IRREVERSIBLE ENTROPY PRODUCTION IN NON-EQUILIBRIUM QUANTUM PROCESSES

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QT60

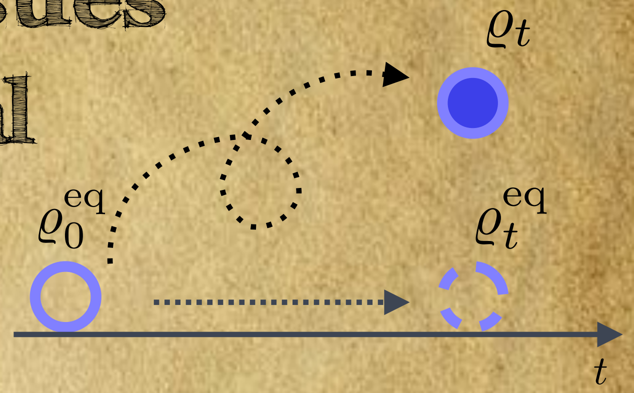
Workshop on thermodynamics, thermoelectrics and transport in quantum devices

Espoo, 20 September 2018

QTεQ QUANTUM TECHNOLOGY at QUEEN'S

PLAN OF THE TALK

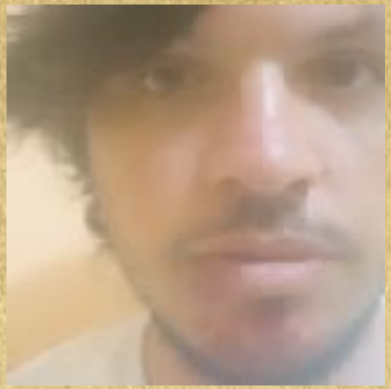
1 Entropy production & current issues in quantum system: a proposal of resolution



Observability of the framework 2

3 Quantumness of the framework $\hbar$

DUE CREDIT



Jader P Santos
(USP, Brazil)

Gabriel T Landi
(USP, Brazil)

Lucas C Celeri
(UFG Goiana, Brazil)

J. P Santos, G. T. Landi, and M. Paternostro Phys. Rev. Lett. 118, 220601 (2017)

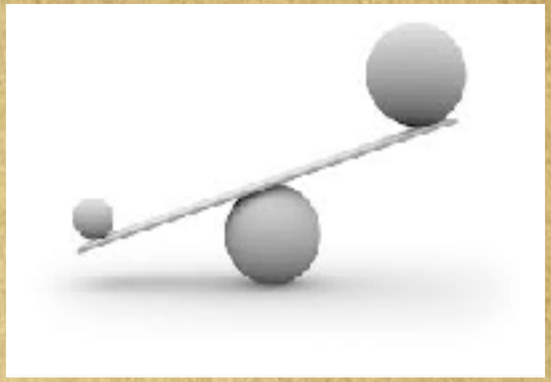
J. P. Santos, L. C. Celeri, F. Brito, G. T. Landi, and M. Paternostro, Phys. Rev. A 97, 052123 (2018)

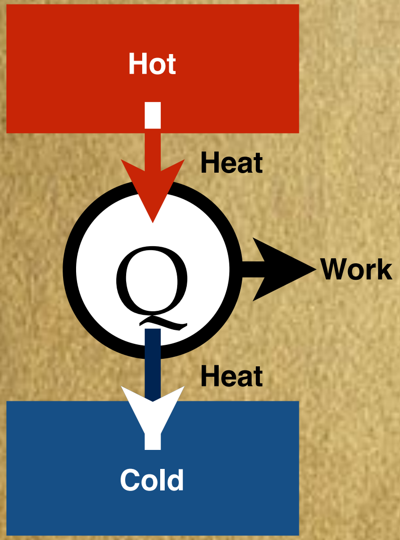
J. P. Santos, A. de Paula, R. Drumond, G. T. Landi, and M. Paternostro, Phys. Rev. A 97, 050101R (2018)

J. P. Santos, L. C. Celeri, G. T. Landi, and M. Paternostro, arXiv:1707.08946 (2017)

Why entropy production?

Non-equilibrium processes dissipate energy. This produces irreversible increase of entropy





Entropy production for estimating the performance of devices (exergy is reduced by irreversibility)

QUANTUM TECHNOLOGY at QUEEN'S

Jukka producing entropy and explaining it

PROGRESS ARTICLES | INSIGHT

PUBLISHED ONLINE: 3 FEBRUARY 2015 | DOI: 10.1038/NPHYS3169

nature physics

Towards quantum thermodynamics in electronic circuits

Jukka P. Pekola

Nature Physics 11, 118 (2015)

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PART 1

CURRENT ISSUES & A PROPOSAL

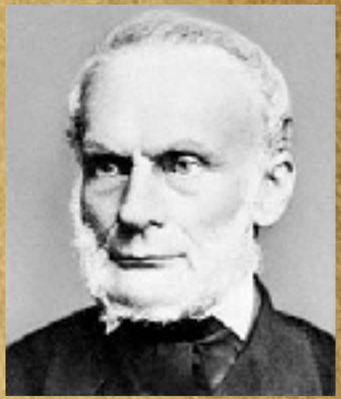
Entropy production

Second Law: $$\Delta S \geq \int \frac{\delta Q}{T}$$

Clausius: "Uncompensated transformation"

