Unexpected links between Stochastic Thermodynamics and criticality at Anderson localization transition

Ivan M. Khaymovich *MPIPKS, Dresden, Germany*

Academy Prof. Jukka P. Pekola
PICO group, Aalto University

Olli-Pentti Saira
Caltech

Jonne V. Koski
ETH Zurich

Prof. Vladimir Kravtsov
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

The Abdus Salam International Centre for Theoretical Physics

QT60 – Workshop on thermodynamics, thermoelectrics and transport in quantum devices, 21 September 2018

My interaction with Jukka

- **<u>2013</u> - 1st talk in PICO group meeting «Bochkov-Kuzovlev and Jarzynski equalities»**
- **<u>Oct 2013 – Oct 2015</u> – PostDoc in PICO group:**
  - ✓ ***5 common publications:***
    - purely theoretical,
    - experimentally realized theoretical ideas,
    - theoretically explained experiments;

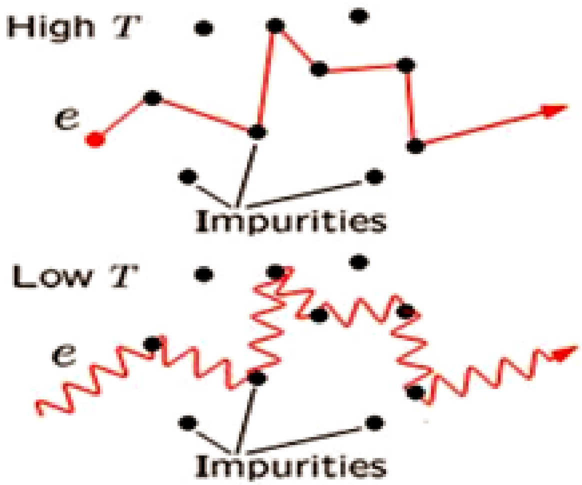
    spread from stochastic thermodynamics, e-pumping, and thermometry to <u>multifractality.</u>
  - ✓ ***Organization of Arctic School (Kilpisjarvi) and 1st Workshop on NonEq Th/dyn and MesoPhys***
  - ✓ ***Common graduate lecture course in Aalto Uni* (+ Erik and Paolo)**
- **<u>2016-now</u> - Further collaboration:**
  - ✓ ***+6 common publications*** *(last is since yesterday in arxiv)*
  - ✓ ***Frequent mutual visits: ~2 times a year***

Outline

- **Introduction:**
  - ✓ Anderson localization transition
  - ✓ Critical wavefunctions
  - ✓ Fluctuation relations in single-electronics
- **Analogy between distributions of dissipated work and multifractal wave functions**
  - ✓ Formal similarity: adiabatic drive and ergodicity
  - ✓ Correspondence and consequences
- **Summary**

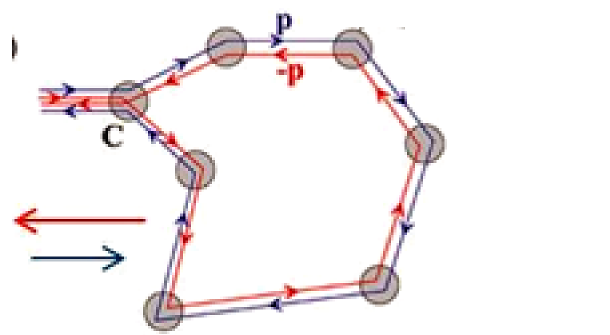
Anderson localization transition

***N* sites in the sample**



Localization transition

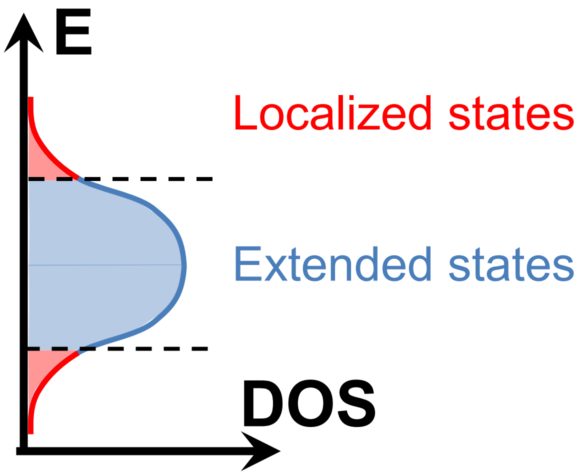
$$\left|\sum_i A_i\right|^2 = \sum_i |A_i|^2 + 2\mathrm{Re}\sum_{i,j} A_i A_j^*$$

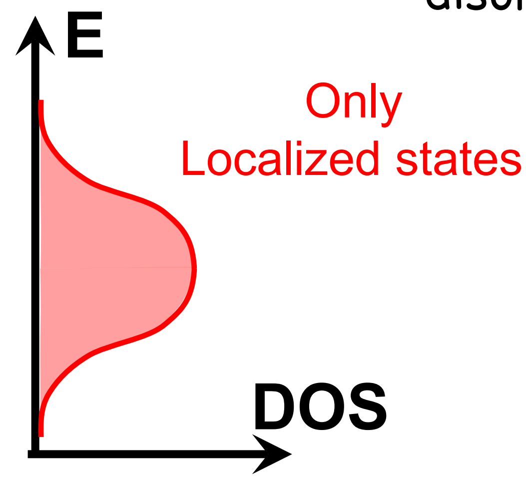


opposing interfering trajectories

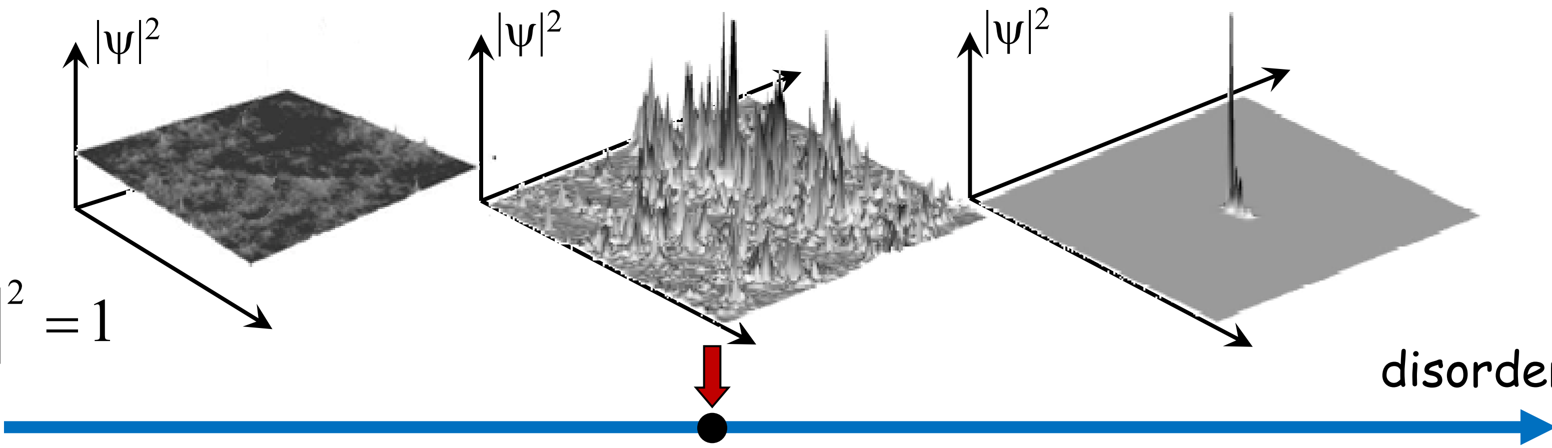
constructive interference of backscattering

