Extreme reductions of entropy in an electronic double dot

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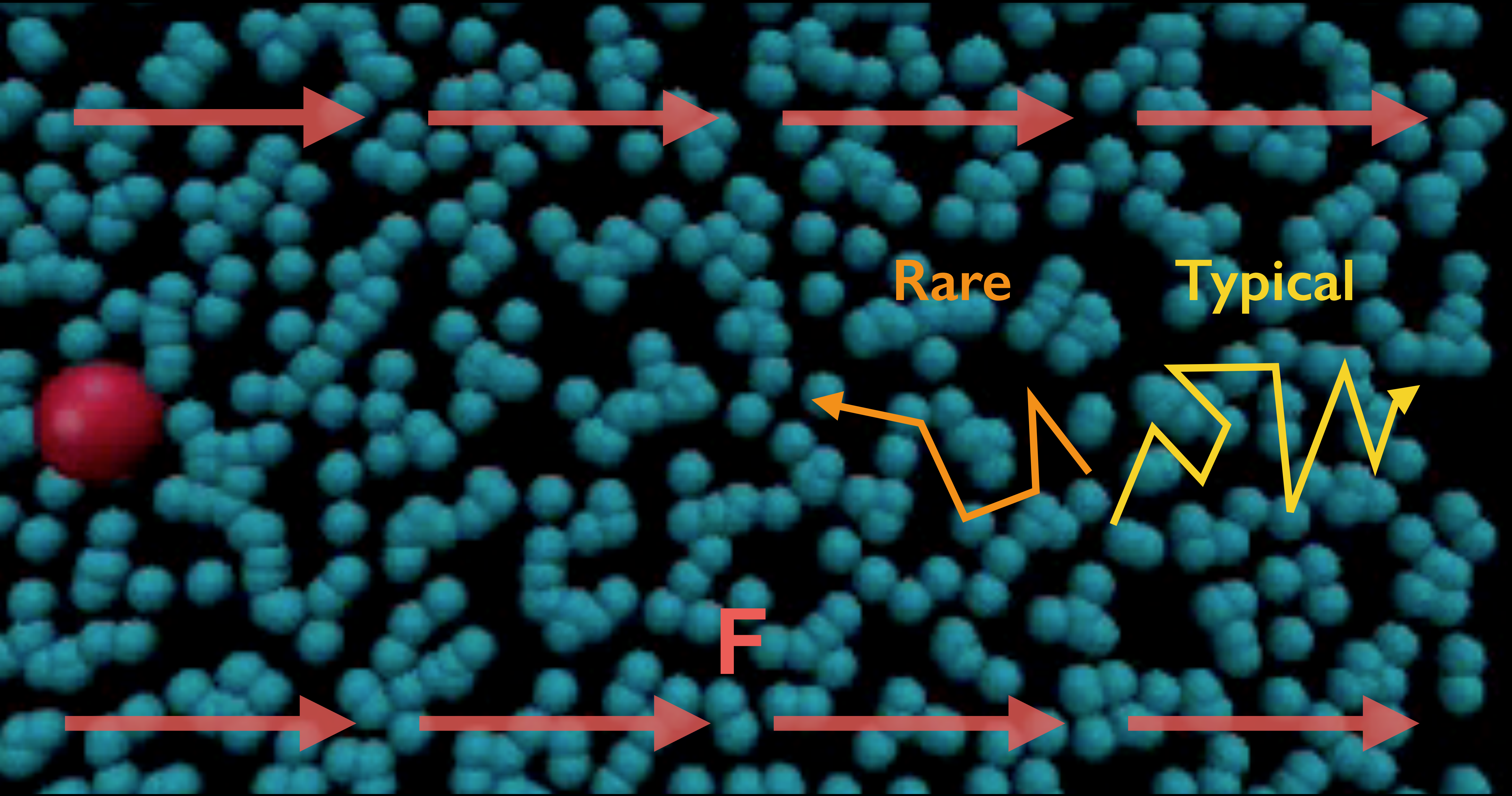
QT60, Espoo (Finland) 19/9/18