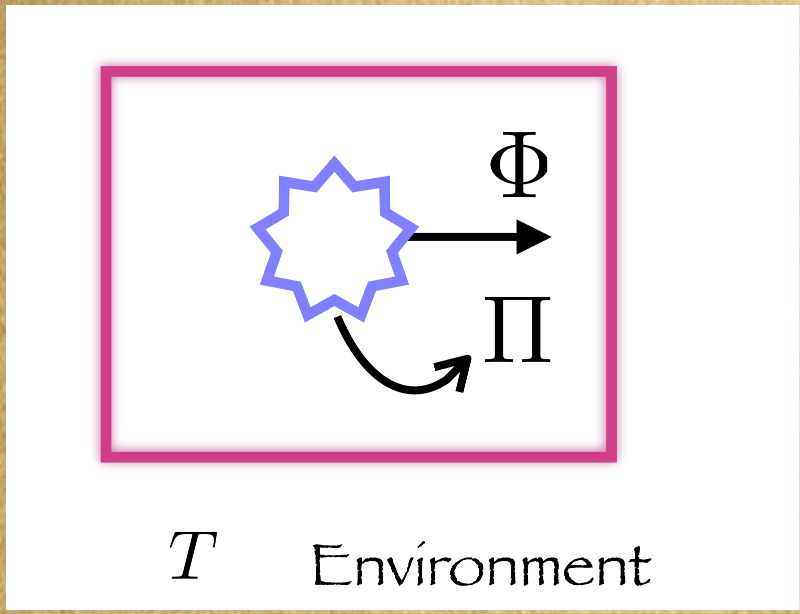
$$\Delta S = \Sigma + \int \frac{\delta Q}{T}$$

Σ → Entropy production



Rudolf Clausius

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \Phi(t) + \Pi(t)$$



$\Pi(t)$ Entropy production rate

$\Phi(t)$ Entropy flux rate

Entropy production

Π, Φ are not observable. No continuity equation for entropy

No unifying theory of entropy production, to date

$\Pi(t)$

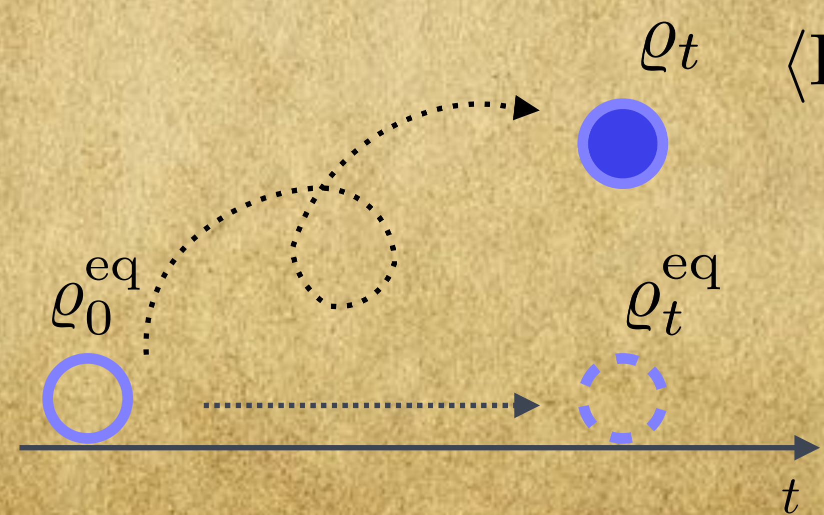
L Onsager
(theory of chemical kinetics)
related to energy currents

J Schnakenberger
transition rates of a system
governed by a master equation

G Gallavotti & E Cohen
integral fluctuation theorem
$\langle e^{-\Sigma} \rangle = 1$

Ch. Jarzynski
& Tasaki-Crooks

$$\frac{P_{\mathrm{f}}(+\Sigma)}{P_{\mathrm{b}}(-\Sigma)} = e^{\Sigma}$$



$$\langle \Pi(t) \rangle = -\partial_t S(\varrho_t || \varrho_t^{\mathrm{eq}}) - \mathrm{Tr}[\varrho_t \partial_t \ln \varrho_t^{\mathrm{eq}}]$$

H Spohn, J Lebowitz
S Deffner, E Lutz
H-P Breuer

Which entropy to use?

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}(\rho)$$

$$\Pi_{vN}(t) = -\partial_t S_{vN}(\rho|\rho_t^*)$$

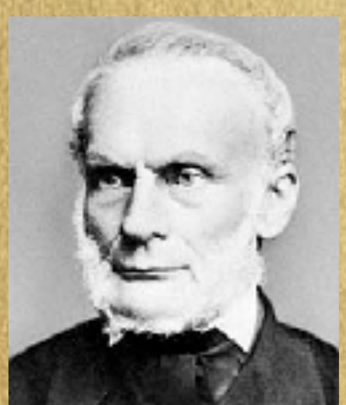
Spohn, Lebowitz

Deffner & Lutz

Donald, Breuer

For a thermal bath: $\Pi_{vN}(t) = \frac{dS_{vN}}{dt} + \Phi_{vN}(t)$

$$= \frac{dS_{vN}}{dt} + \frac{\Phi_E(t)}{T}$$



Rudolf Clausius

Energy flux from system to environment

Which entropy to use?