disorder



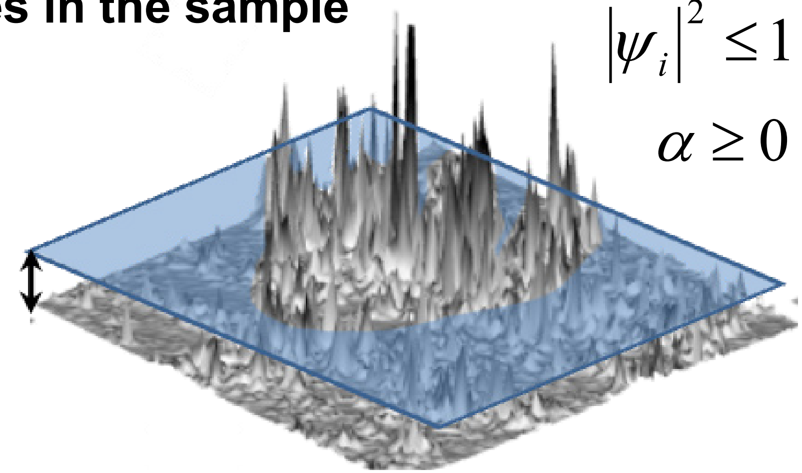


Anderson localization transition and multifractality



$$\sum_{i=1}^{N} |\psi_i|^2 = 1$$

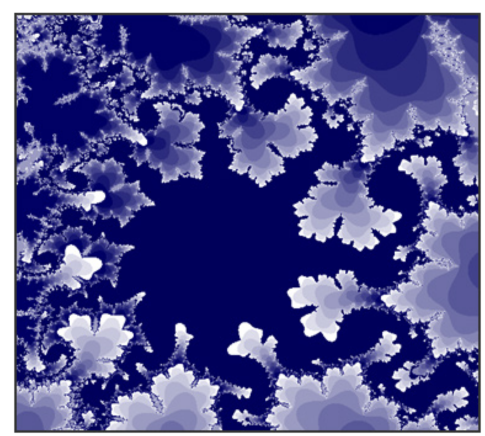
***N* sites in the sample**



$$|\psi_i|^2 \leq 1$$

$$\alpha \geq 0$$

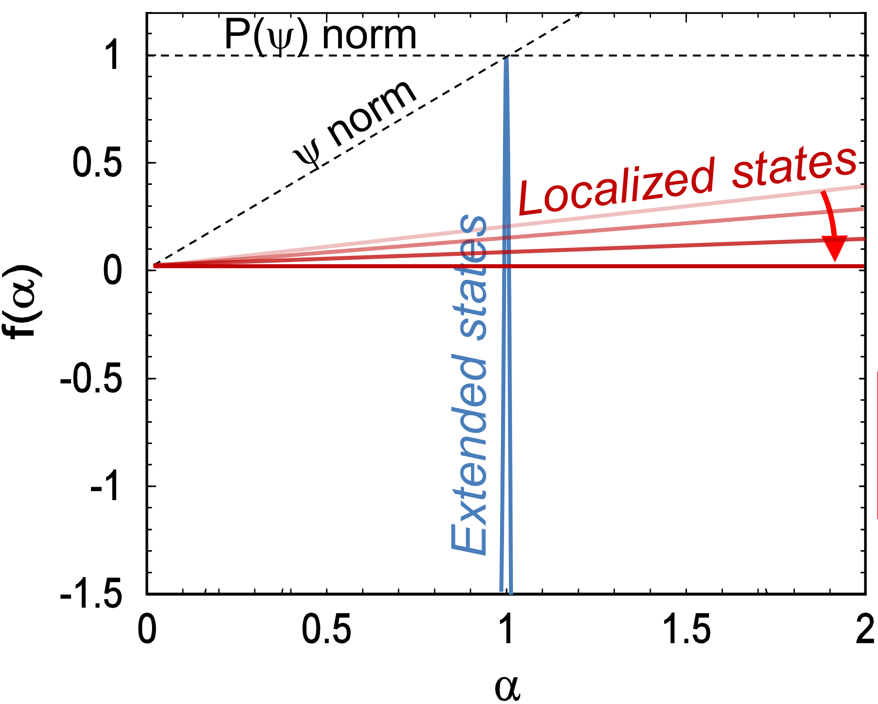
Map of the regions with amplitude larger than the chosen level



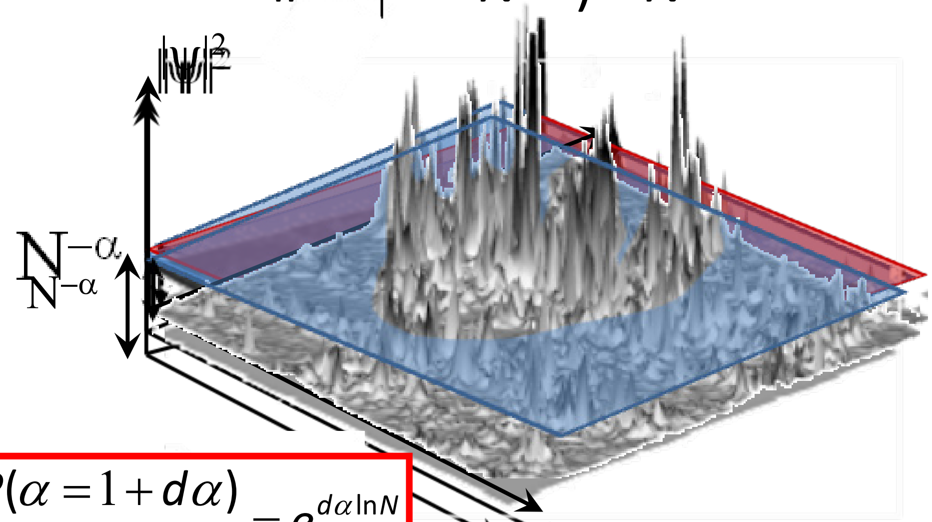
V. Kravtsov "Multifractality of wave functions: basic concepts and experiment", lecture in Moscow, Dec 2011

Spectrum of fractal dimensions

Evers, Mirlin RMP **80**, 1355 (2008)



$$\#(|\psi|^2 \propto N^{-\alpha}) \sim N^{f(\alpha)}$$



$$\frac{P(\alpha = 1 + d\alpha)}{P(\tilde{\alpha} = 1 - d\alpha)} = e^{d\alpha \ln N}$$

$$d|\psi|^2 \sim N^{f(\alpha)-1} d\alpha$$

Exact symmetry

$$\boxed{f(\alpha)} = f(2-\alpha) + \alpha - 1$$

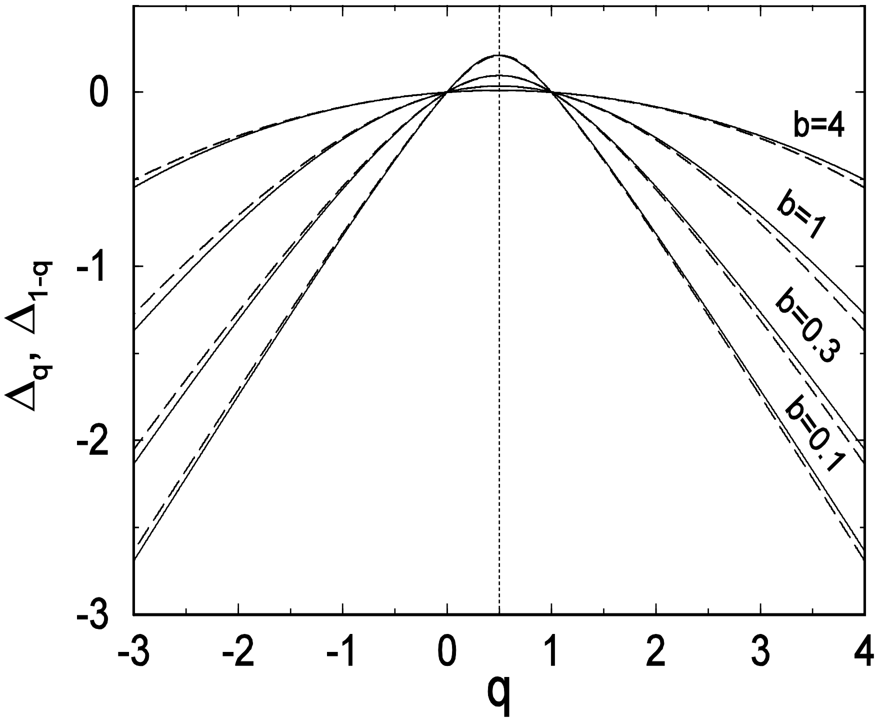
$$0 \le \alpha \le 2$$

Mirlin et al PRL **97**, 046803 (2006)

