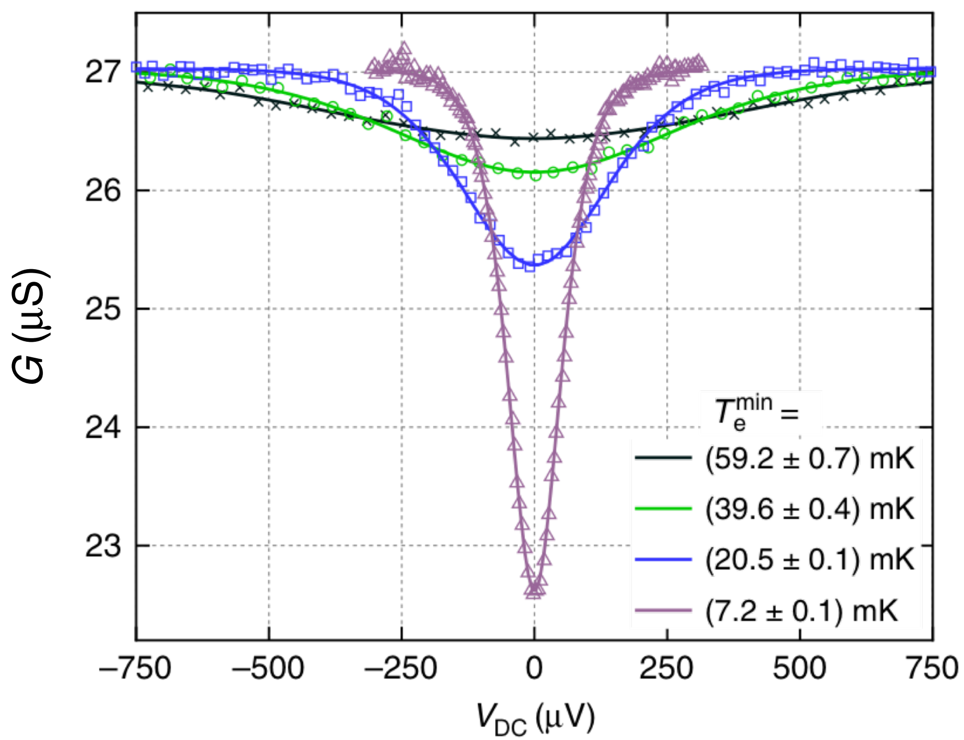


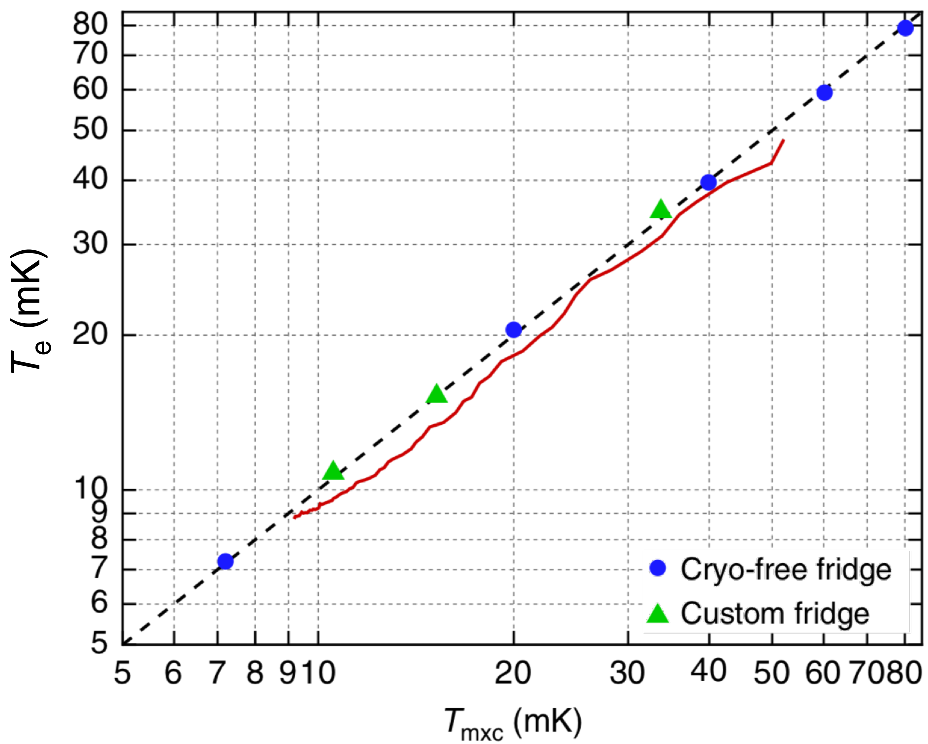




D.J. Bradley, *et al*., Nature Commun. **7**, 1455 (2016)

Recent Advances of CBTs





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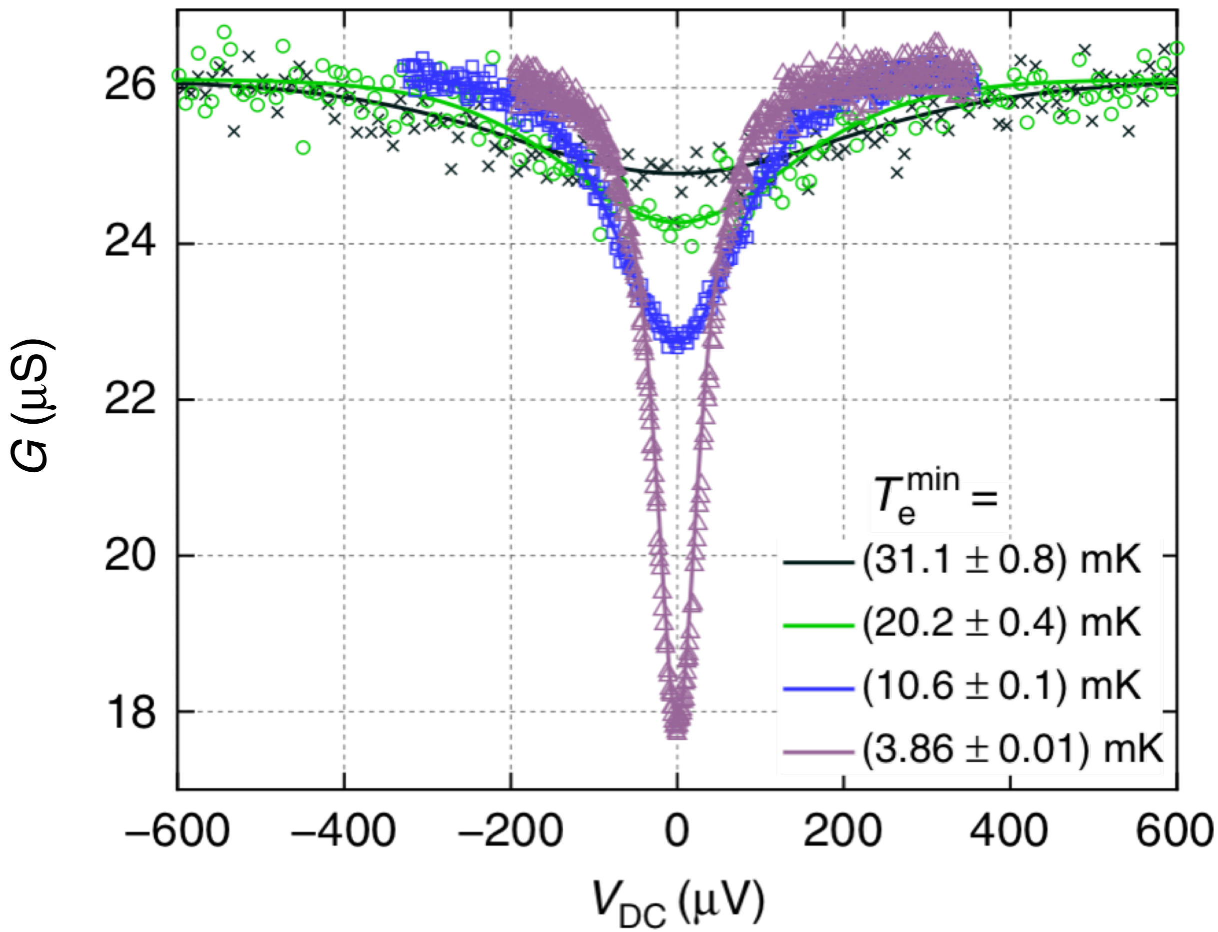
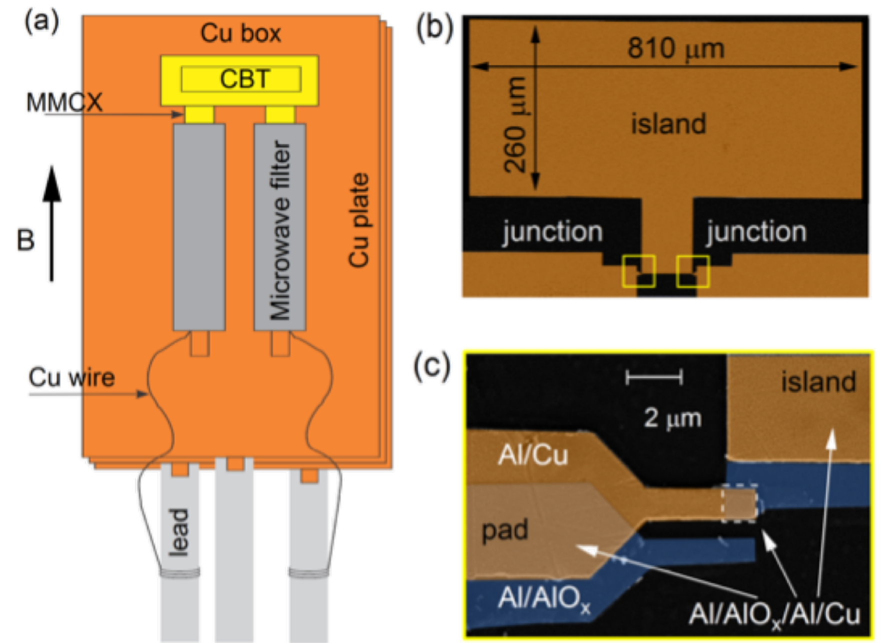
Recent Advances of CBTs

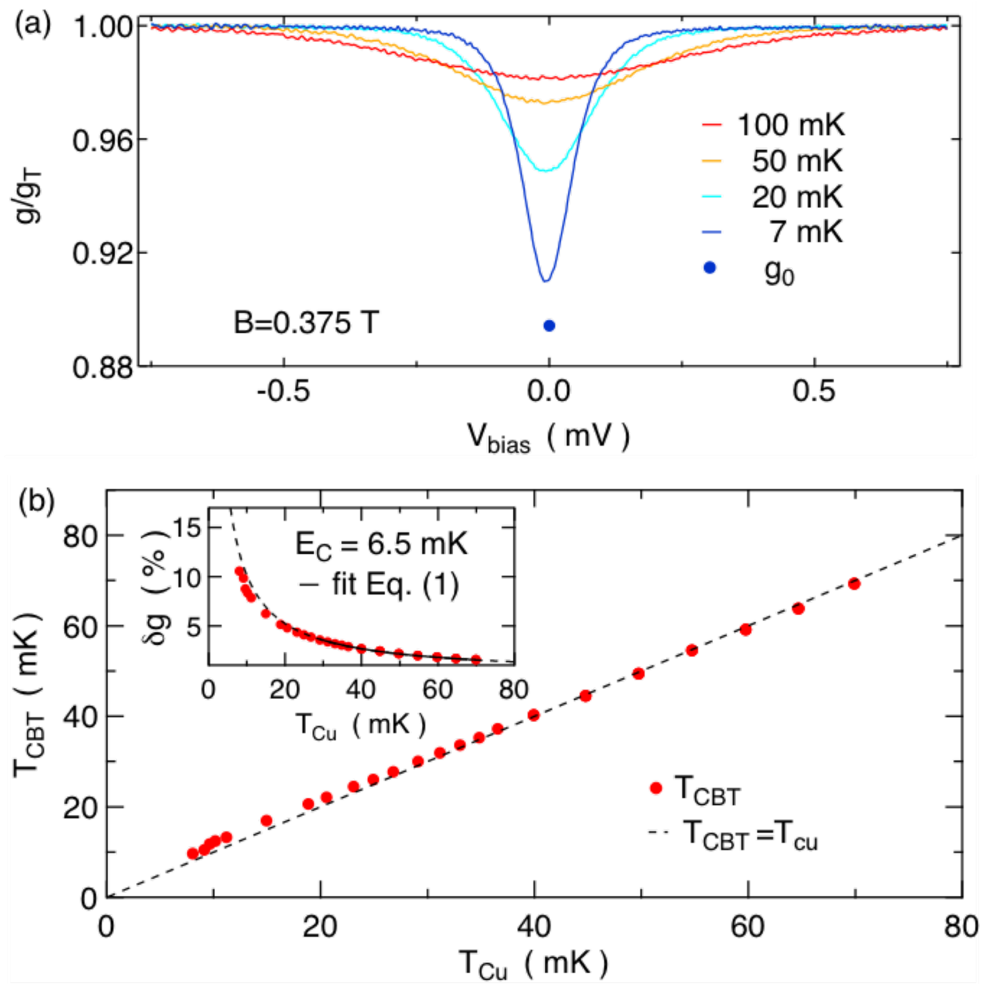


- temperature of electron gas below 4 mK

D.J. Bradley, *et al*., Nature Commun. **7**, 1455 (2016)