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Spohn, Lebowitz
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$$= \frac{dS_{vN}}{dt} + \frac{\Phi_E(t)}{T}$$

$\Pi(t), \Phi(t)$ diverge as $T \to 0$

Idealised large heat reservoirs

Energy flux from system to environment

The Belfast-Sao Paulo proposal

$$S = -\int d^2\alpha \, W(\alpha) \ln W(\alpha)$$

Entropy of the Wigner function

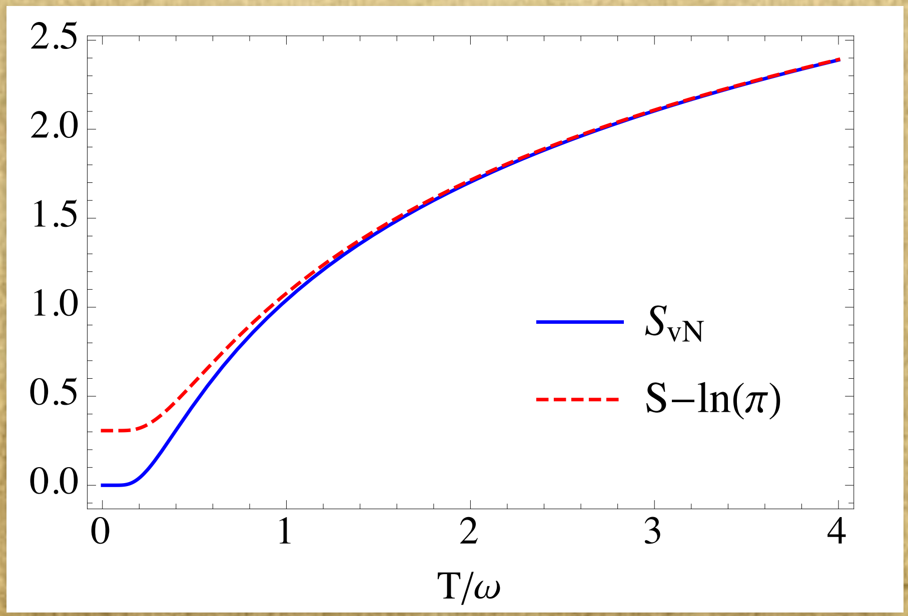
For Gaussian states:

- coincides with Rènyi-2 entropy $S_2(\varrho) = -\ln \mathrm{Tr}\varrho^2$ → can be directly related to free energy difference

J C Baez, arXiv 1182.2098 (2011)

- satisfies the strong sub-additivity inequality → can be used to construct correlation measures $\mathcal{I}_2(\varrho_{a:b})$

- for thermal states:



$$\Pi(t) = -\partial_t S(W(t)|W_{eq}) \geq 0 \quad \text{(Gaussian states)}$$

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

Why it makes sense

$$\Pi(t) = -\int \mathrm{d}^2\alpha\, \mathcal{D}(W)\ln(W/W_{\mathrm{eq}})$$

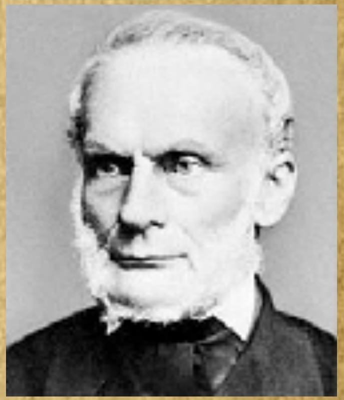
For a single harmonic oscillator in a thermal bath:

$$\Phi(t) = \frac{\gamma}{\bar{n}+1/2}(\langle a^\dagger a\rangle - \bar{n})$$

Observable!!

$$= \frac{\Phi_E}{\omega(\bar{n}+1/2)} \simeq \frac{\Phi_E}{T}$$

but no divergence at zero-temperature



Rudolf Clausius

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

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How about non-Gaussian states?

The formalism makes sense for Gaussian states.

How about non-Gaussian ones?

$$S = \int d\Omega\, Q(\Omega) \ln Q(\Omega)$$

Use Wherl entropy!

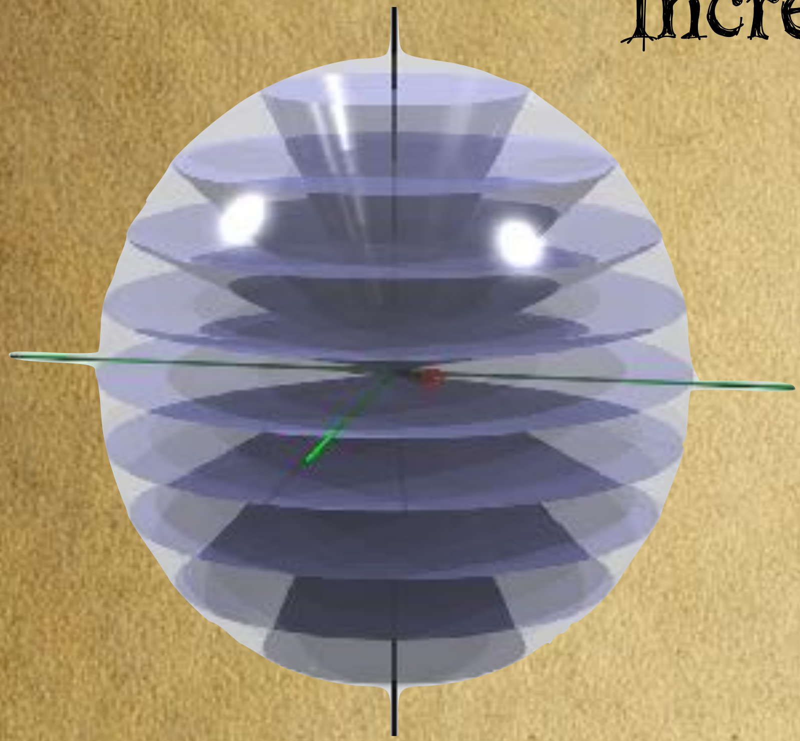
Husimi function
positive even for non-Gaussian states

Incredibly useful for spin dynamics!