$$|\psi|^2 = N^{-\alpha} \Longleftrightarrow \alpha = -\ln|\psi|^2 / \ln N$$

$$|\psi_i|^2 \le 1 \Longleftrightarrow \alpha \ge 0$$

Wavefunction moments

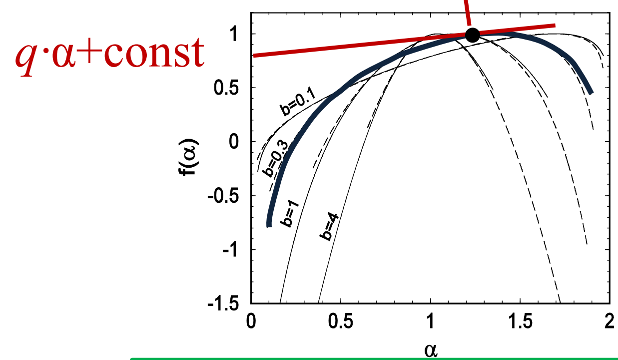


$$q=0:\ \frac{1}{N}\sum_{i=1}^{N}1=1;\quad q=1:\ \sum_{i=1}^{N}|\psi_i|^2=1$$

$$|\psi|^2=N^{-\alpha}$$

$$\boxed{q-1+\Delta_q=\min_{\alpha}[q\alpha-f(\alpha)]}$$

$$\left\langle \sum_i |\psi_i|^{2q}\right\rangle \sim \int N^{f(\alpha)}N^{-q\alpha}d\alpha \sim N^{-(q-1+\Delta_q)}$$



Exact symmetry

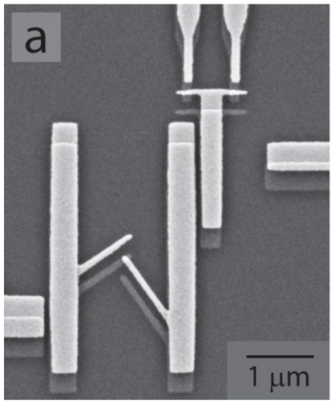
$$f(\alpha)=f(2-\alpha)+\alpha-1$$

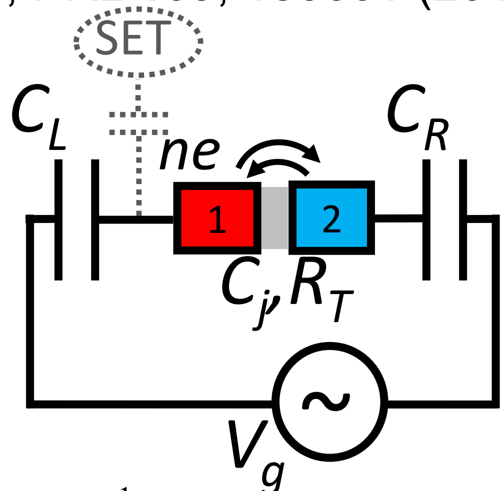
$$\boxed{\Delta_q}=\Delta_{1-q}$$

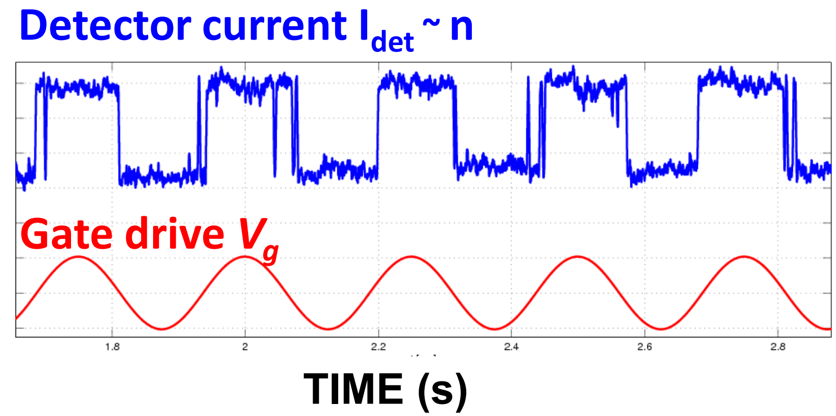
Mirlin et al PRL **97**, 046803 (2006)

Work distribution in driven SEB

O.-P. Saira et al., PRL **109**, 180601 (2012); J.V. Koski et al., Nat. Phys. **9**, 644 (2013).