Recent Advances of CBTs

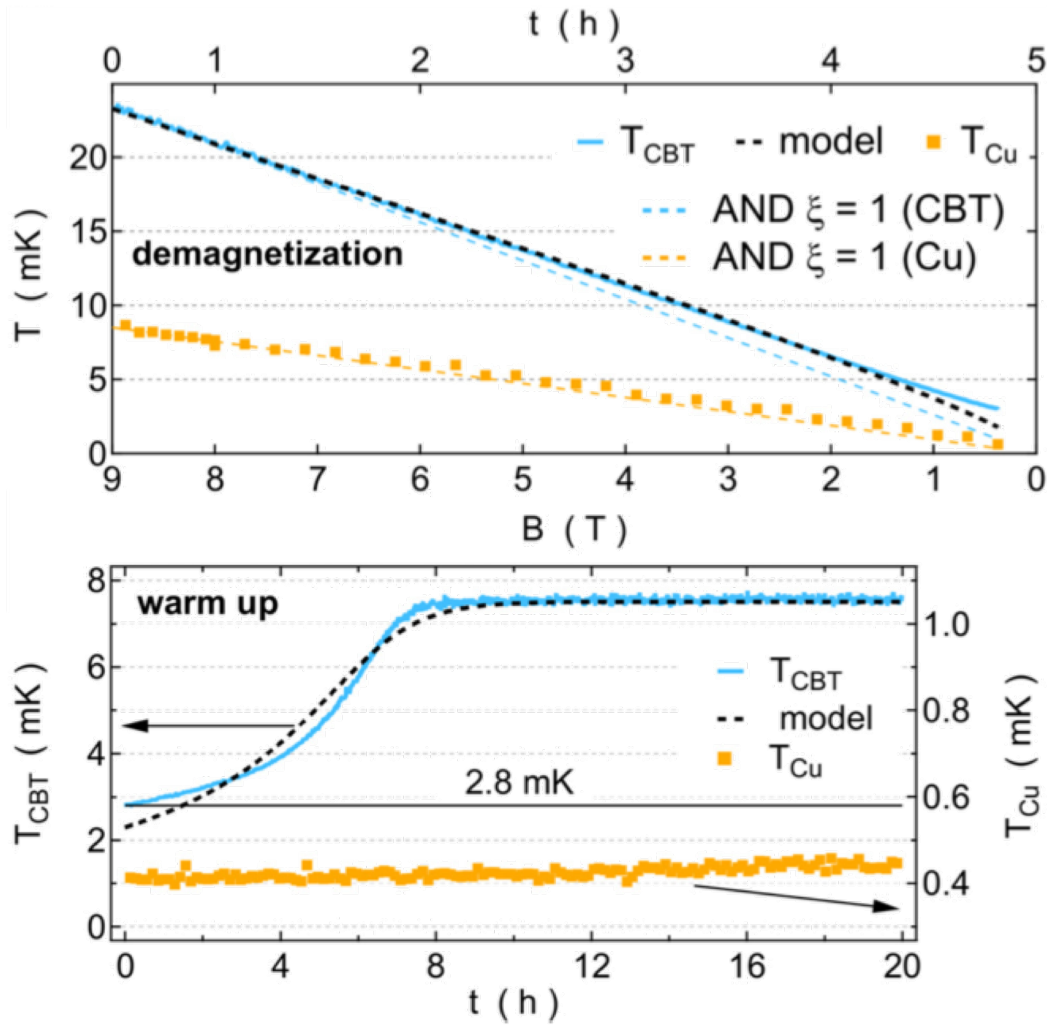




adiabatic demagnetization of both the electronic leads and the large metallic islands

M. Palma, *et al.*, Appl. Phys. Lett. **111**, 253105 (2017)

Recent Advances of CBTs



EMP

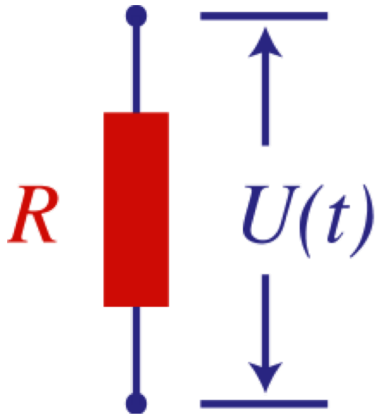
minimum temperature of electron gas below 2.8 mK

M. Palma, *et al.*, Appl. Phys. Lett. **111**, 253105 (2017)

Noise Thermometry

thermal voltage fluctuations across a conductor

1927



$$S_U = \frac{\langle U^2 \rangle}{\Delta f} = 4 k_\mathrm{B} T R$$

John Bertrand "Bert" Johnson

quantum corrections

$$S_U = 4hfR\left[\frac{1}{2} + \frac{1}{e^{hf/k_\mathrm{B}T} - 1}\right]$$

$$\simeq 4k_\mathrm{B}TR\left[1 + \frac{1}{12}\left(\frac{hf}{k_\mathrm{B}T}\right)^2\right]$$

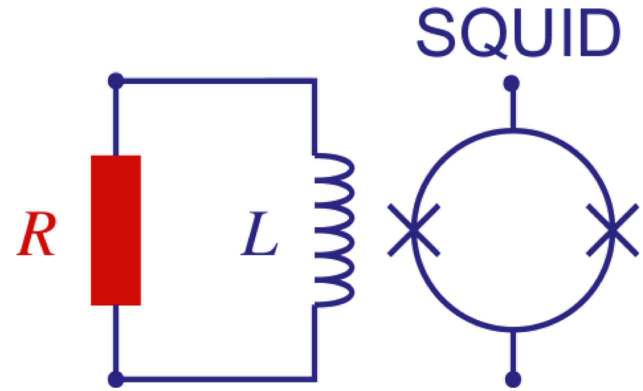
1928

Harry Nyquist

can be neclegted since
(T > 100 µK, f < 1 kHz)

$$\frac{hf}{k_\mathrm{B}T} < 5 \times 10^{-4}$$

first suggested by R.A. Webb, et al. JLTP **13**, 383 (1973)



current noise

$$S_I = \frac{4k_\mathrm{B}T}{R}$$

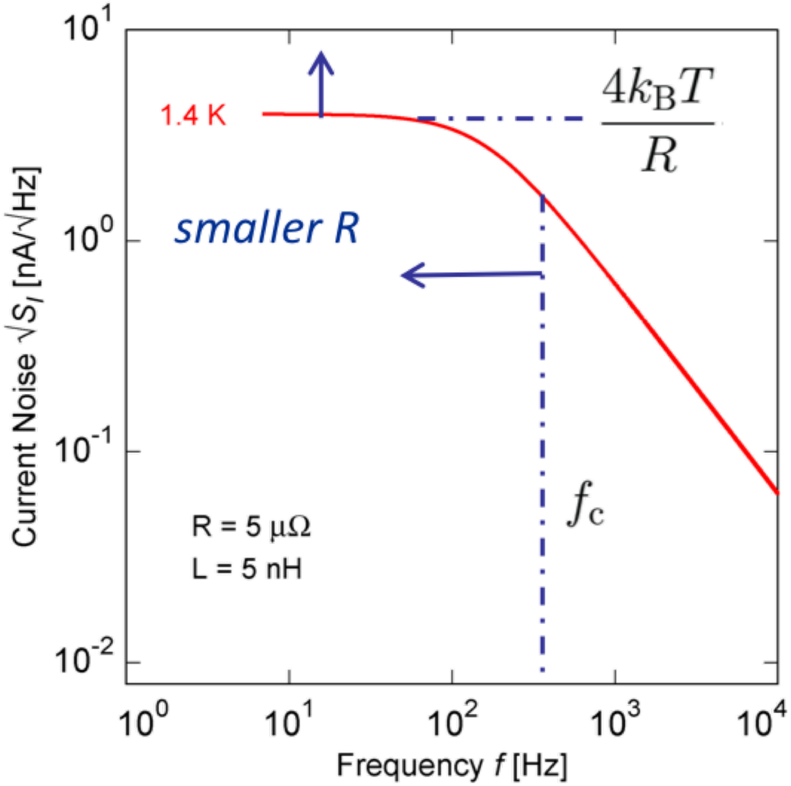
For R ~ mΩ even at T ~ 1mK
large compared to SQUID current sensitivity

Finite bandwidth due to reactance $i\omega L$:

$$S_I = \frac{4k_\mathrm{B}T}{R}\frac{1}{1+(f/f_0)^2} \quad \text{with} \quad f_0 = \frac{1}{2\pi}\frac{R}{L}$$

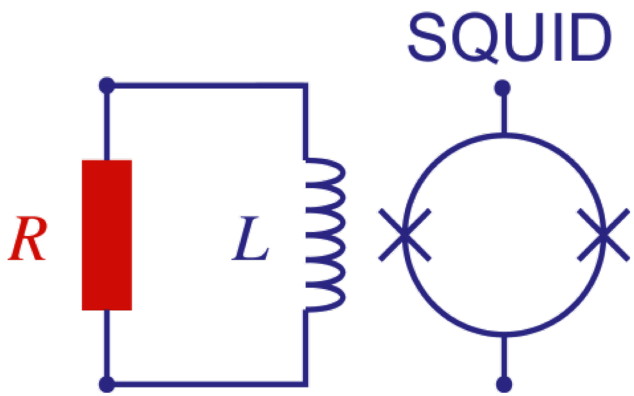
Coil = <u>one</u> degree of freedom, thus

$$\overline{E} = \int_0^\infty \frac{1}{2} L\, S_I\, \mathrm{d}f = \frac{1}{2}k_\mathrm{B}T$$