J. P. Santos, L. C. Celeri, F. Brito, G. T. Landi, and M. Paternostro, Phys. Rev. A 97, 052123 (2018)

Angular momenta!

Incredibly useful for spin dynamics!



$$|\Omega\rangle = e^{-i\phi J_z} e^{-i\theta J_y} e^{-i\psi J_z} |J, J\rangle$$

Spin coherent states

$$\mathcal{Q}(\Omega) = \langle \Omega | \rho | \Omega \rangle$$ Husimi function

$$\partial_t \rho = -i[H, \rho] + \mathcal{D}(\rho)$$

$$\partial_t \mathcal{Q} = U(\mathcal{Q}) + \mathcal{D}(\mathcal{Q})$$

Fokker-Planck equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{(2J+1)}{4\pi} \int \mathrm{d}\Omega \, \mathcal{D}(\mathcal{Q}) \ln \mathcal{Q}$$

J. P. Santos, L. C. Celeri, F. Brito, G. T. Landi, and M. Paternostro, Phys. Rev. A 97, 052123 (2018)

Dephasing channel

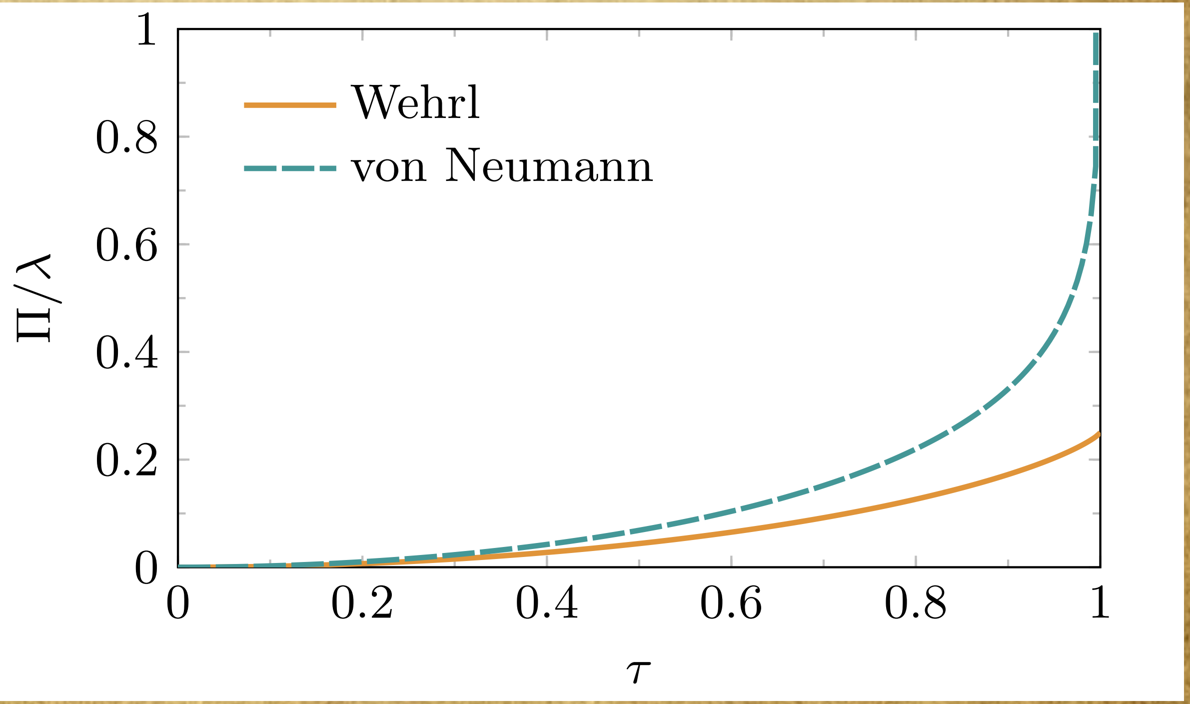
$$\mathcal{D}(\mathcal{Q}) = -\frac{\lambda}{2}\mathcal{J}_z(\mathcal{J}_z(\mathcal{Q}))$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \Pi = \frac{\lambda}{2}\left(\frac{2J+1}{4\pi}\right)\int \mathrm{d}\Omega\, \frac{|\mathcal{J}_z(\mathcal{Q})|^2}{\mathcal{Q}} \geq 0$$

No associated entropy flux!