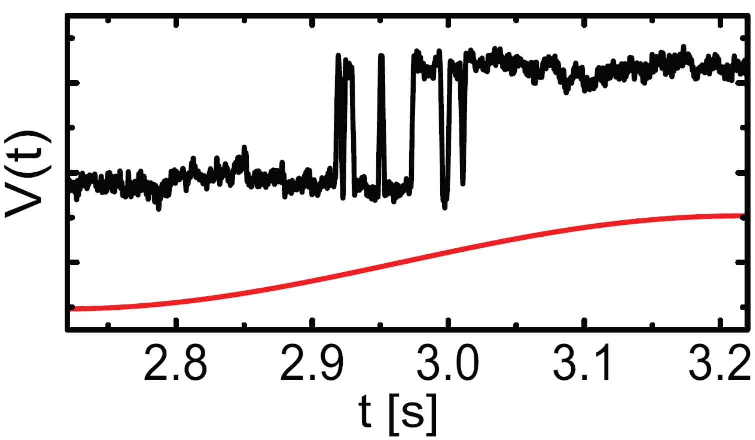


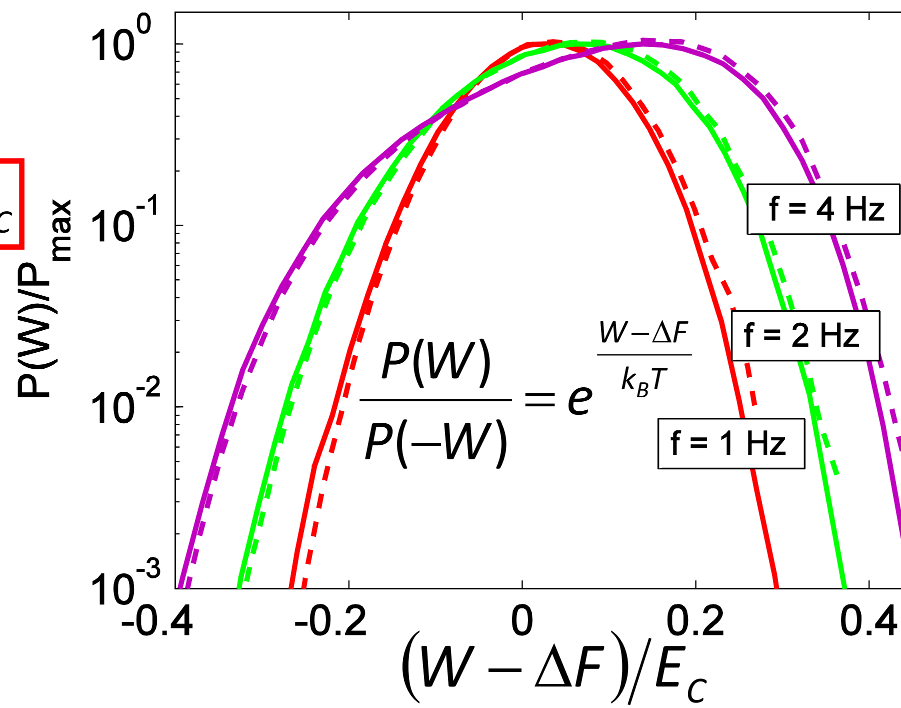


$$W-\Delta F=-2E_C\int_0^1 (n_g-n)\,dn_g$$

Low temperature limit

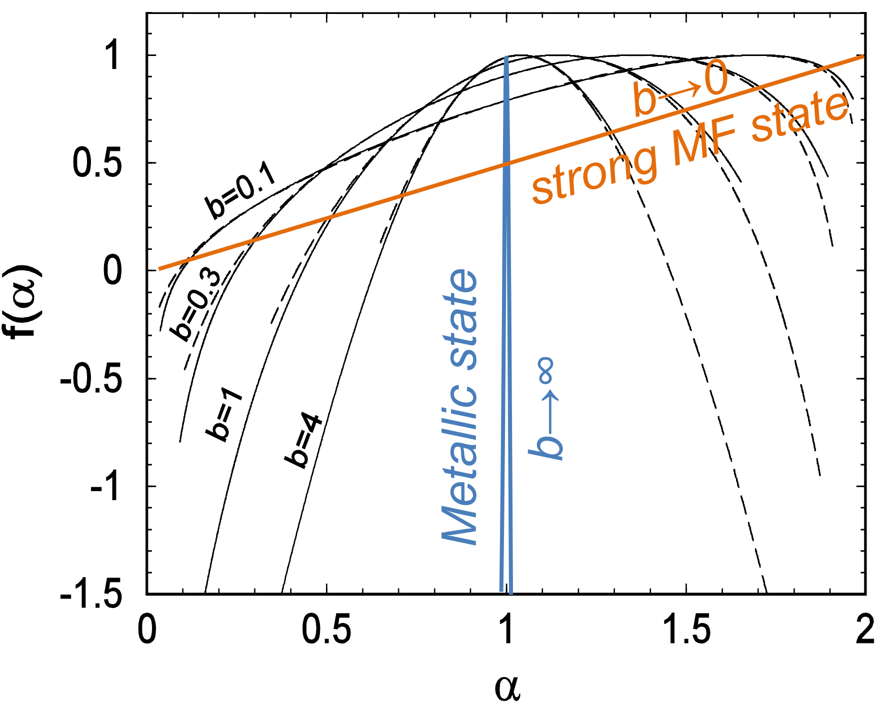
$E_C >> k_B T \quad n=0,1 \quad \boxed{|W-\Delta F|\le E_C}$





Multifractal spectrum and work distribution

Evers, Mirlin RMP **80**, 1355 (2008)



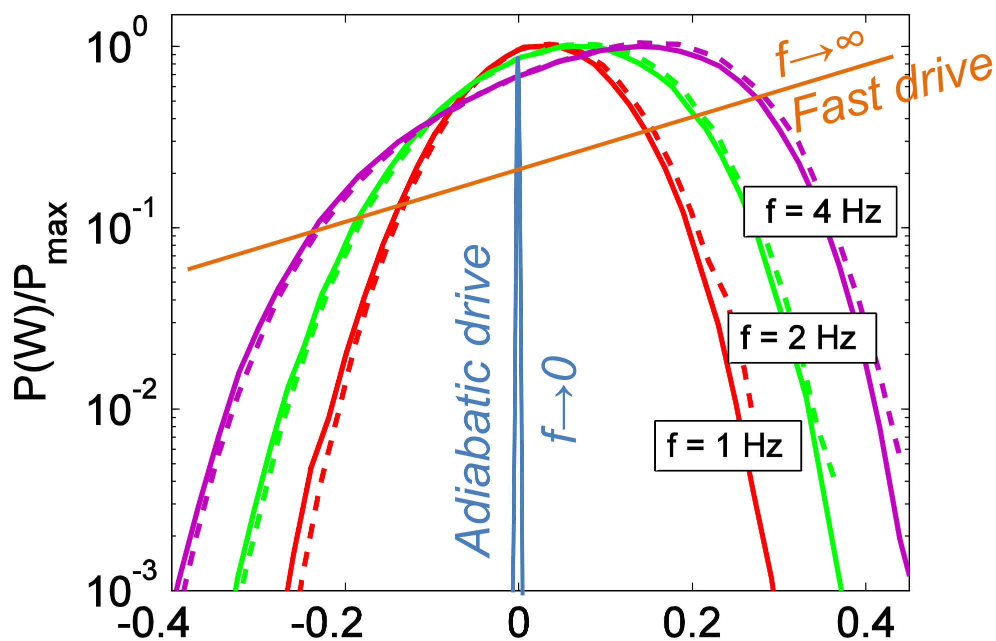
$$P(|\psi|^2)d|\psi|^2 \sim N^{f(\alpha)-1}d\alpha$$

$$|\psi|^2 = N^{-\alpha} \Longleftrightarrow \alpha = -\ln|\psi|^2 / \ln N$$

$$0 \le \alpha \le 2$$

Nat. Comm. **6**, 7010 (2015)

O.-P. Saira et. al. PRL **109** 180601 (2012)