Current Sensing Noise Thermometry

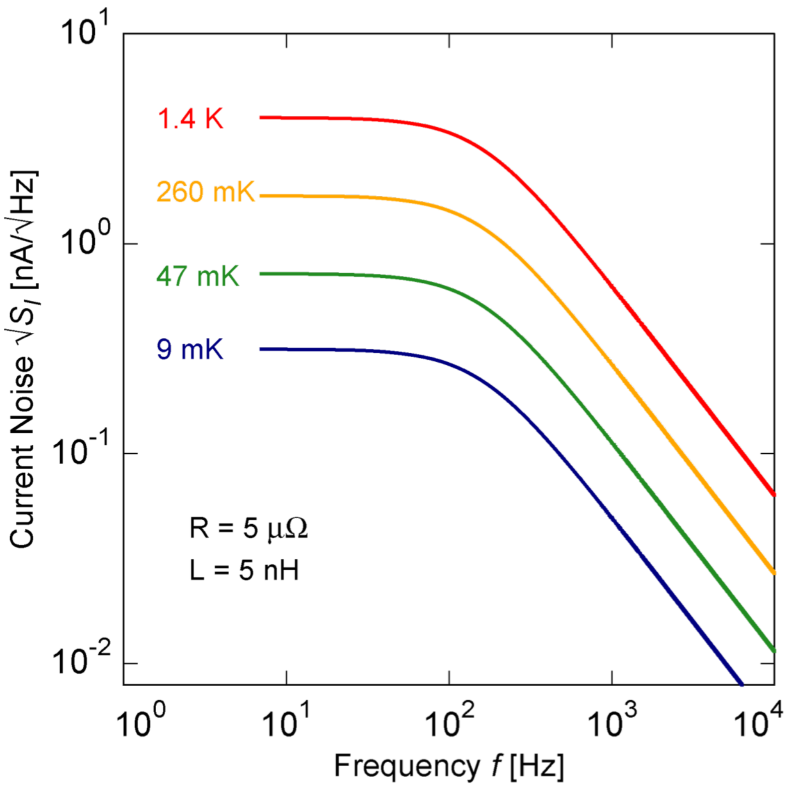
first suggested by R.A. Webb, et al. JLTP **13**, 383 (1973)



current noise

$$S_I = \frac{4k_\mathrm{B}T}{R}$$

For $R \sim \mathrm{m}\Omega$ even at T ~ 1mK
large compared to SQUID current sensitivity

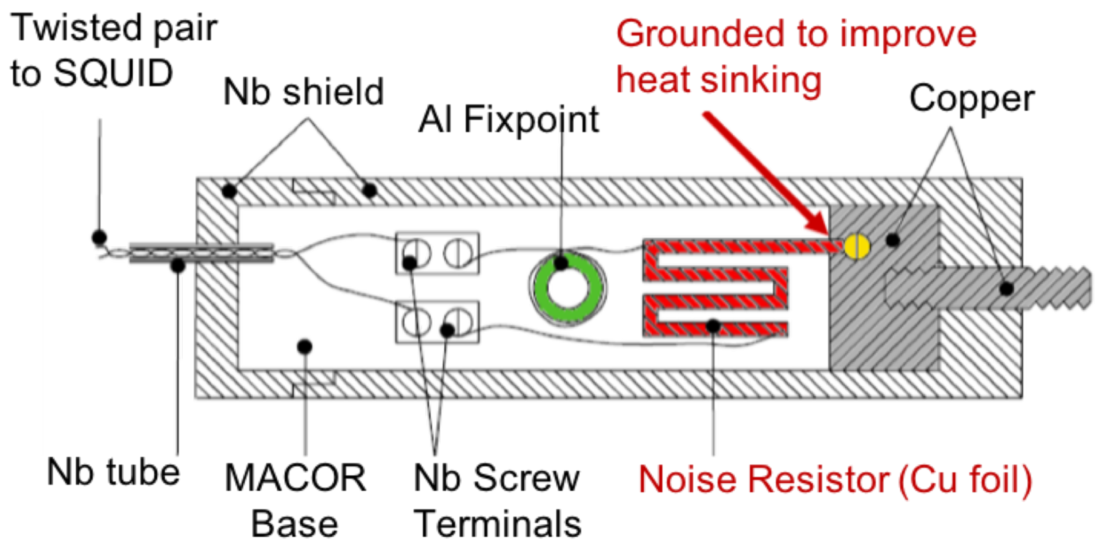


Finite bandwidth due to reactance *iωL*:

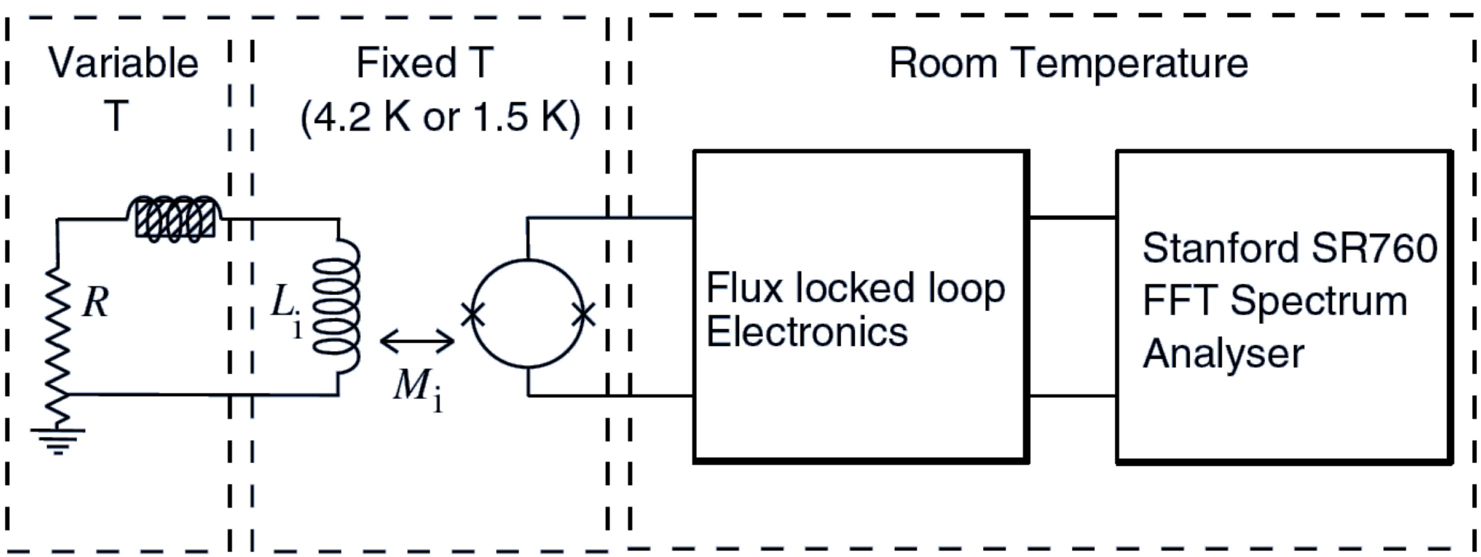
$$S_I = \frac{4k_\mathrm{B}T}{R}\frac{1}{1+(f/f_0)^2} \quad \text{with} \quad f_0 = \frac{1}{2\pi}\frac{R}{L}$$

Coil = <u>one</u> degree of freedom, thus

$$\overline{E} = \int_0^\infty \frac{1}{2}\,L\,S_I\,\mathrm{d}f = \frac{1}{2}k_\mathrm{B}T$$