Dephasing as a unital map: entropy only increases

spin-1/2 case



J. P. Santos, L. C. Celeri, F. Brito, G. T. Landi, and M. Paternostro, Phys. Rev. A 97, 052123 (2018)

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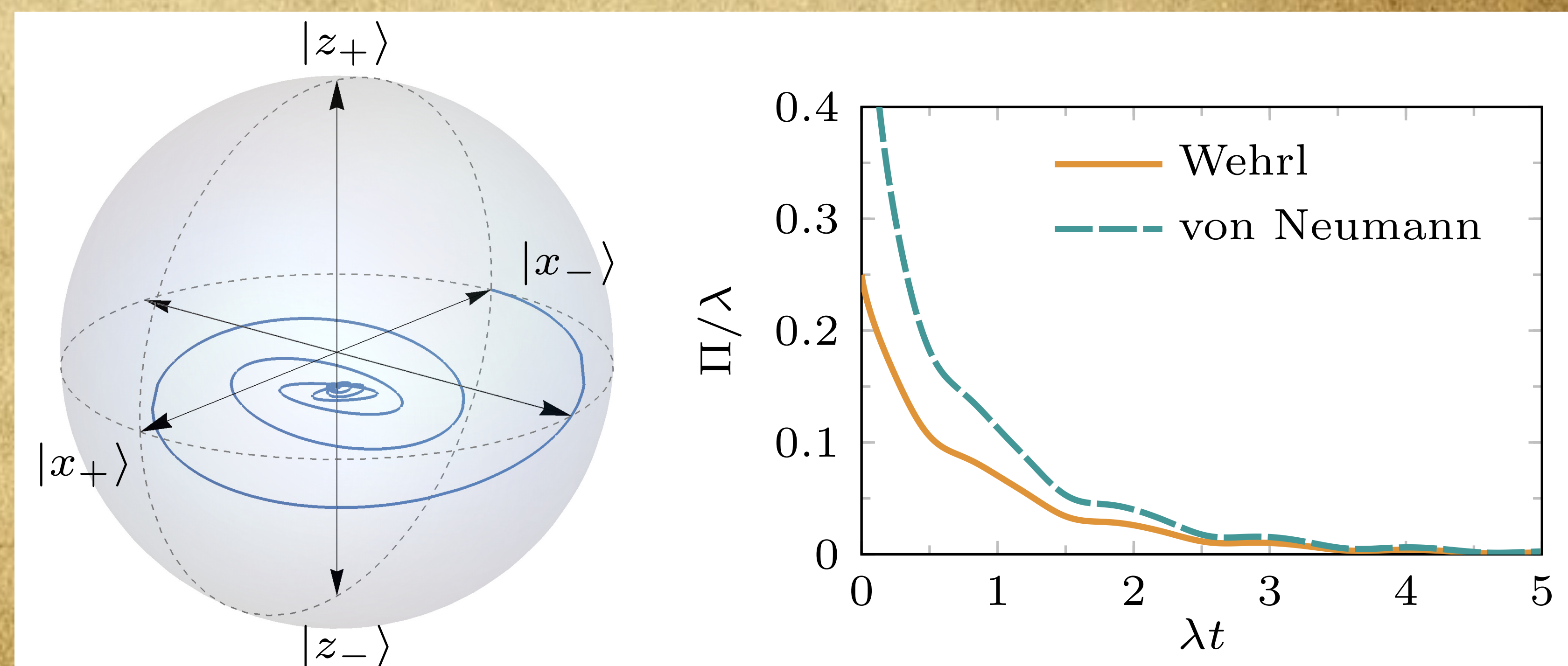
No associated entropy flux!

Dephasing as a unital map: entropy only increases

spin-1/2 case in a rotating magnetic field



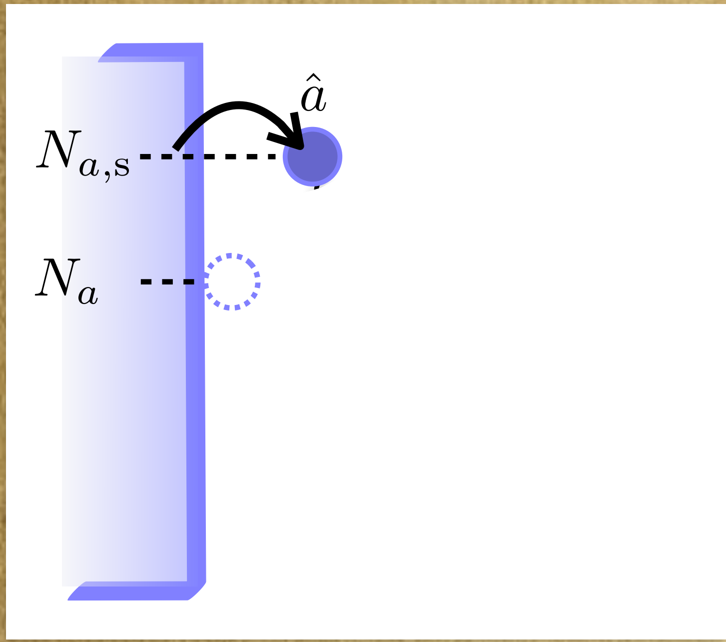
J. P. Santos, L. C. Celeri, F. Brito, G. T. Landi, and M. Paternostro, Phys. Rev. A 97, 052123 (2018)

PART

2

OBSERVABILITY

Link to observables!



For a single harmonic oscillator in a thermal bath:

$$\Pi_s = 2\kappa_a\left(\frac{\langle \hat{q}_a^2\rangle_s + \langle \hat{p}_a^2\rangle_s}{2N_a+1} - 1\right)$$

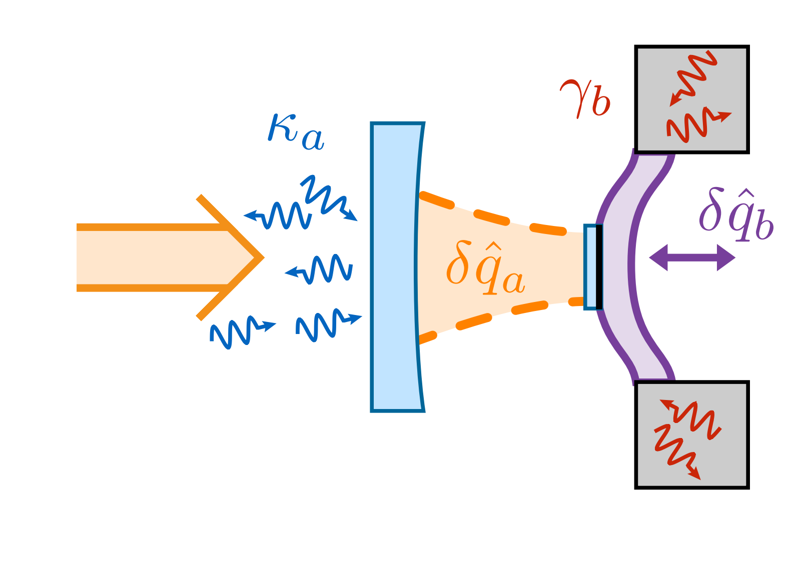
$$\Pi_\mathrm{s} = 2\kappa_a\left(\frac{\langle \hat{q}_a^2\rangle_\mathrm{s} + \langle \hat{p}_a^2\rangle_\mathrm{s}}{2N_a+1} - 1\right) + 2\kappa_b\left(\frac{\langle \hat{q}_b^2\rangle_\mathrm{s} + \langle \hat{p}_b^2\rangle_\mathrm{s}}{2N_b+1} - 1\right)$$

Experimentally testable (and indeed tested!)