$$(W - \Delta F)/E_C$$

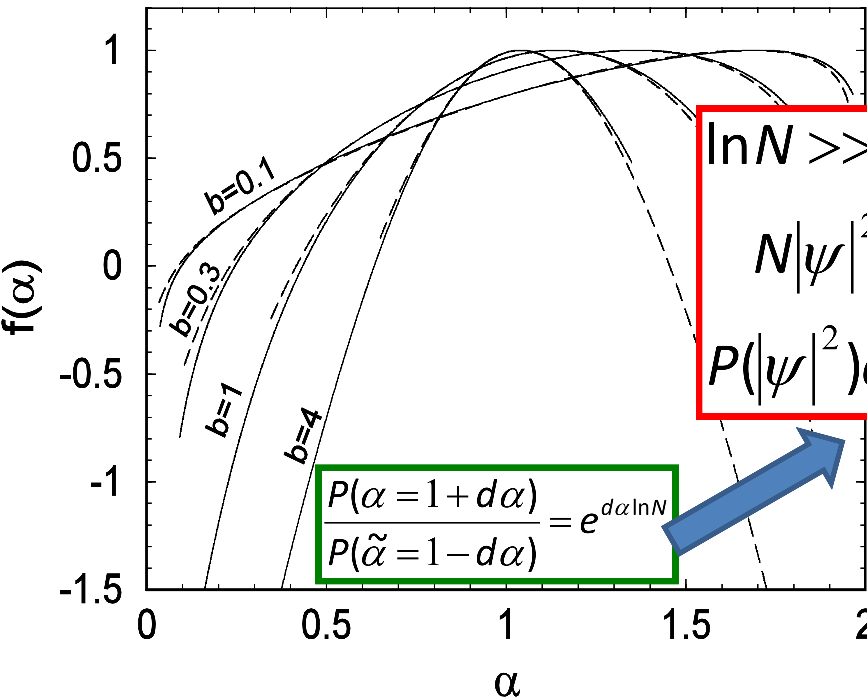
$$y = (W - \Delta F)/(n_w k_B T)$$

$$-E_C \le W - \Delta F \le E_C$$

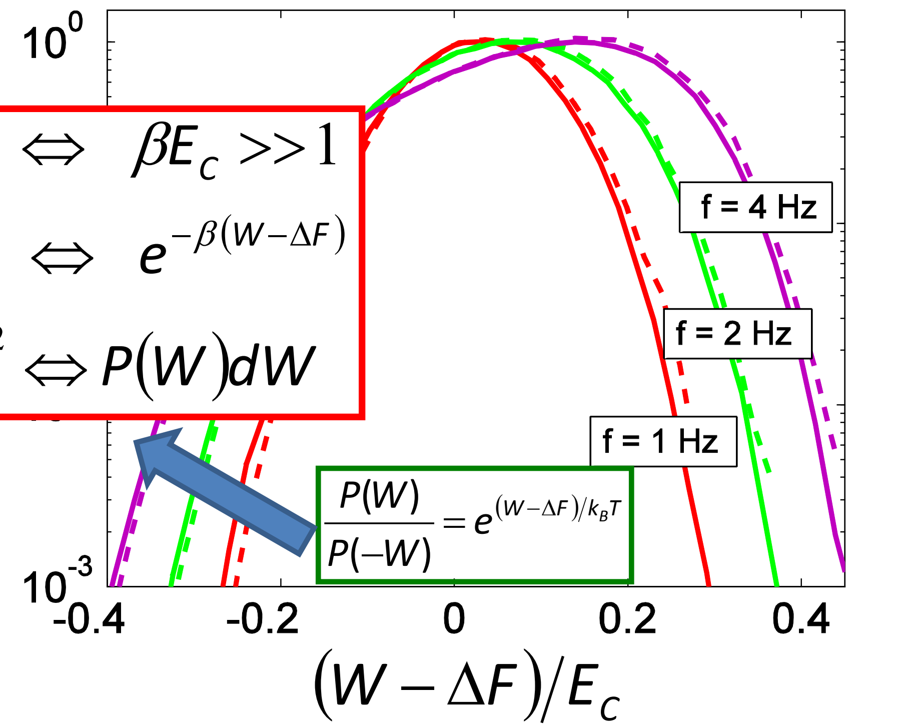
Multifractal spectrum and work distribution

Evers, Mirlin RMP **80**, 1355 (2008)

O.-P. Saira et. al. PRL **109** 180601 (2012)



$$\frac{P(\alpha = 1 + d\alpha)}{P(\tilde{\alpha} = 1 - d\alpha)} = e^{d\alpha \ln N}$$



$$\frac{P(W)}{P(-W)} = e^{(W-\Delta F)/k_B T}$$

$$\ln N >> 1 \quad \Leftrightarrow \quad \beta E_C >> 1$$

$$N|\psi|^2 \quad \Leftrightarrow \quad e^{-\beta(W-\Delta F)}$$

$$P(|\psi|^2)d|\psi|^2 \Leftrightarrow P(W)dW$$

$$P(|\psi|^2)d|\psi|^2 \sim N^{f(\alpha)-1}d\alpha$$

$$|\psi|^2 = N^{-\alpha} \quad \Leftrightarrow \quad \alpha = -\ln|\psi|^2 / \ln N$$

$$0 \le \alpha \le 2$$

$$d\alpha = \left(W - \Delta F\right)/E_C$$

$$-E_C \le W - \Delta F \le E_C$$

$$0 \le d\alpha + 1 \le 2$$

Nat. Comm. **6**, 7010 (2015)

Multifractal spectrum and work distribution

Multifractality theory

Evers, Mirlin

RMP **80**, 1355 (2008)

$$\ln N \to \infty$$

$$N|\psi|^2$$

$$f(\alpha)$$

Mirlin-Fyodorov symmetry

$$f(\alpha) = f(2-\alpha) + \alpha - 1$$

$$0 \le \alpha \le 2$$

$$N^{q-1}\left\langle \sum_i |\psi_i|^{2q} \right\rangle = \left\langle \left(N|\psi|^2\right)^q \right\rangle \sim N^{-\Delta_q}$$

Work statistics

O.-P. Saira et. al.

PRL **109** 180601 (2012)

$$\beta E_C >> 1$$

$$e^{-\beta(W-\Delta F)}$$

$$1 + \ln\left[P(W)/P_{\max}\right]/\beta E_C$$

Crooks relation

$$\ln P(W) = \ln P(-W) + \beta(W - \Delta F)$$

$$-E_C \le W - \Delta F \le E_C$$

$$\left\langle e^{-q\beta(W-\Delta F)} \right\rangle_\beta = e^{-\beta E_C \cdot \Delta_q^w(T)}$$

$$\beta E_C \to \infty$$

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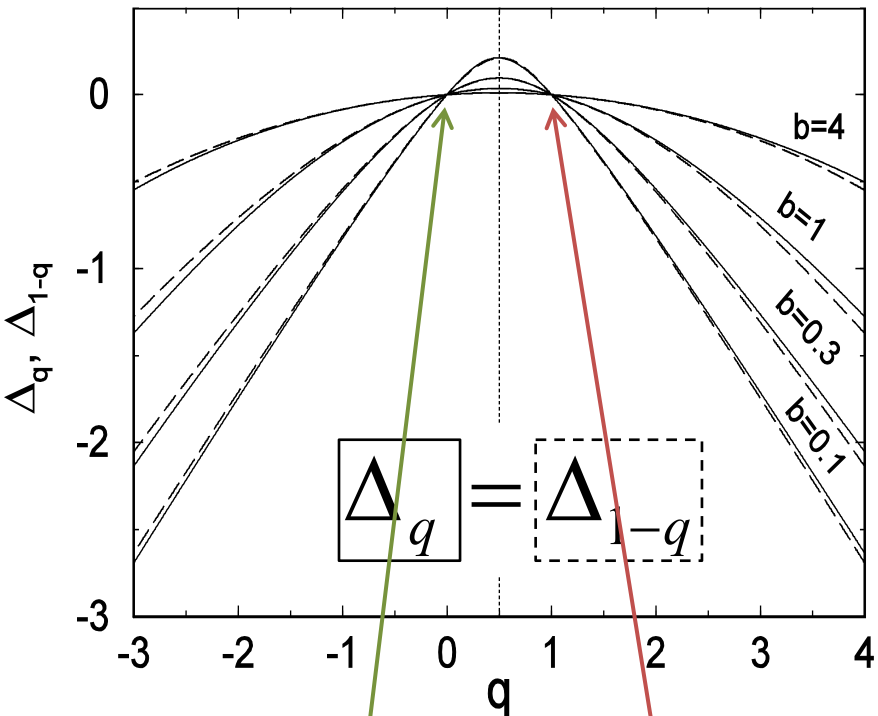
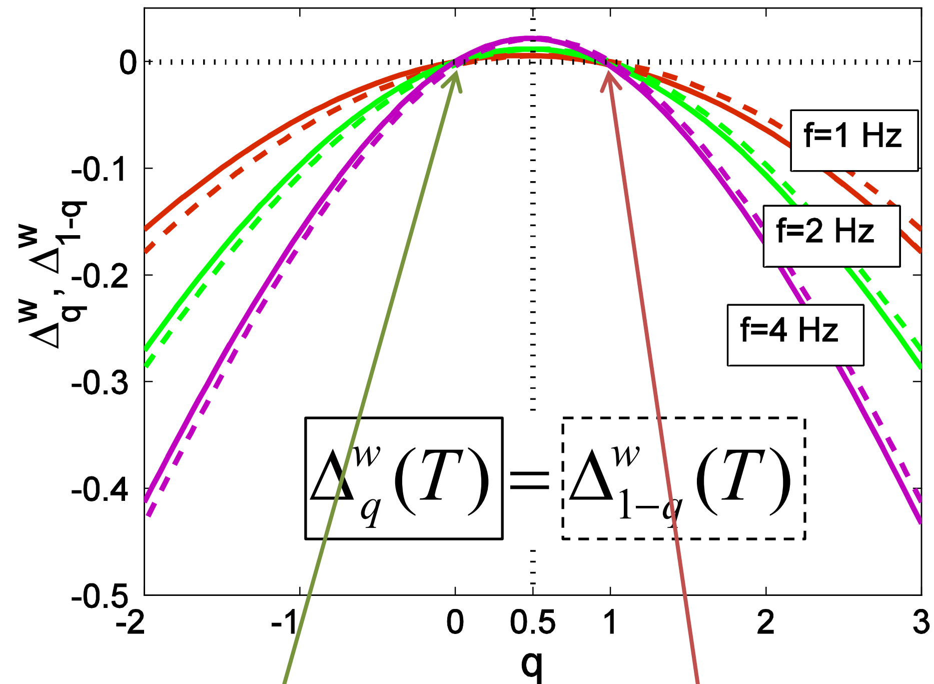
$$\boxed{\Delta_q^w(T) \equiv -\frac{\ln\left\langle e^{-q\beta(W-\Delta F)}\right\rangle}{\beta E_C} \to \Delta_q^w}$$