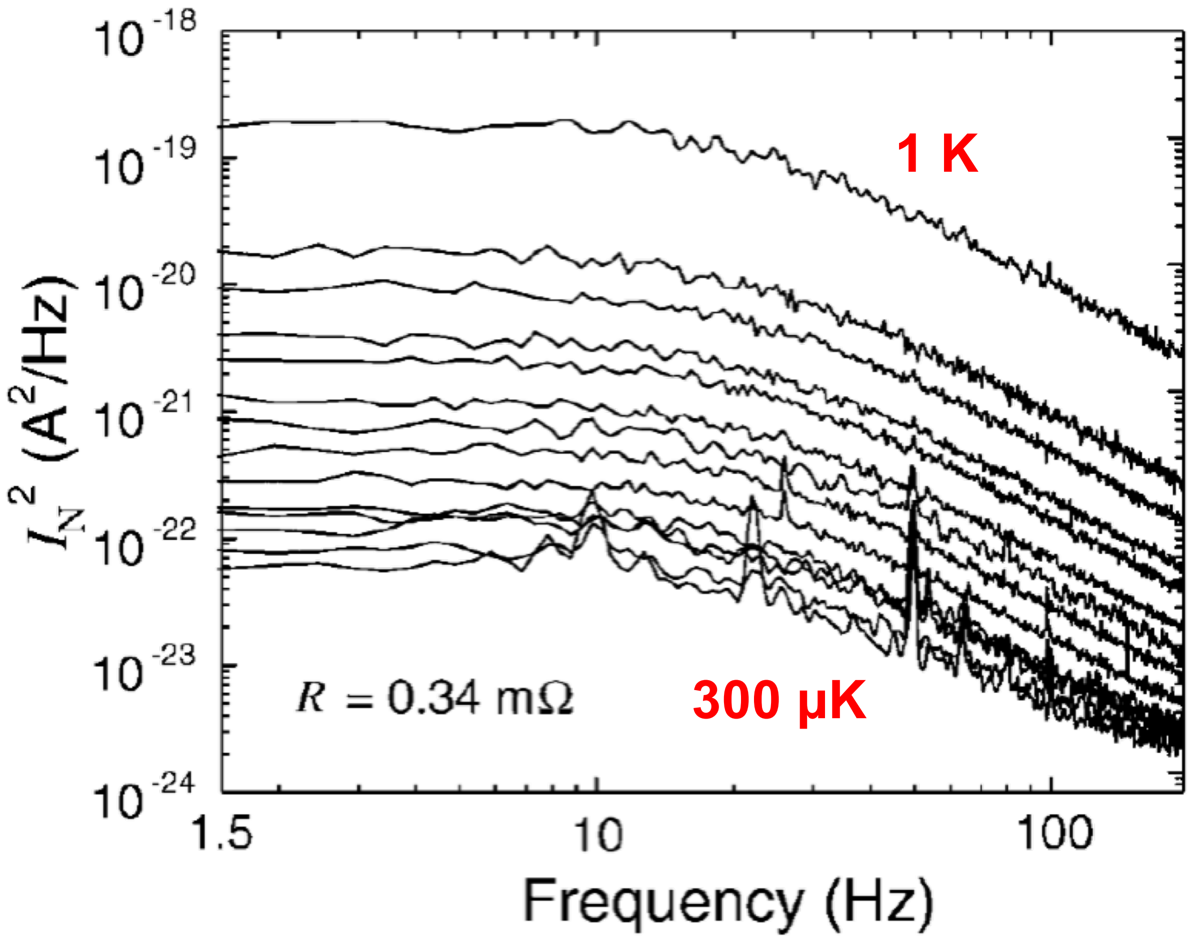


Readout

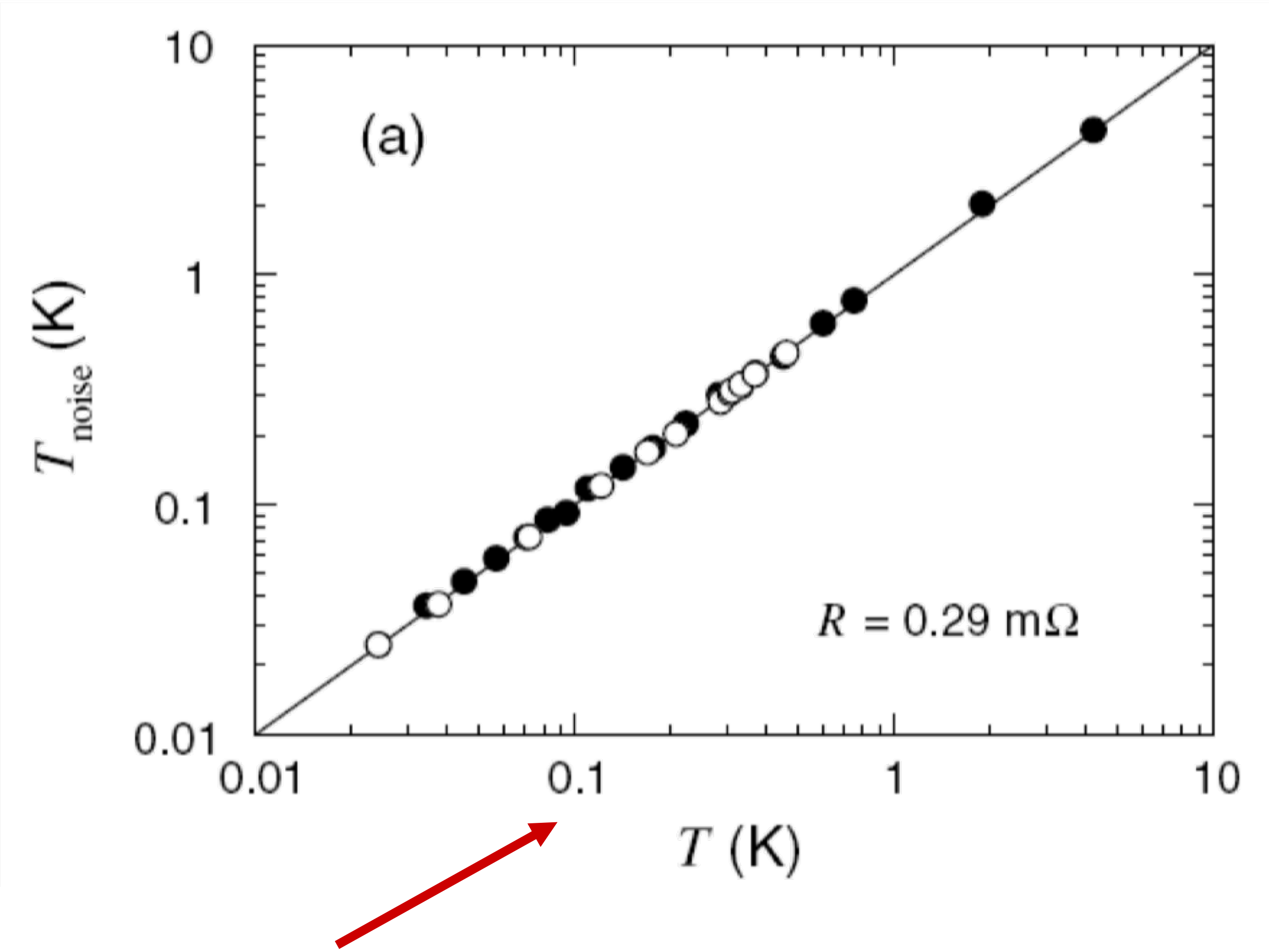


C.P. Lusher *et al*., Meas. Sci. Tech. **12**(1), 1 (2001)

Noise Spectra



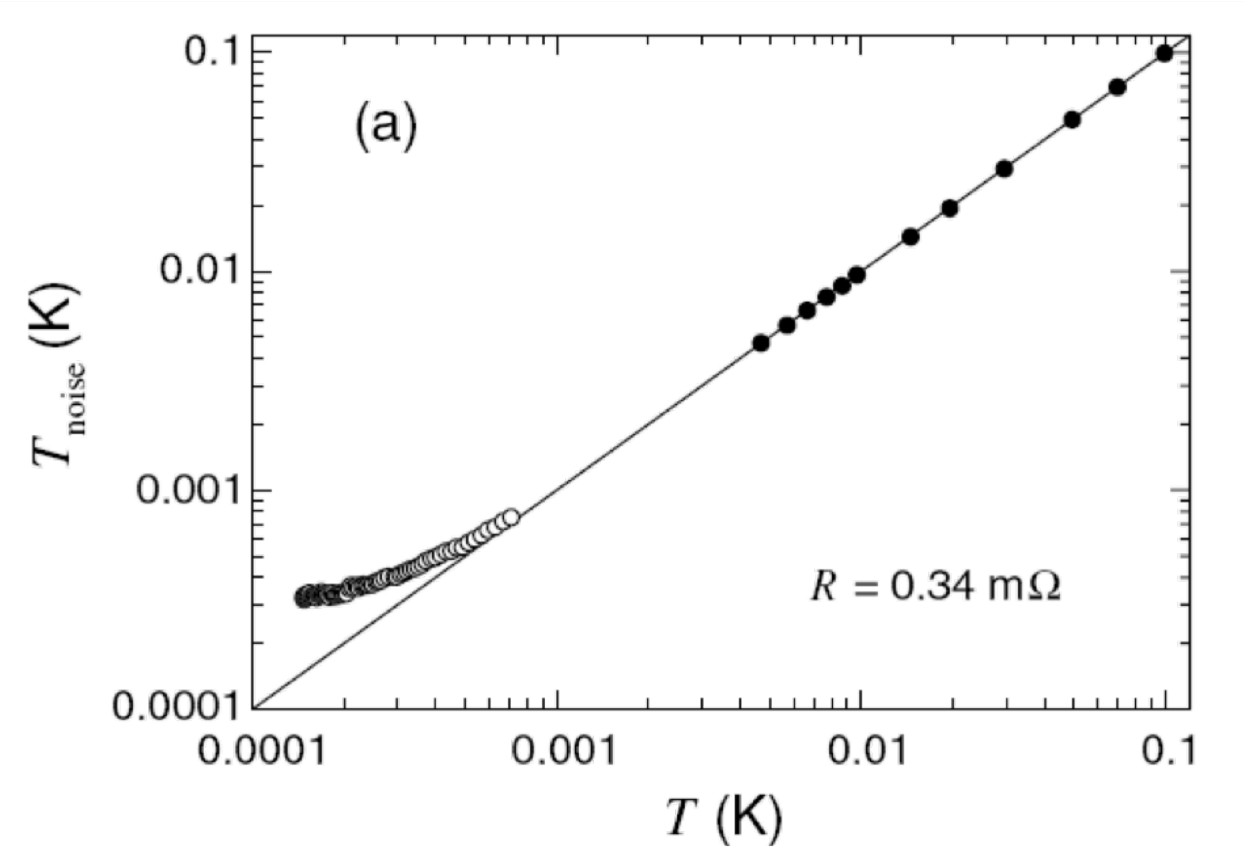
C.P. Lusher *et al*., Meas. Sci. Tech. **12**(1), 1 (2001)



[3]He melting curve thermometer (PTB)

and calibrated resistance thermometers

C.P. Lusher *et al*., Meas. Sci. Tech. **12**(1), 1 (2001)

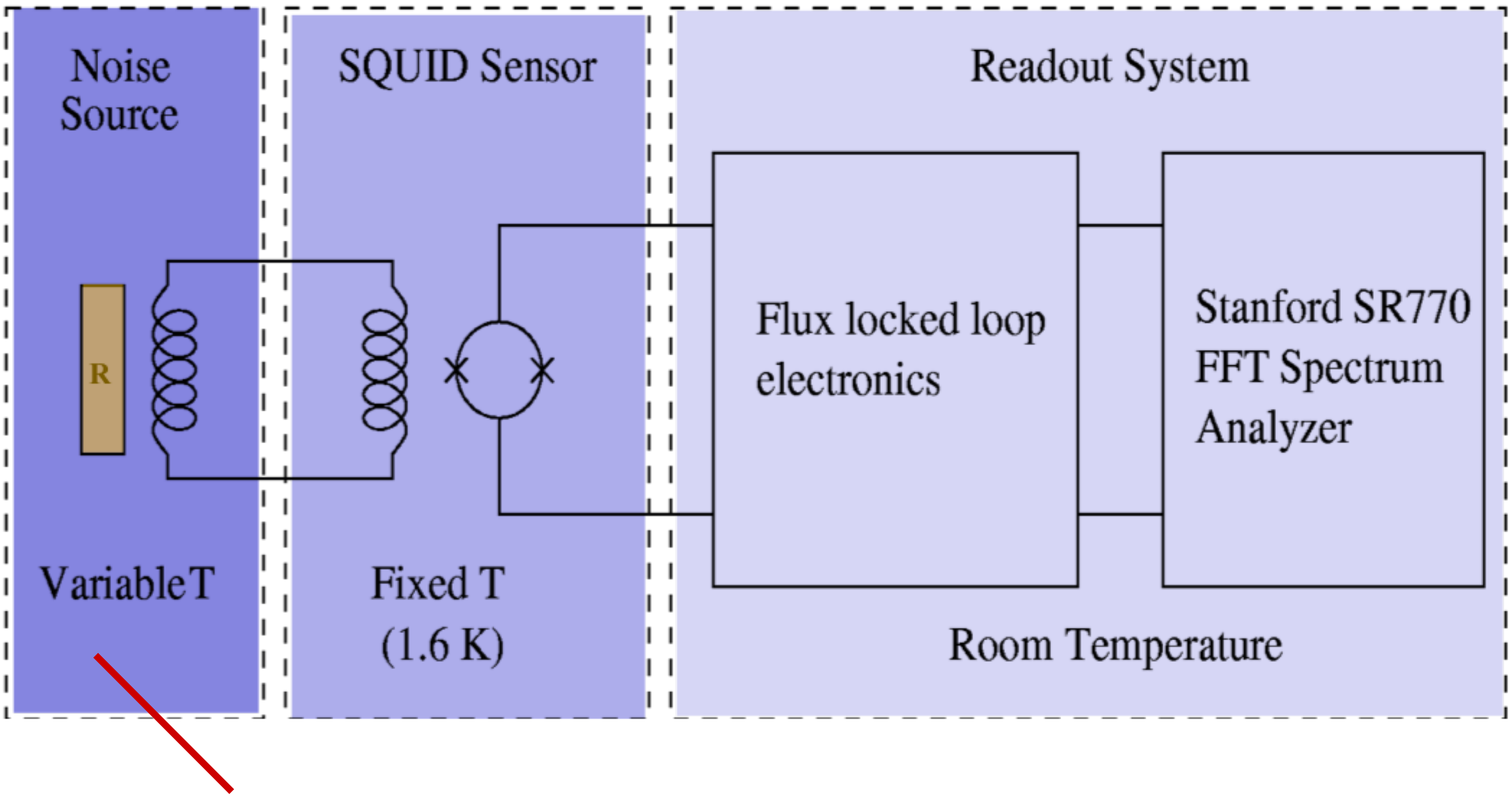


lowest noise power corresponds to: $T = 300\ \mu K$

Decoupling caused by parasitic heat flow of few pW through the 0.3 mW resistor

C.P. Lusher *et al*., Meas. Sci. Tech. **12**(1), 1 (2001)