M Brunelli et al. arXiv:1602.06958 (2016), to appear in Phys Rev Lett

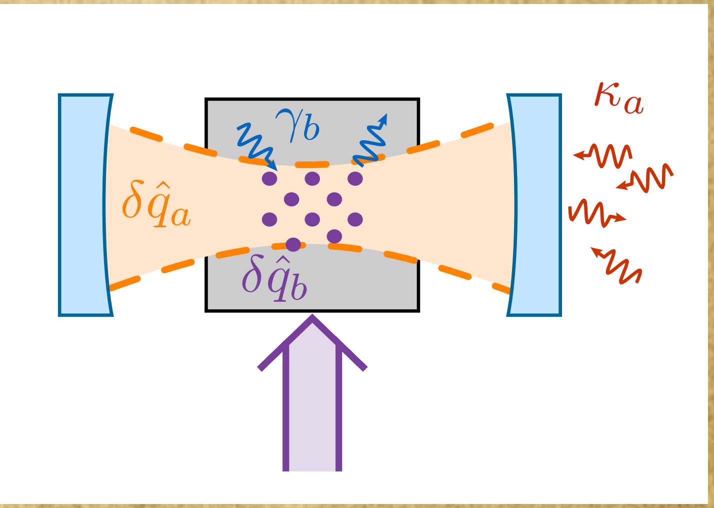
Entropy production in mesoscopics

Nano-optomechanics



$$H = \frac{\hbar\omega}{2}(p^2+q^2) + \hbar(\omega_c - gq)a^\dagger a + i\hbar\mathcal{E}(a^\dagger e^{-i\omega_0 t} - a e^{i\omega_0 t})$$