Moments of wavefunctions and work generating function





$$\left\langle \left(N|\psi|\right)^{2q} \right\rangle \sim N^{-\Delta_q}$$

$$\frac{1}{N}\sum_{i=1}^{N} 1 = 1; \quad \sum_{i=1}^{N} |\psi_i|^2 = 1$$

Normalization and Jarzynski equality

$$\langle 1 \rangle = 1, \quad \left\langle e^{-\beta(W-\Delta F)} \right\rangle = 1$$

$$\Delta_q^w(T) = -\ln\left\langle e^{-q\beta(W-\Delta F)} \right\rangle \Big/ \beta E_C$$

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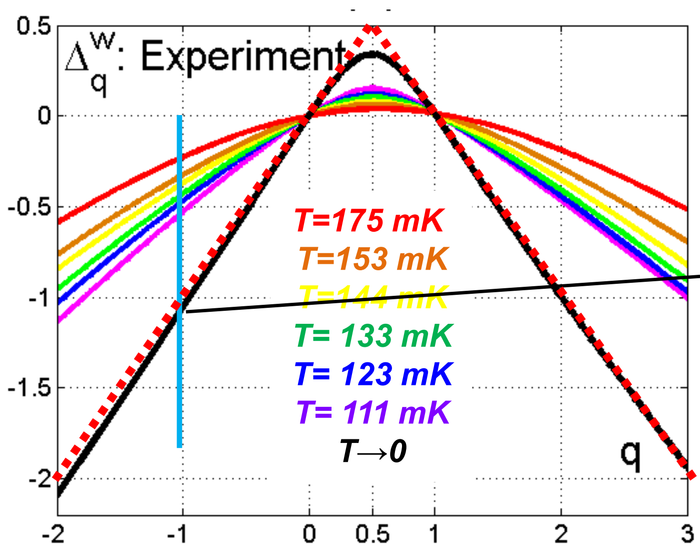
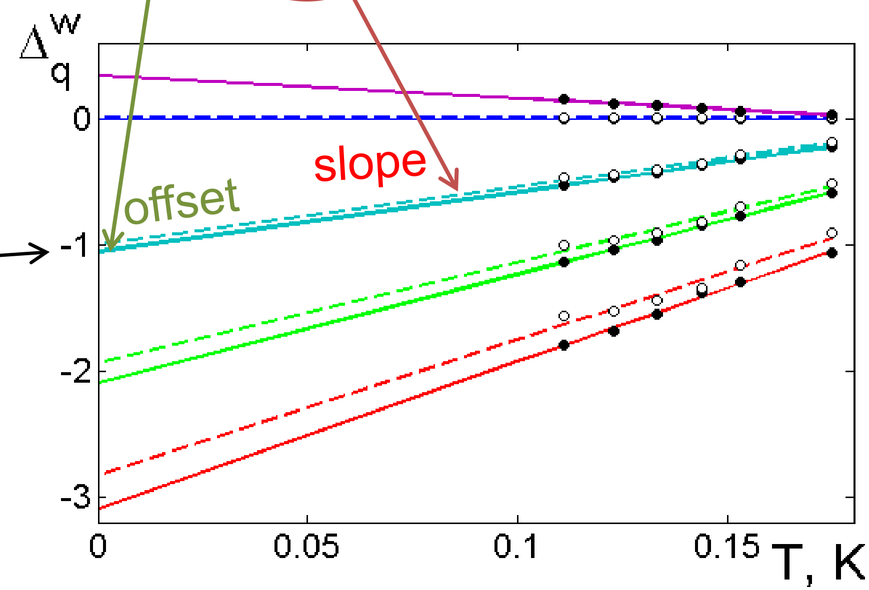
Results

Experiment $\Delta_q^w(T) = -\frac{k_B T}{E_C}\ln\left\langle e^{q\beta(\Delta F - W)}\right\rangle = \Delta_q^w + \frac{k_B T}{E_C}\ln c_q^0 \rightarrow \Delta_q^w$ **Theory**





1. Generalization of Jarzynski equality

$$\left\langle e^{(\Delta F - W)/T}\right\rangle = e^{-\beta E_C \Delta_1^w} = 1$$

$$\left\langle e^{q\beta(\Delta F - W)}\right\rangle = c_q(T) e^{-\beta E_C \Delta_q^w}$$

$$\boxed{\Delta_q^w(T)} = \boxed{\Delta_{1-q}^w(T)}$$

3. Universal linear behavior for q<0 or q>1 for any system

$$\boxed{\Delta_q^w \approx \tfrac{1}{2} - \left|q - \tfrac{1}{2}\right| + O\left(\frac{T}{E_C}\right)}$$

Theoretical result T→0
q>1 or q<0

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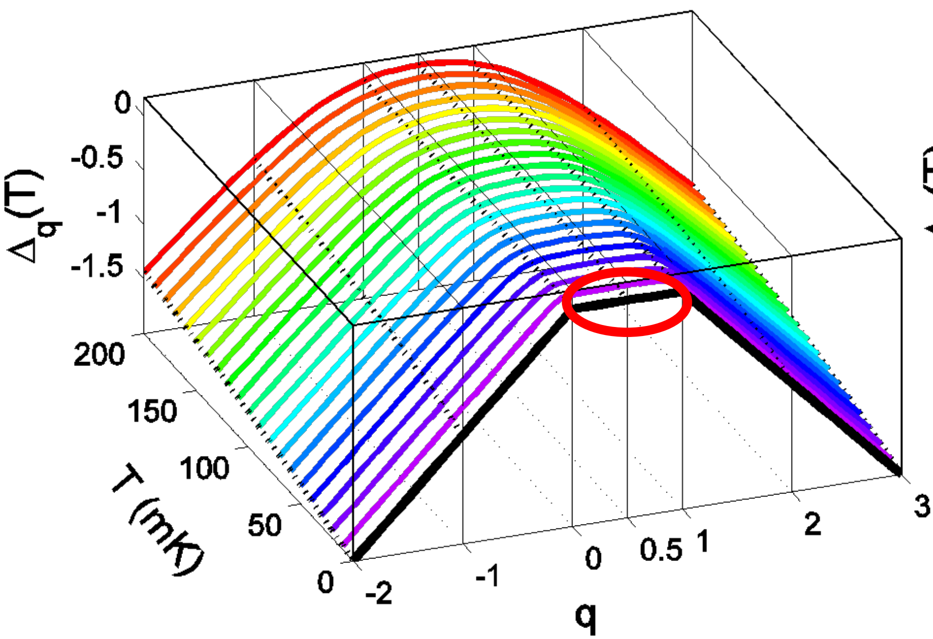
Theoretical Results