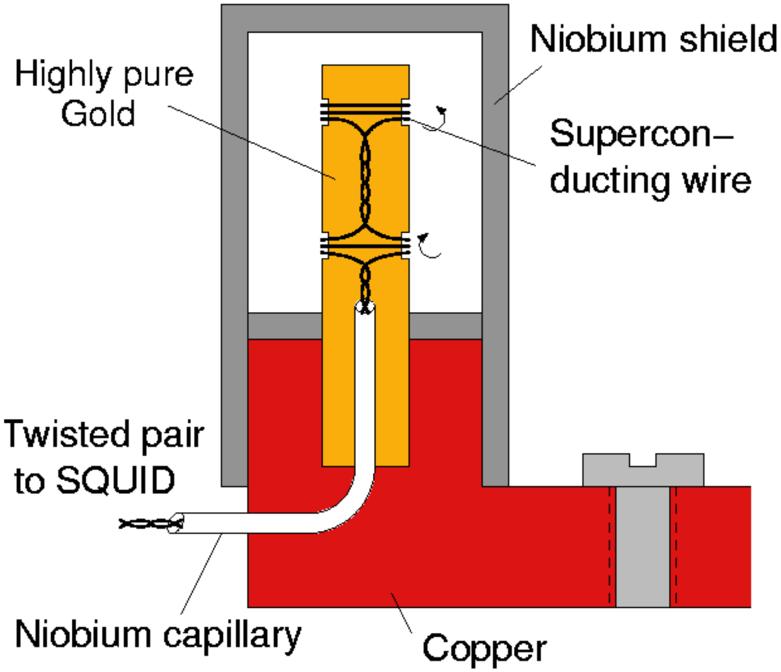
Inductive Readout: Flux Noise

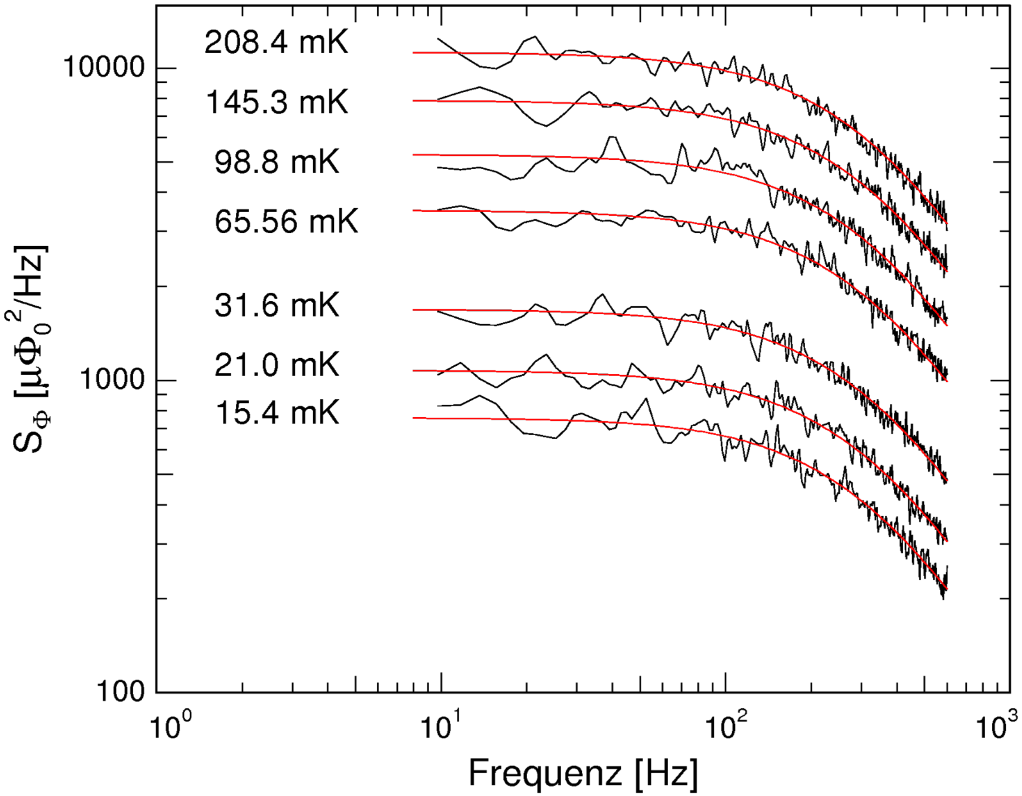


$$Z = R(\omega) + i\omega L(\omega)$$

A. Netsch, E. Hassinger, C. E.,
A. Fleischmann, AIP **850**, 1593 (2006)

Flux-Sensing Noise Thermometer MFFT





Noise source :

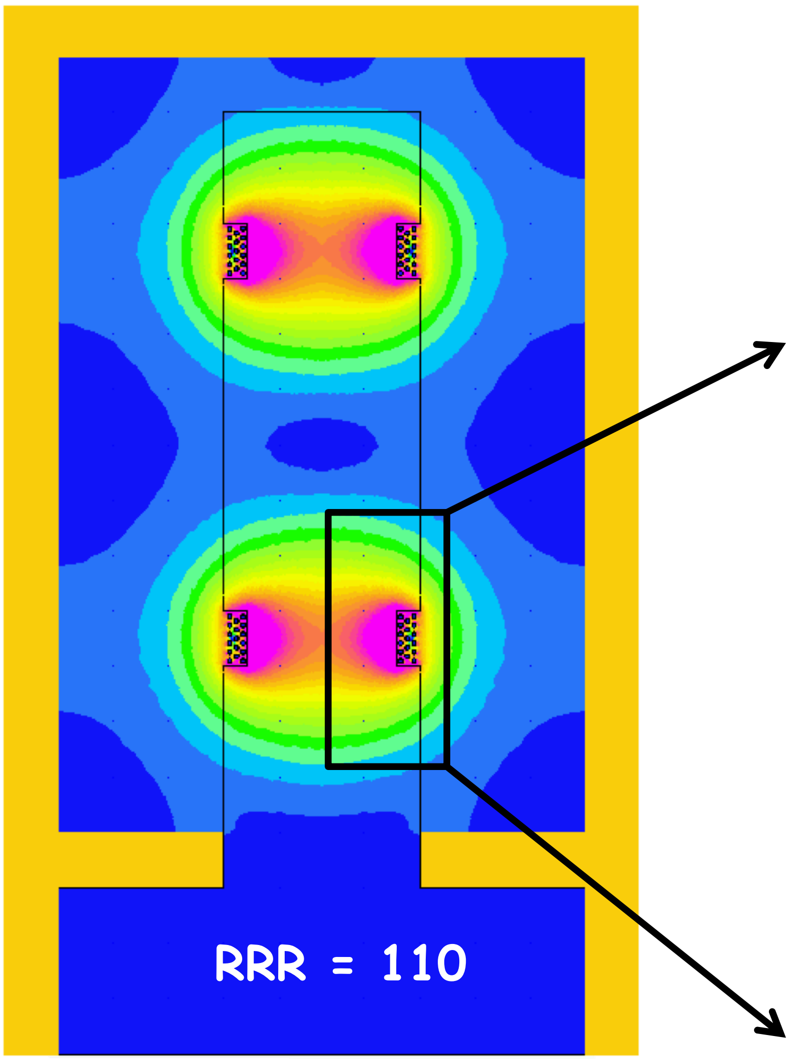
Gold cylinder ∅ 2 mm
purity > 99,999%, RRR = 110

A. Netsch, E. Hassinger, C. E.,
A. Fleischmann, AIP **850**, 1593 (2006)

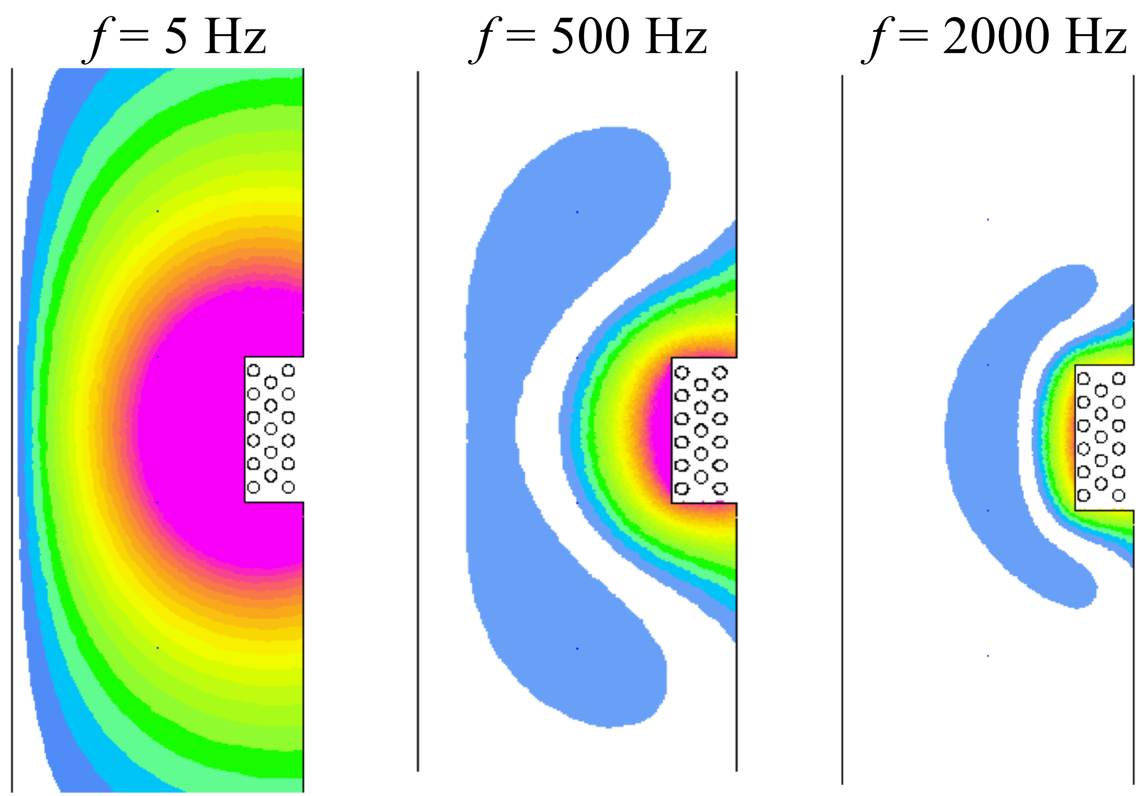
- Spectral shape independent of temperature !
- SQUID noise corresponds to T_N = 150 µK

FEM-Calculation of the Complex Impedance

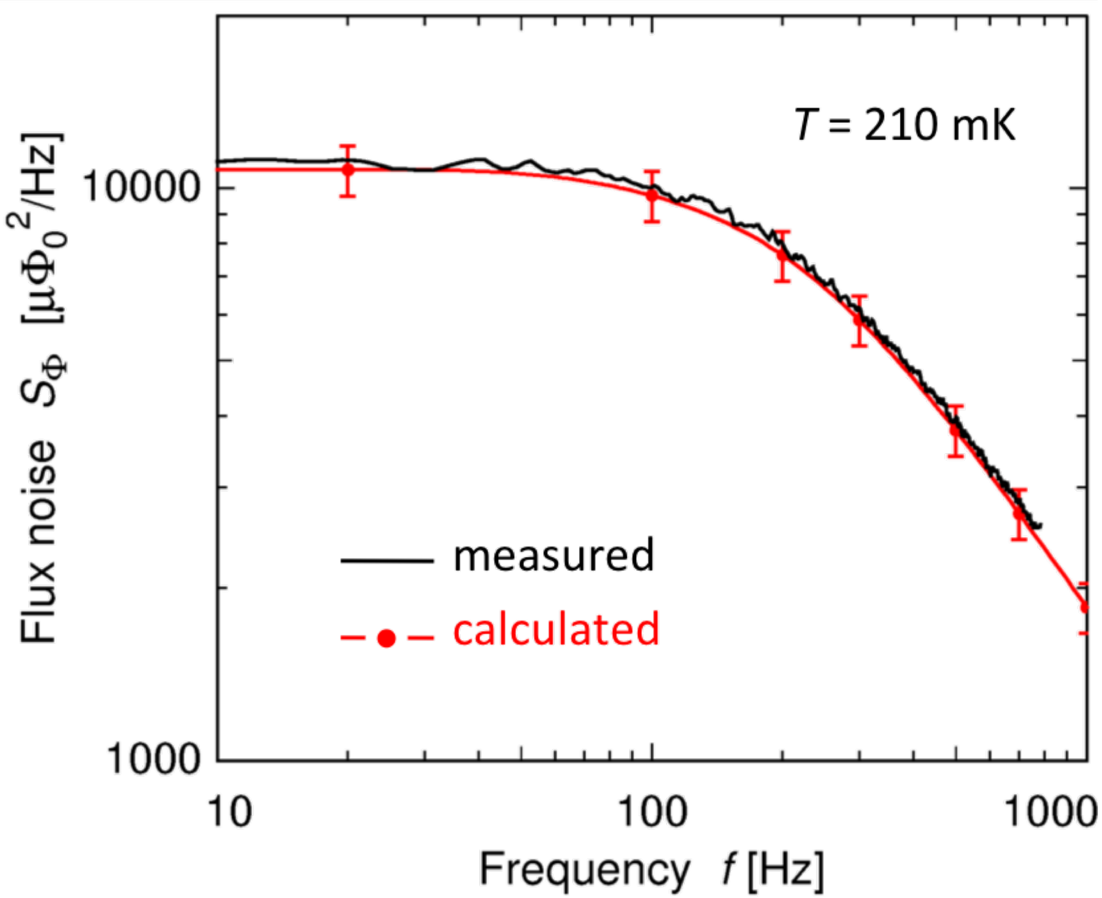
$|\mathbf{B}|$ at $f = 0$



Out-of-phase Eddy-current density (→ losses)