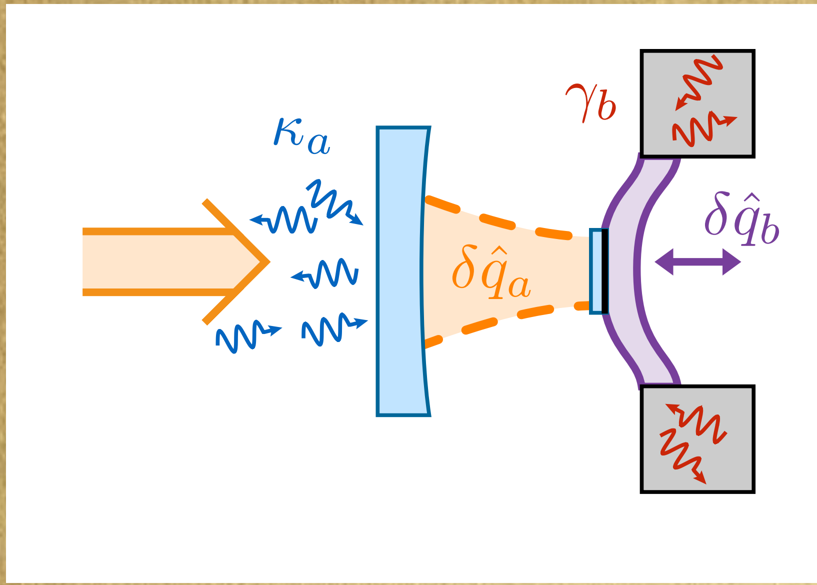
Intra-cavity atomic systems

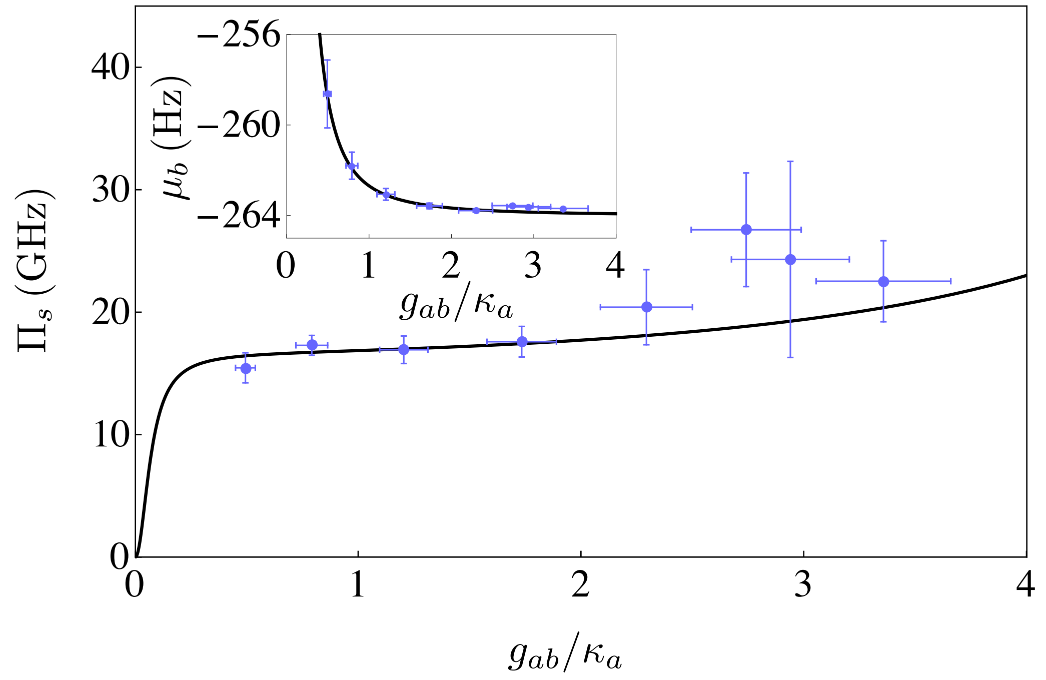


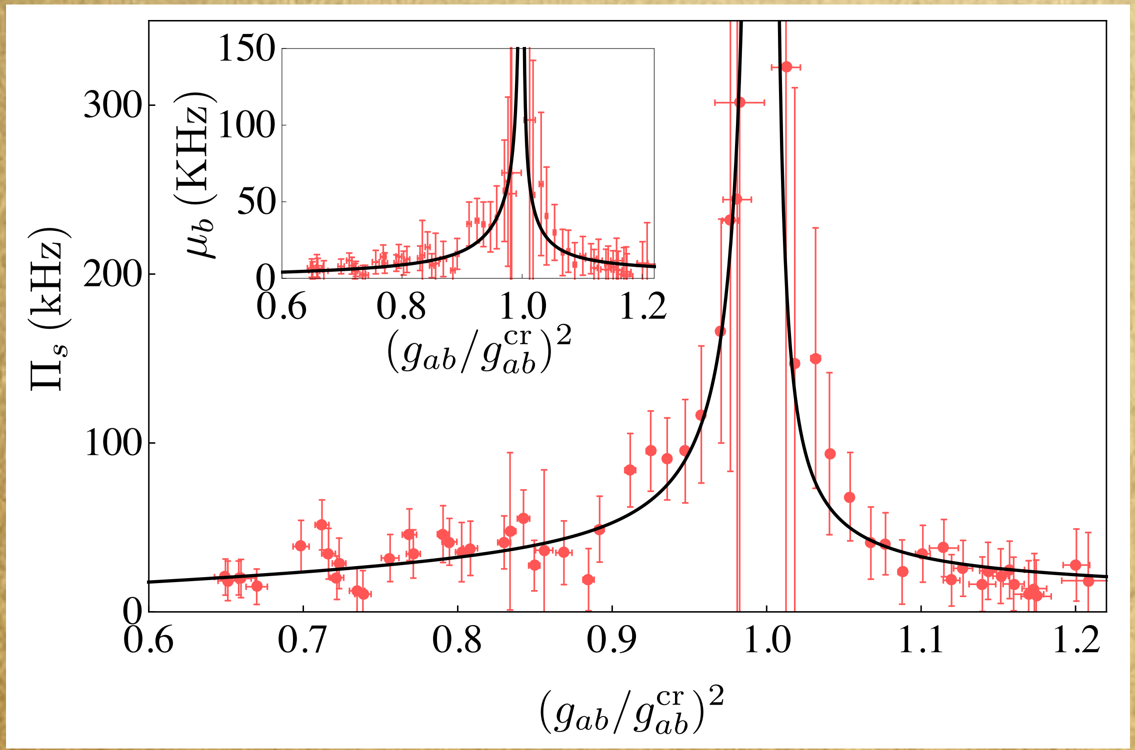
$$\hat{H} = \omega_0 \hat{J}_z + \omega \hat{a}^\dagger \hat{a} + \frac{2\lambda}{\sqrt{N}}\left(\hat{a} + \hat{a}^\dagger\right)\hat{J}_x$$

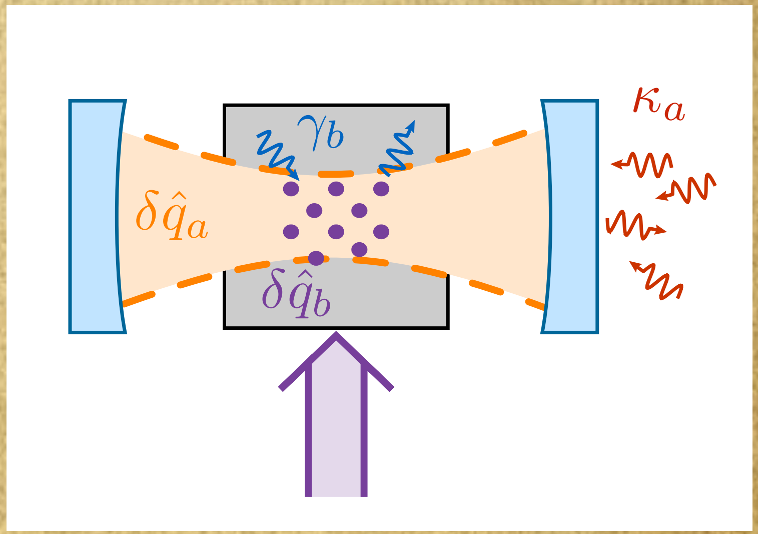
M Brunelli et al. arXiv:1602.06958 (2016), to appear in Phys Rev Lett

Entropy production in mesoscopics









M Brunelli et al. arXiv:1602.06958 (2016), to appear in Phys Rev Lett

PART

3

QUANTUMNESS

What makes this scenario quantum?