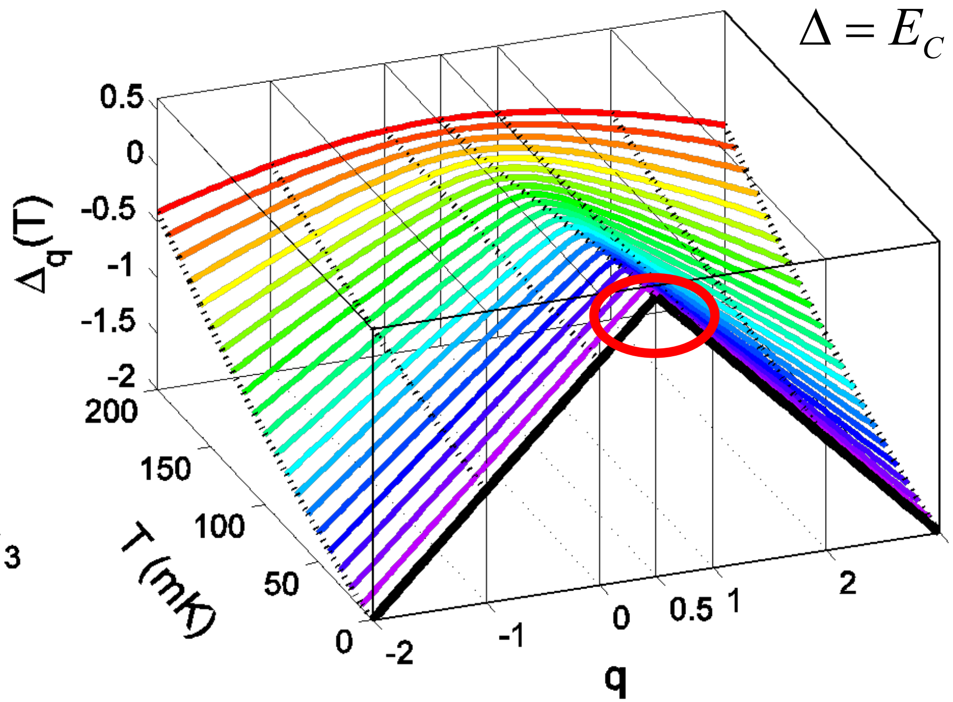
NIN SEB

$$e^2 R_T \Gamma[U] = \frac{U}{1 - e^{-U/T}}$$



SIN SEB

$$e^2 R_T \Gamma[U] \approx \sqrt{\frac{\pi T \Delta}{2}} e^{-\Delta/T} \left[1 + e^{U/T}\right]$$



All system specific things are within 0<q<1

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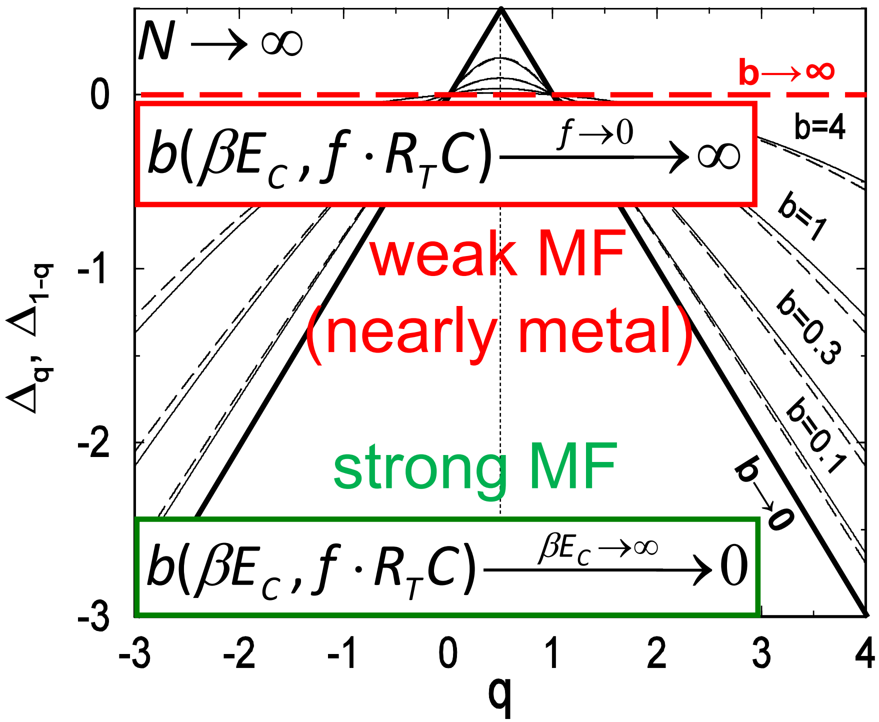
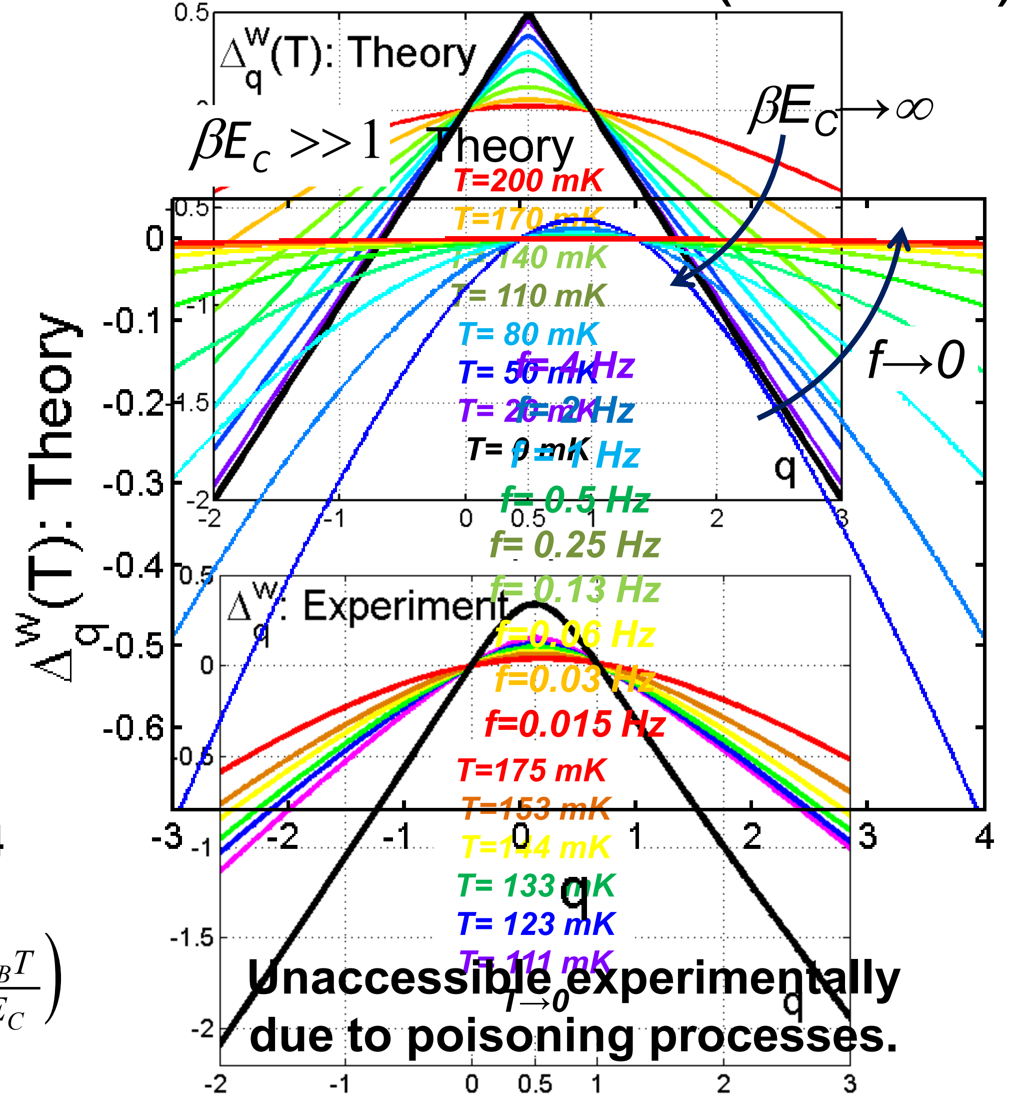
T→0 limit = only strong MF?