Thermometer with Au-cylinder, RRR = 110

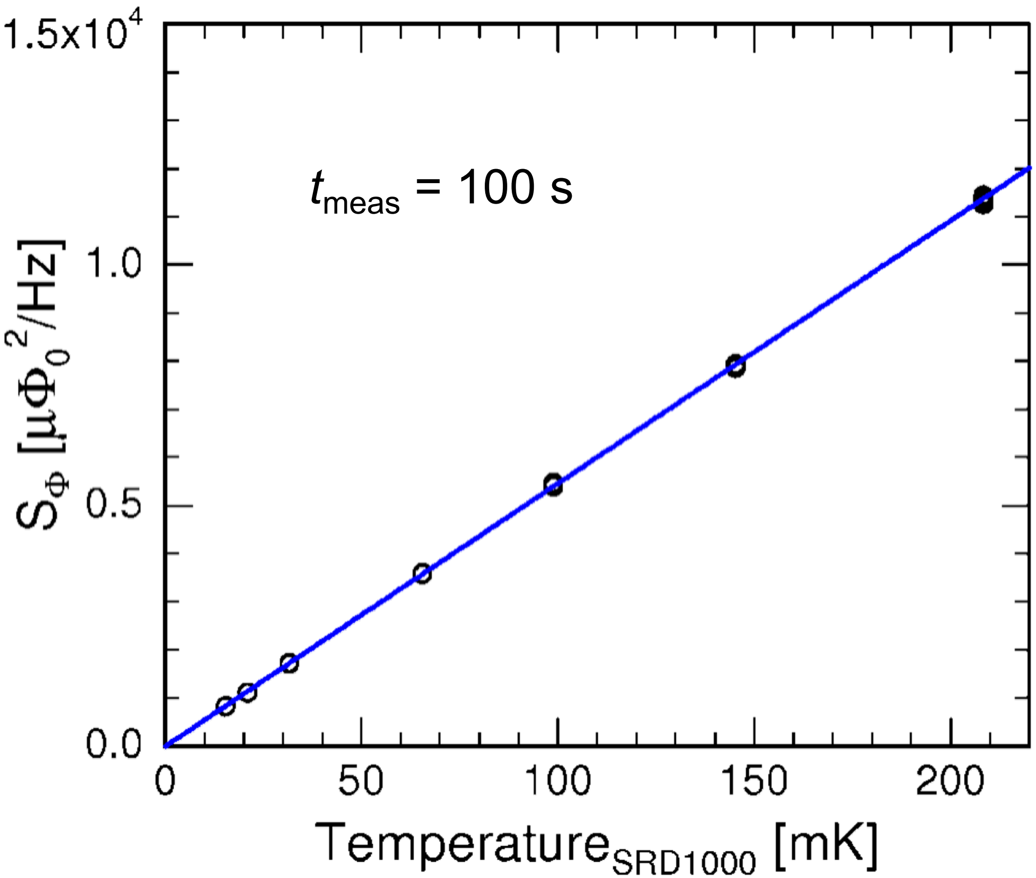


Spectral shapes agree very well !

Non-trivial shape
depends on geometry and resistivity

< 5% uncertainty
of calculated absolute values
due to stray inductances

For thermometry:
One-point calibration necessary



linear temperature dependence of noise power

$$S_\Phi \sim T$$

A. Netsch, E. Hassinger, C. E.,
A. Fleischmann, AIP **850**, 1593 (2006)

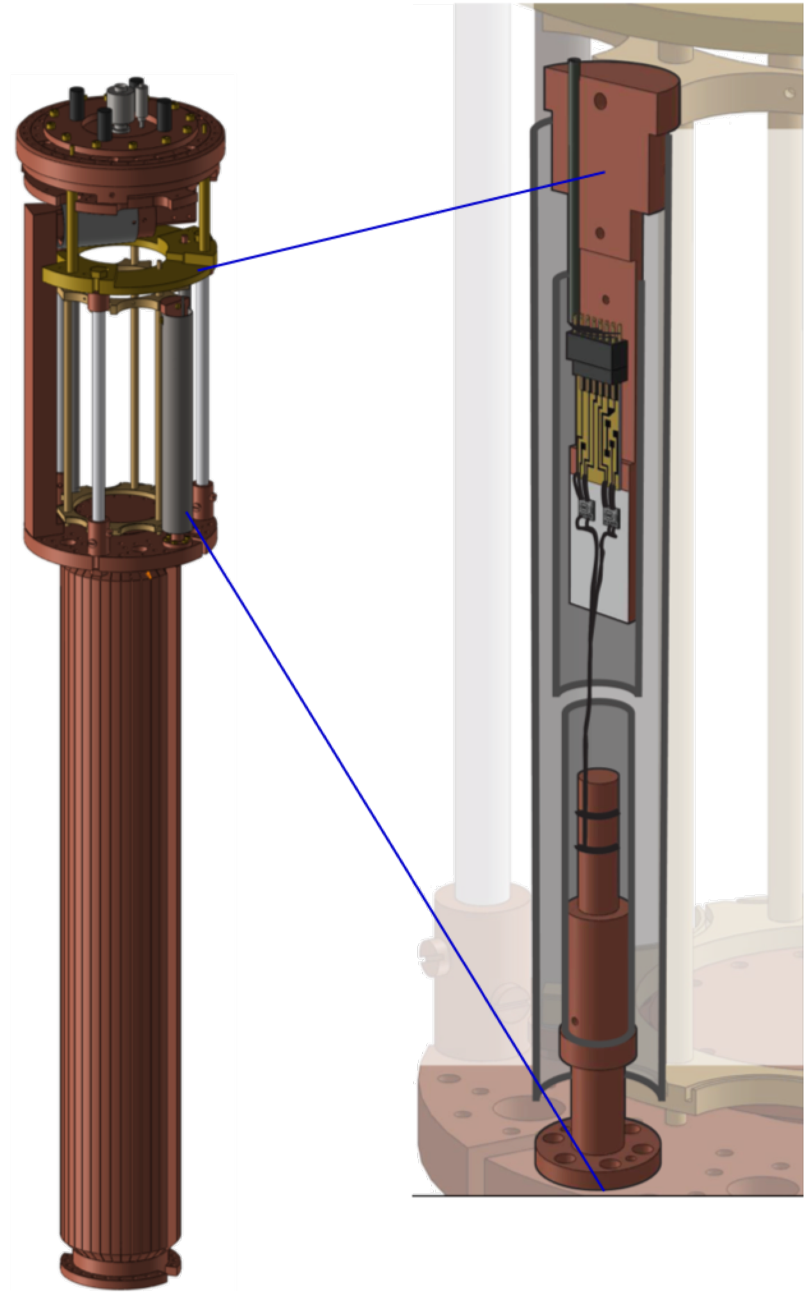
Problem:
noise amplitudes become very small

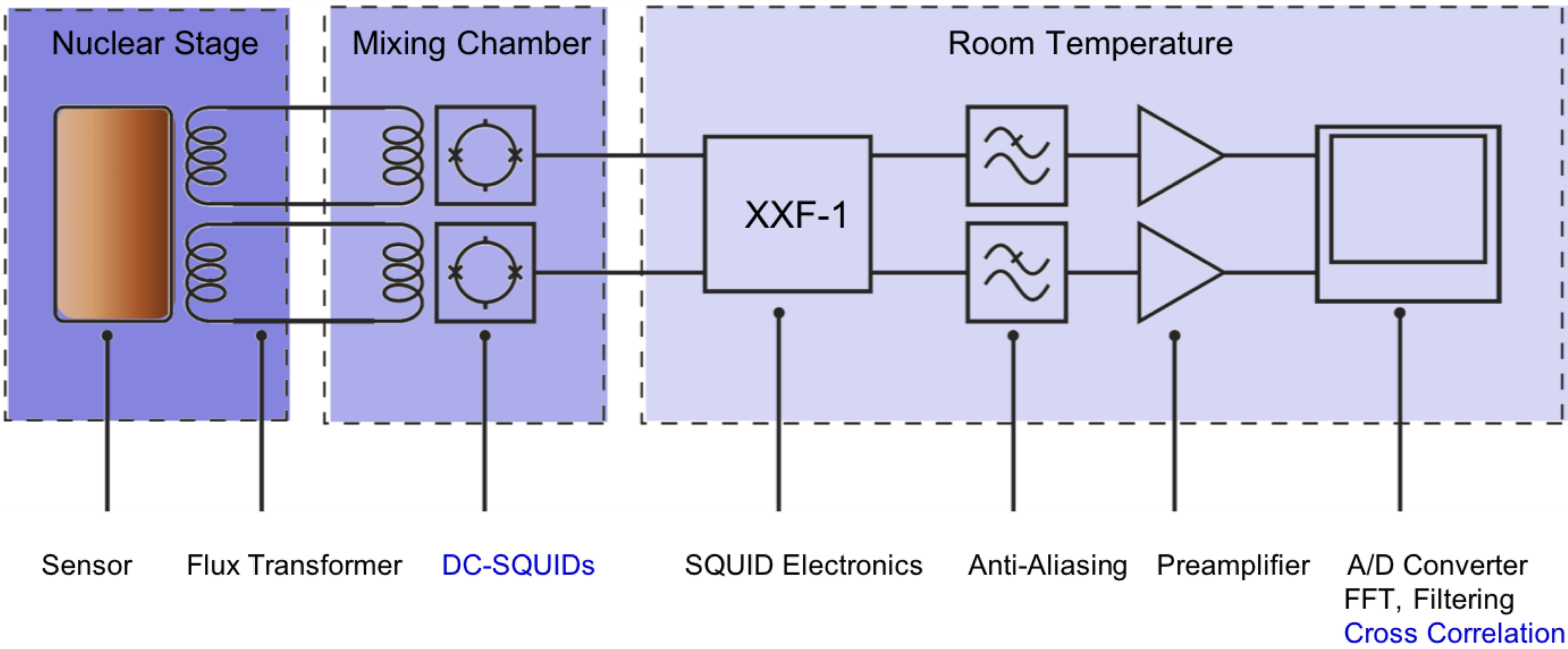
Requirement for noise source:
high conductivity → large signal
low conductivity → wide bandwidth
constant conductivity at low temperatures

Our approach:
high purity copper (5N),
free of Kondo-impurities
additional heat treatment to release hydrogen
→ very high conductivity (RRR ~1000)

optimizing the RRR by cold working

→ cut-off frequency ~ 100 Hz





D. Rothfuß, A. Reiser, A. Fleischmann, C.E.
Appl. Phys. Lett. **103**, (2013)

Cross Correlation for Noise Reduction

Channel 1: $A_1(t) = U(t) + N_1(t)$

Channel 2: $A_2(t) = U(t) + N_2(t)$

if $U(t), N_1(t), N_2(t)$
pairwise uncorrelated

Cross Correlation:

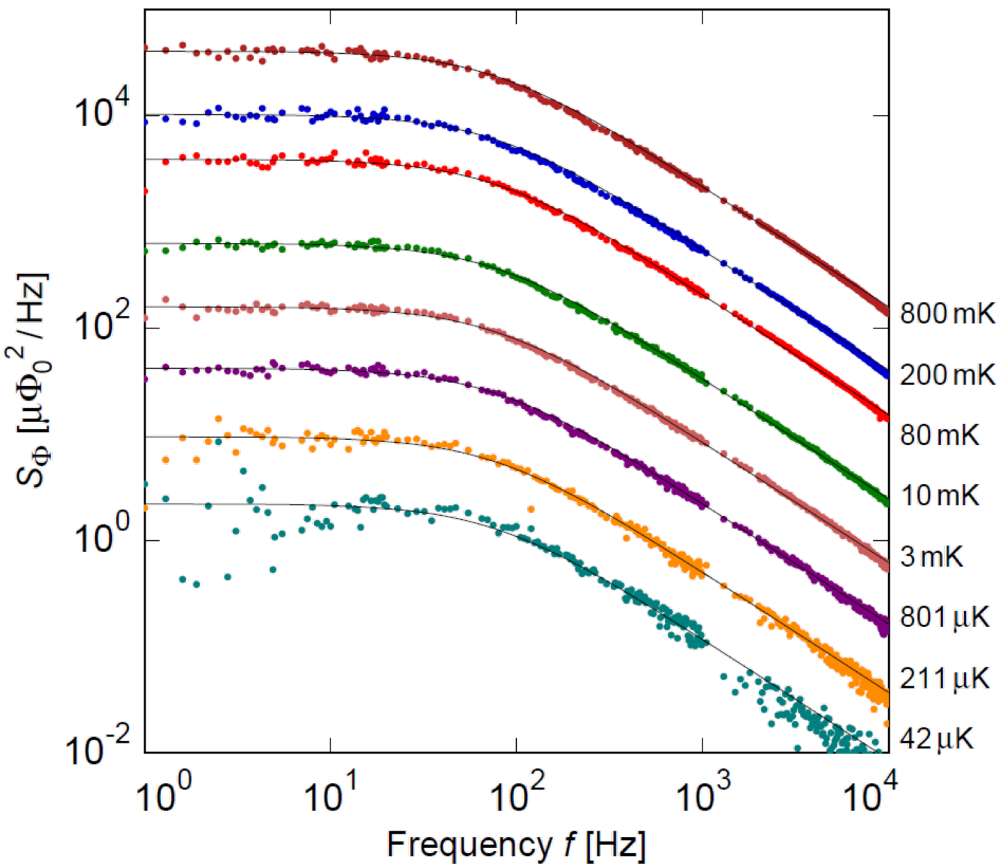
$$C(t') = \lim_{T_\mathrm{W}\to\infty} \frac{1}{T_\mathrm{W}} \int_0^{T_\mathrm{W}} A_1(t) A_2(t+t') \mathrm{d}t \overset{\downarrow}{=} R(t)$$

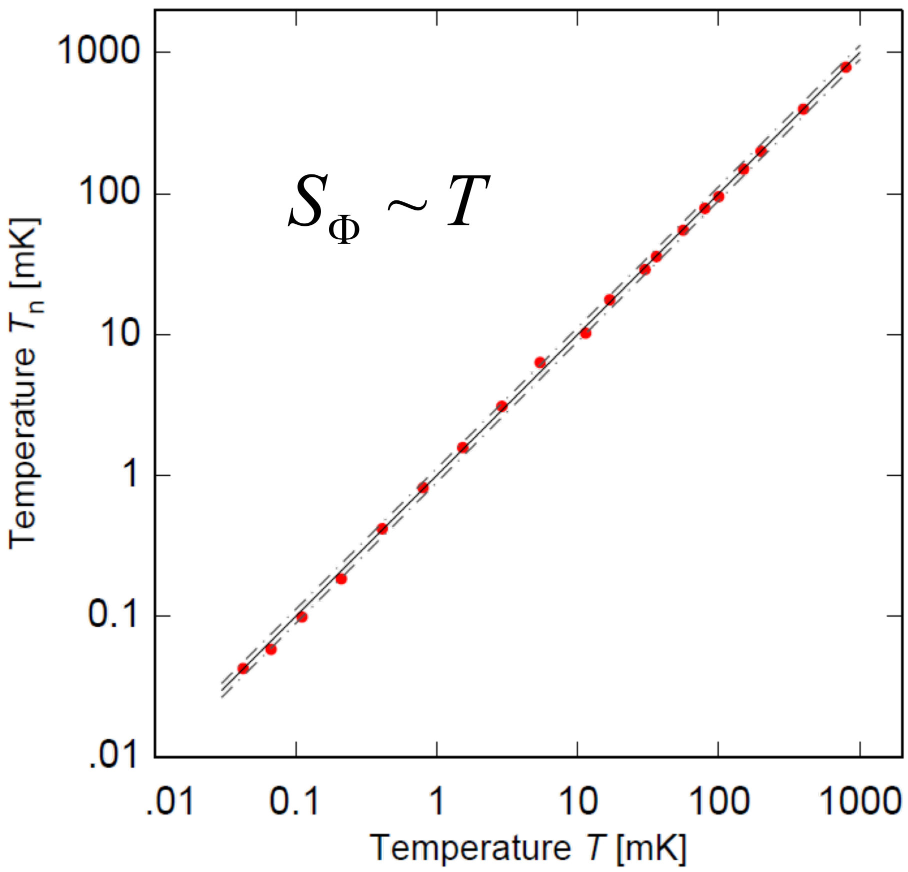
Auto Correlation

Spectral Power Density via Wiener-Khinchin Theorem

$$S(\omega) = 2 \int_{-\infty}^{\infty} R(t)\, \mathrm{e}^{-i\omega t} \mathrm{d}t$$

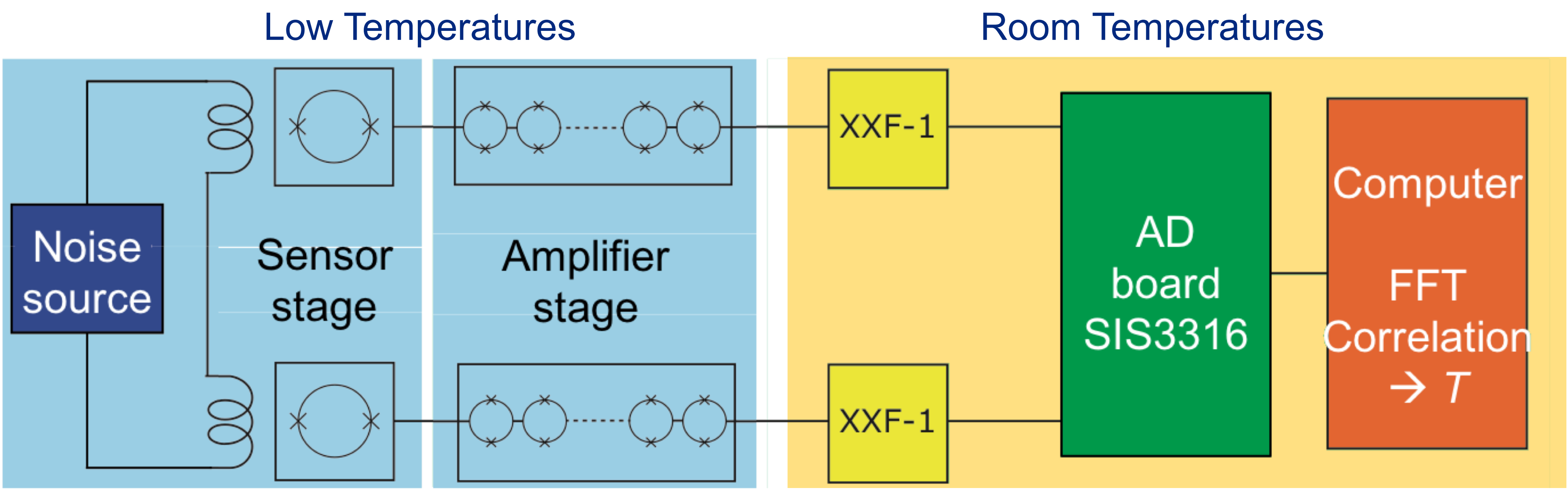
Spectral Power Density: After Cross Correlation