$$F(\rho) = F_{\mathrm{eq}} + TS(\rho||\rho_{\mathrm{eq}})$$ Non-equilibrium free energy

$$F(\rho) \geq F_{\mathrm{eq}}$$ Equilibration implies decrease of free energy

$$\Pi = -\frac{1}{T}\frac{\mathrm{d}F(\rho)}{\mathrm{d}t} \geq 0 \quad \Pi = 0 \text{ iff } \rho = \rho_{\mathrm{eq}}$$

$$S(\rho||\rho_{\mathrm{eq}}) = \mathcal{S}(p||p_{\mathrm{eq}}) + \mathcal{C}(\rho)$$

diagonal entropy

relative entropy of coherence (Baumgratz, Cramer, Plenio)

J. P. Santos, L. C. Celeri, G. T. Landi, and M. Paternostro, arXiv:1707.08946 (2017) to appear in Nature Quantum Information

What makes this scenario quantum?

$$\Pi = \Pi_d + \Upsilon$$

$$\Pi_d = -\tfrac{\mathrm{d}}{\mathrm{d}t}\mathcal{S}(p||p_{\mathrm{eq}}) \qquad \Upsilon = -\frac{\mathrm{d}\mathcal{C}(\rho)}{\mathrm{d}t}$$

Interpretation to the mismatch between entropy production in quantum and classical settings

$$\Phi = \Pi - \frac{\mathrm{d}S}{\mathrm{d}t} = \sum_n \frac{\mathrm{d}p_n}{\mathrm{d}t} \ln p_{\mathrm{eq}}^n$$

entropy flux has no contribution arising from quantum coherences

J. P. Santos, L. C. Celeri, G. T. Landi, and M. Paternostro, arXiv:1707.08946 (2017) to appear in Nature Quantum Information

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The Belfast crew

Bread on tables..

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QUANTUM TECHNOLOGY at QUEEN'S

Department for the Economy

Science Foundation Ireland
sfi
For what's next

THE FRAMEWORK PROGRAMME FOR RESEARCH AND INNOVATION
HORIZON 2020

Alexander von Humboldt
Stiftung / Foundation

THE ROYAL SOCIETY

ROYAL COMMISSION
EXHIBITION OF 1851

EPSRC
Pioneering research and skills

MSCA
COFUND
@MSCActions

MARIE CURIE ACTIONS

Fondazione
Angelo Della Riccia

newton
international fellowships

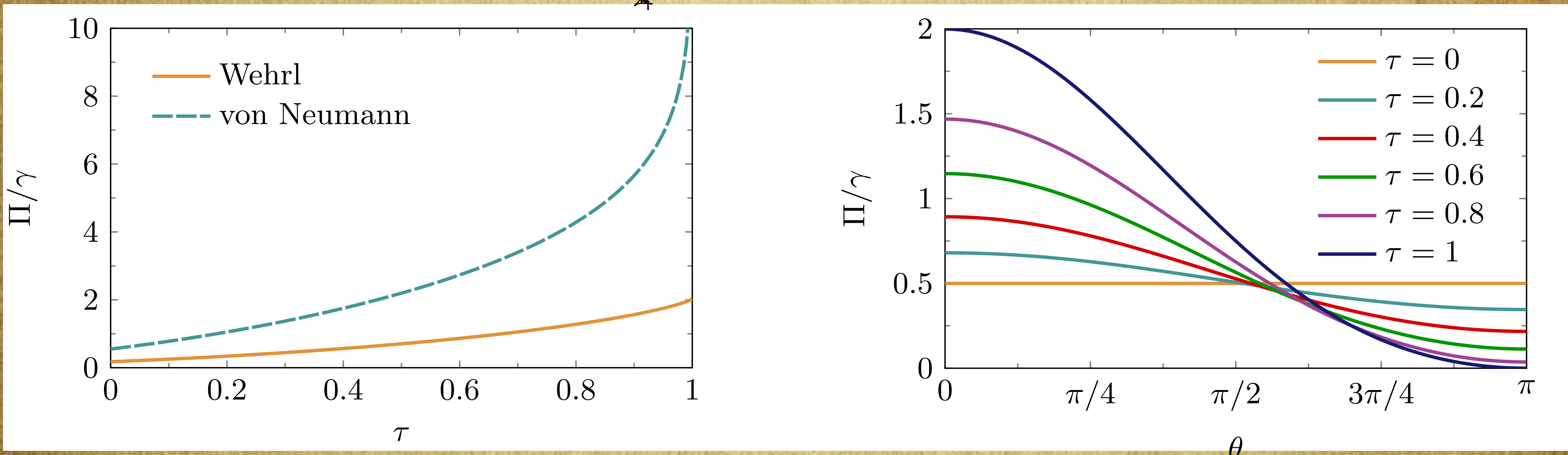
THANK YOU & MANY HAPPY RETURNS, JUKKA

Amplitude channel

$$\mathcal{D}(\mathcal{Q}) = \frac{\gamma}{2}\left\{\mathcal{J}_-(f(\mathcal{Q})) - \mathcal{J}_+(f^*(\mathcal{Q}))\right\}$$

$$f(\mathcal{Q}) = \frac{1}{2}\left[2J\mathcal{Q} - \mathcal{J}_z(Q)\right]e^{i\phi}\sin\theta + \frac{1}{2}\left[\cos\theta - (2\bar{n}+1)\right]\mathcal{J}_+(\mathcal{Q})$$

spin-1/2 case



while divergence of the von Neumann at T=0

J. P. Santos, L. C. Celeri, F. Brito, G. T. Landi, and M. Paternostro, Phys. Rev. A 97, 052123 (2018)