Critical exponents in MF theory



$$\Delta_q^w = \Delta_{1-q}^w \qquad \Delta_q^w \approx \frac{1}{2} - \left|q - \frac{1}{2}\right| + O\left(\frac{k_B T}{E_C}\right)$$

SIN SEB (f = 4 Hz)



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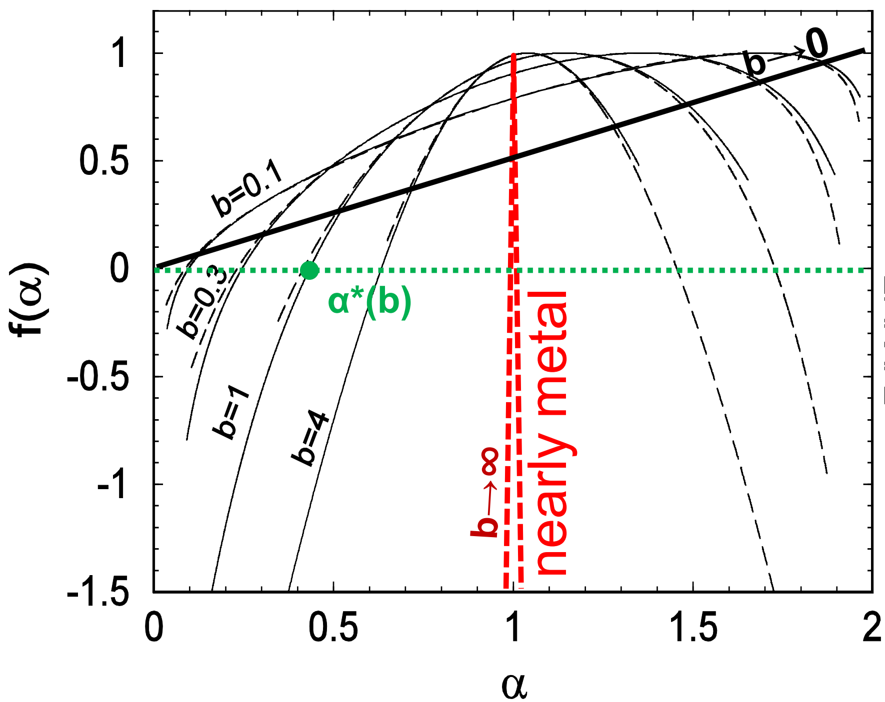
Results. Comparison with MF

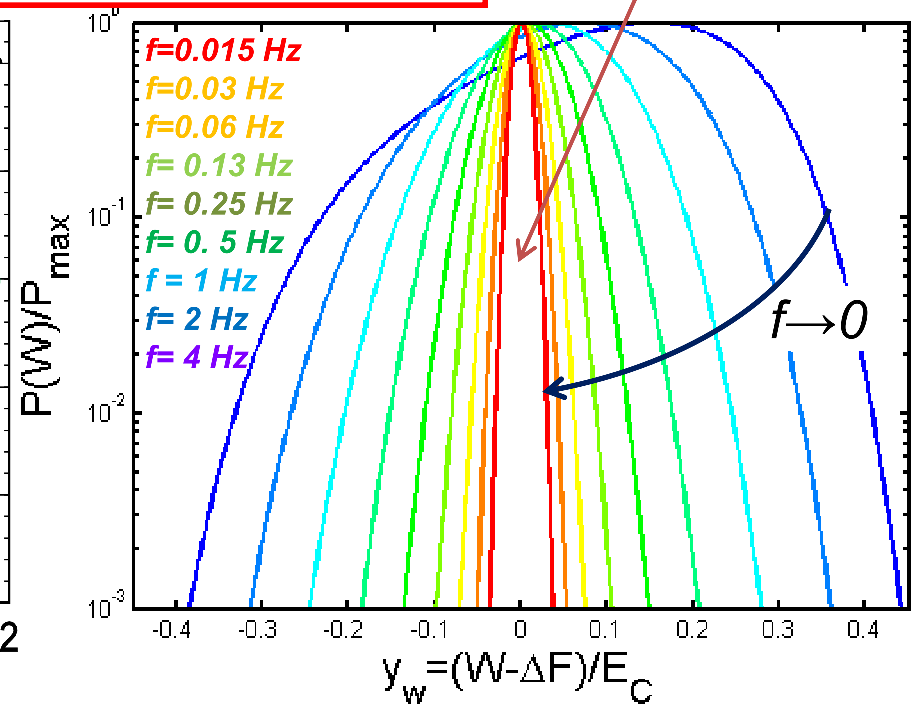
Critical exponents in MF theory

SIN SEB

$$\boxed{b(\beta E_C, f\cdot R_T C)\xrightarrow{f\to 0}\infty}$$

nearly adiabatic





$$P(\alpha^*)\propto 1/N$$

$$\alpha^*(b)=\begin{cases}1.0208\cdot b, & b<<1\\ 1-(2\pi\beta_{ens}b)^{-1}, & b>>1\end{cases}$$

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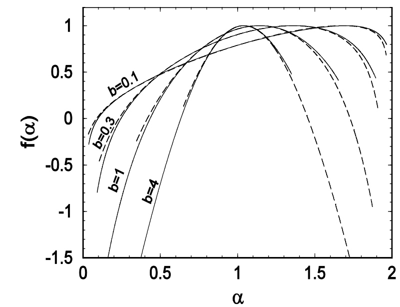
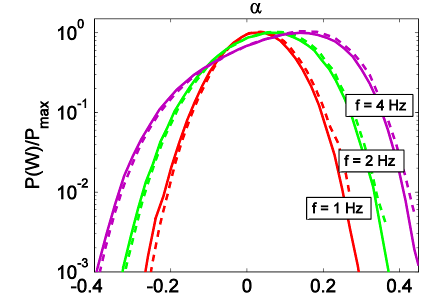
NIN SEB. General result

$$\beta E_C\cdot f\cdot R_T C=\frac{(1-\alpha^*)^2}{\alpha^*}=\begin{cases}0.98\cdot b^{-1}, & b<<1\\ (2\pi\beta_{ens}b)^{-2}, & b>>1\end{cases}$$

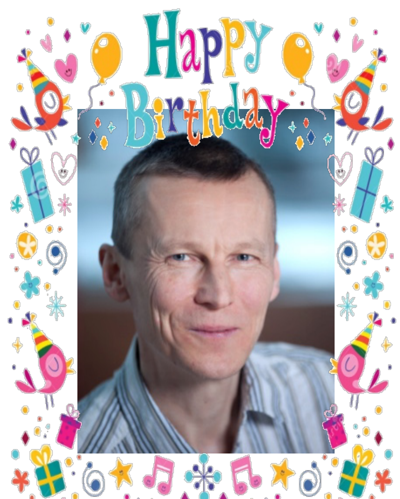
Summary





Non-trivial analogy between work statistics in SEB and multifractal statistics in ALT is uncovered:

- Jarzynski equality is generalized
- Possible reason of f(a) symmetry is suggested
- Statistics of critical wavefunctions as stochastic dynamics (Loewner – Schramm eqs?)



Thank you for attention!

Happy birthday, Jukka!