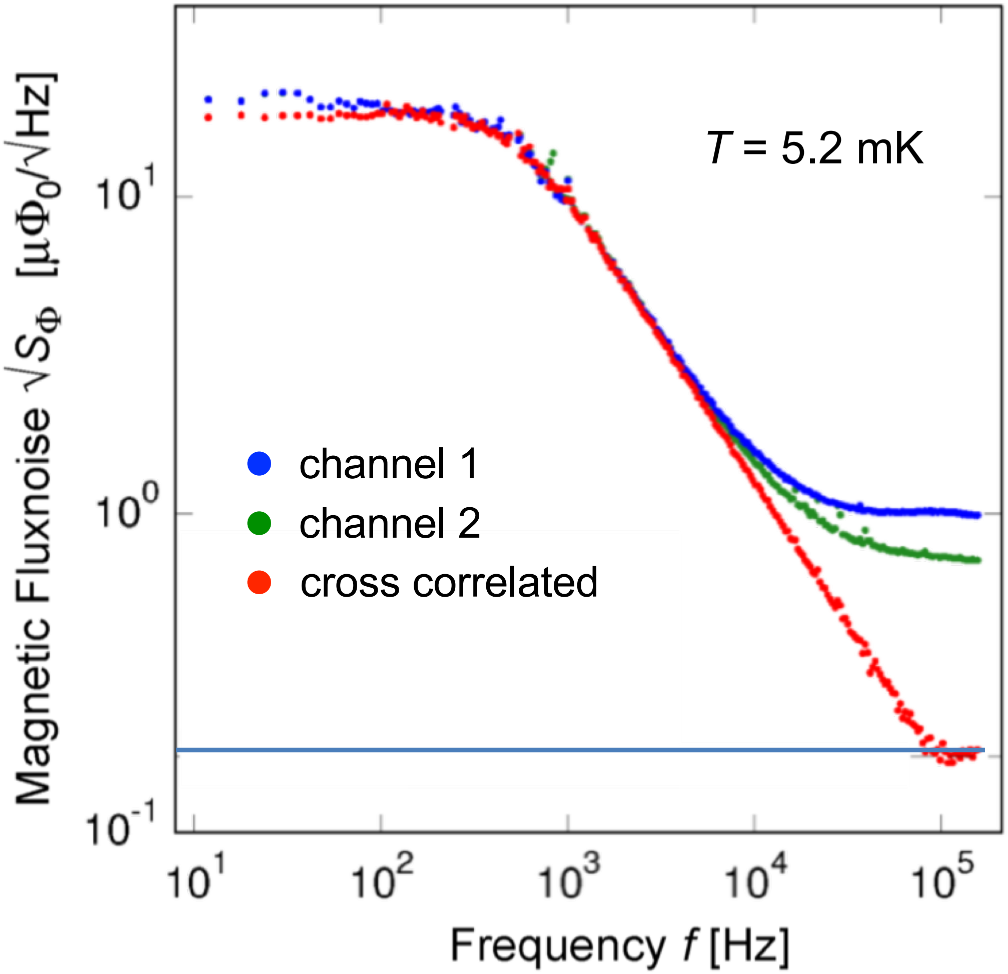


D. Rothfuß, A. Reiser, A. Fleischmann, C.E.
Appl. Phys. Lett. **103**, (2013)

Double Channel and Double Stage Readout



M. Hempel, C. Ständer, A. Fleischmann, S. Kempf, C. E. to be published

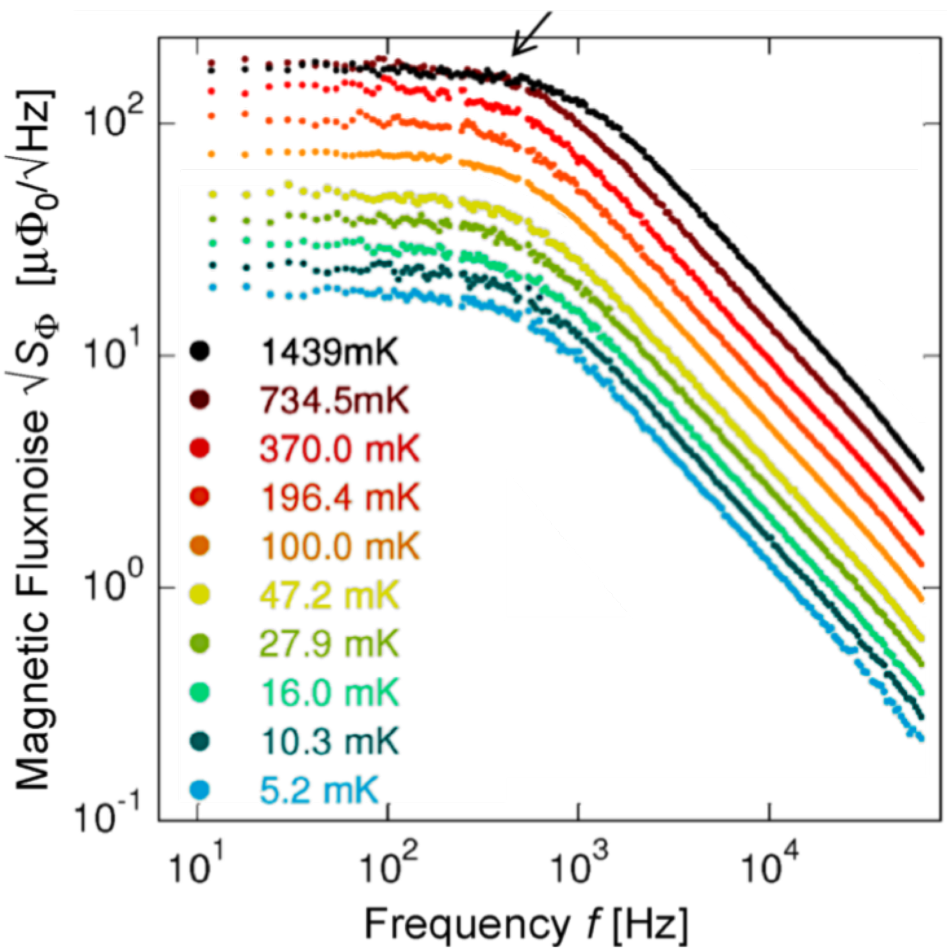


0.5 µK

C. Ständer, Bachelor Thesis 2018

Performance of C3NT

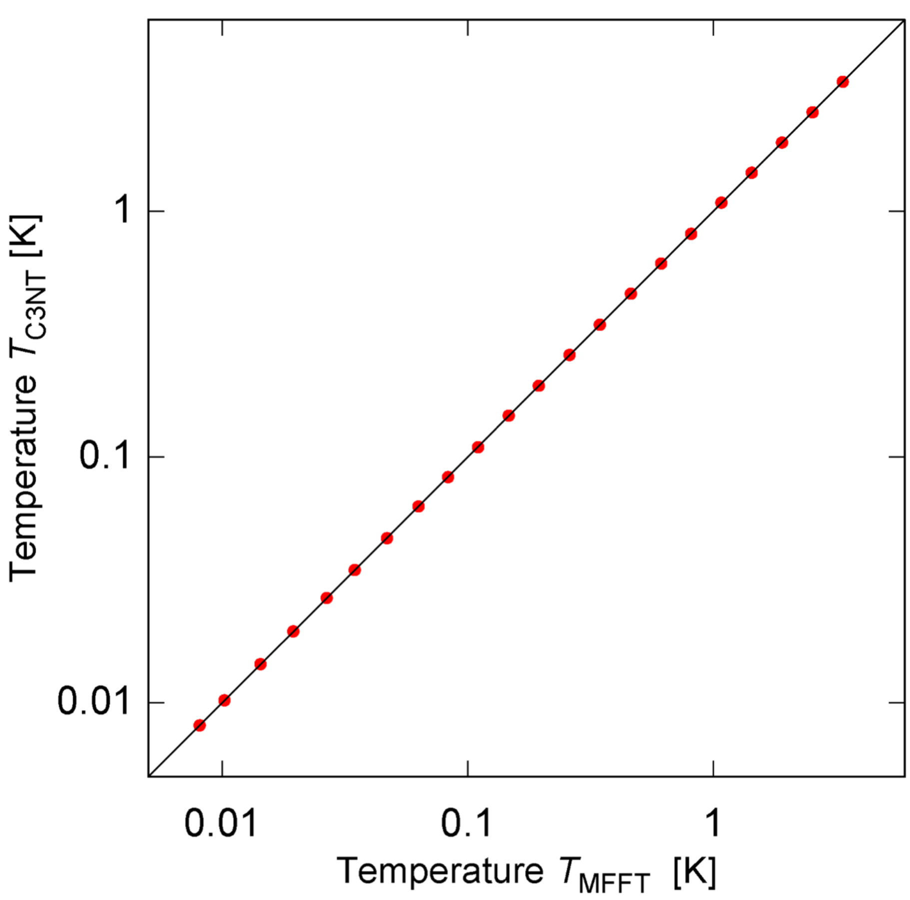
Spectral form changes because of Al bond wires

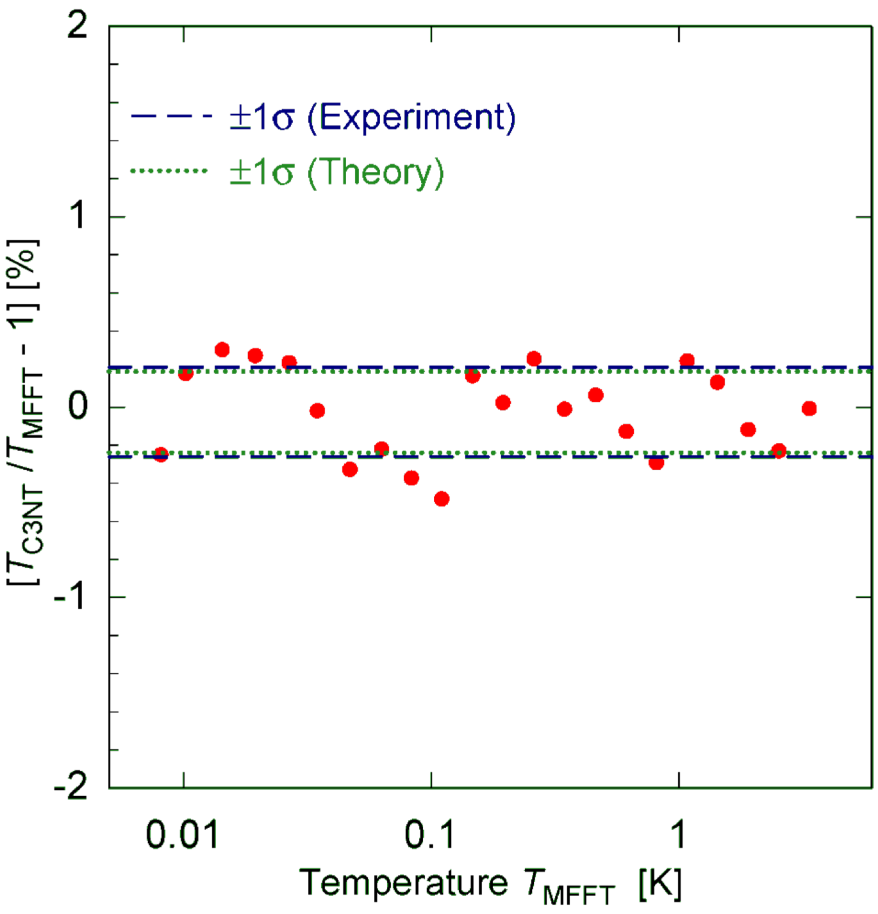


- no change of spectral form below 800 mK and for frequencies up to 10^5 Hz
- measured at 40 different temperatures

C. Ständer, Bachelor Thesis 2018

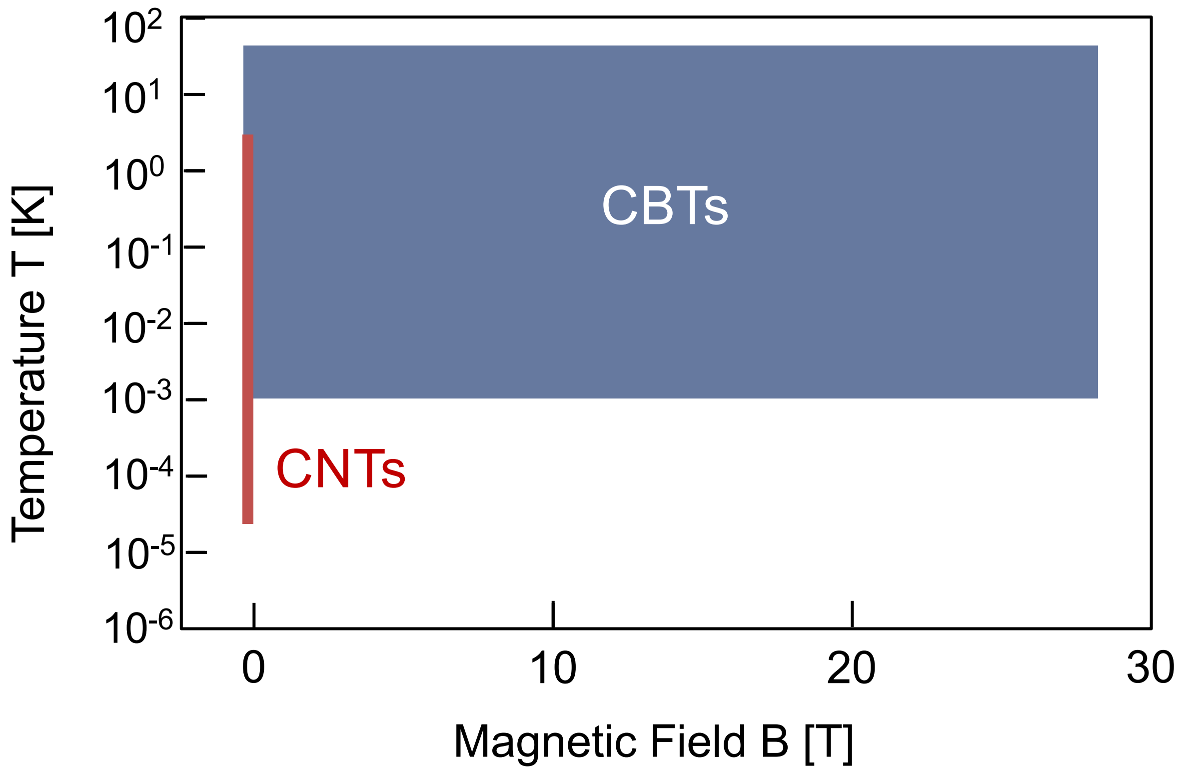
C3NT vs MFFT

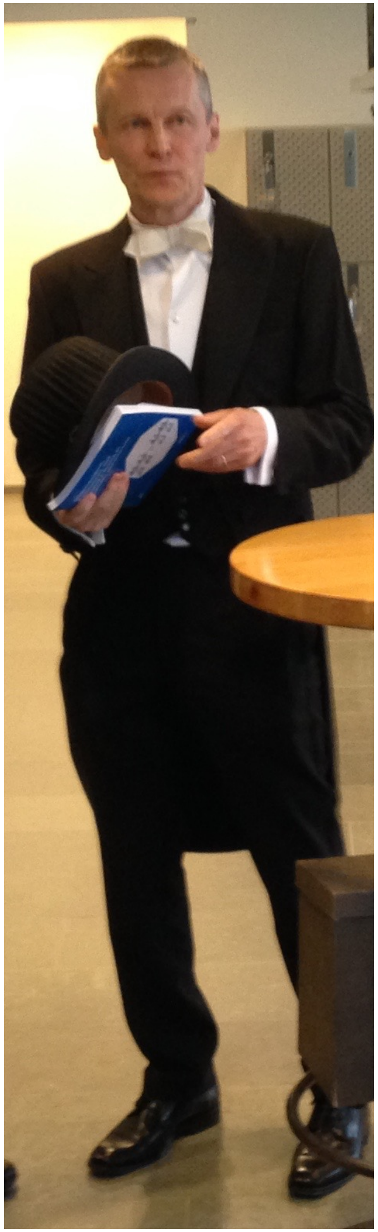




- Agreement between thermometers better than 0.3%
- Deviations purely of statistical nature

M. Hempel, C. Ständer, A. Fleischmann,
S. Kempf, C. E. to be published





Happy Birthday Jukka!