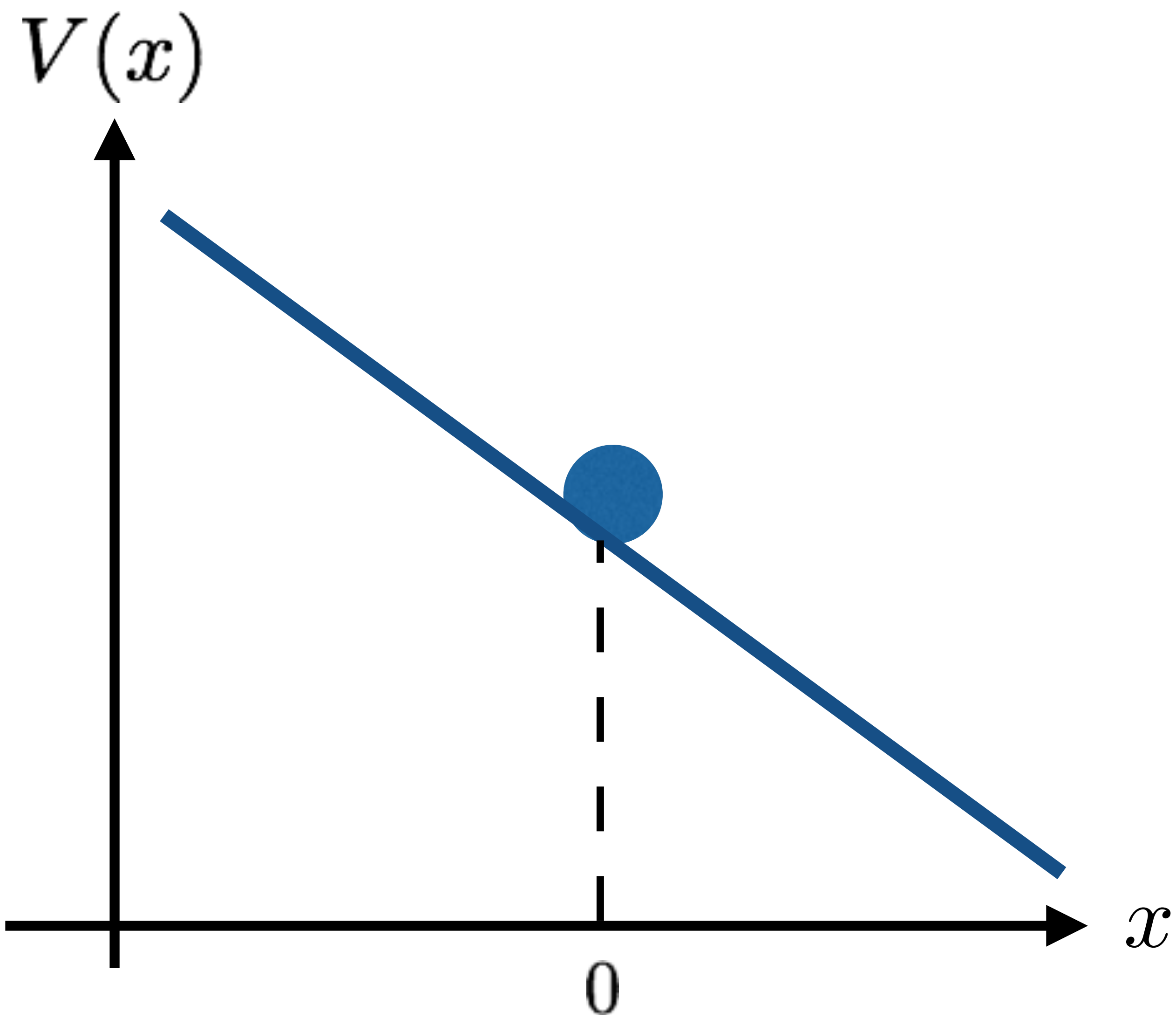
Jukka Pekola's 60th Birthday

Nonequilibrium steady states



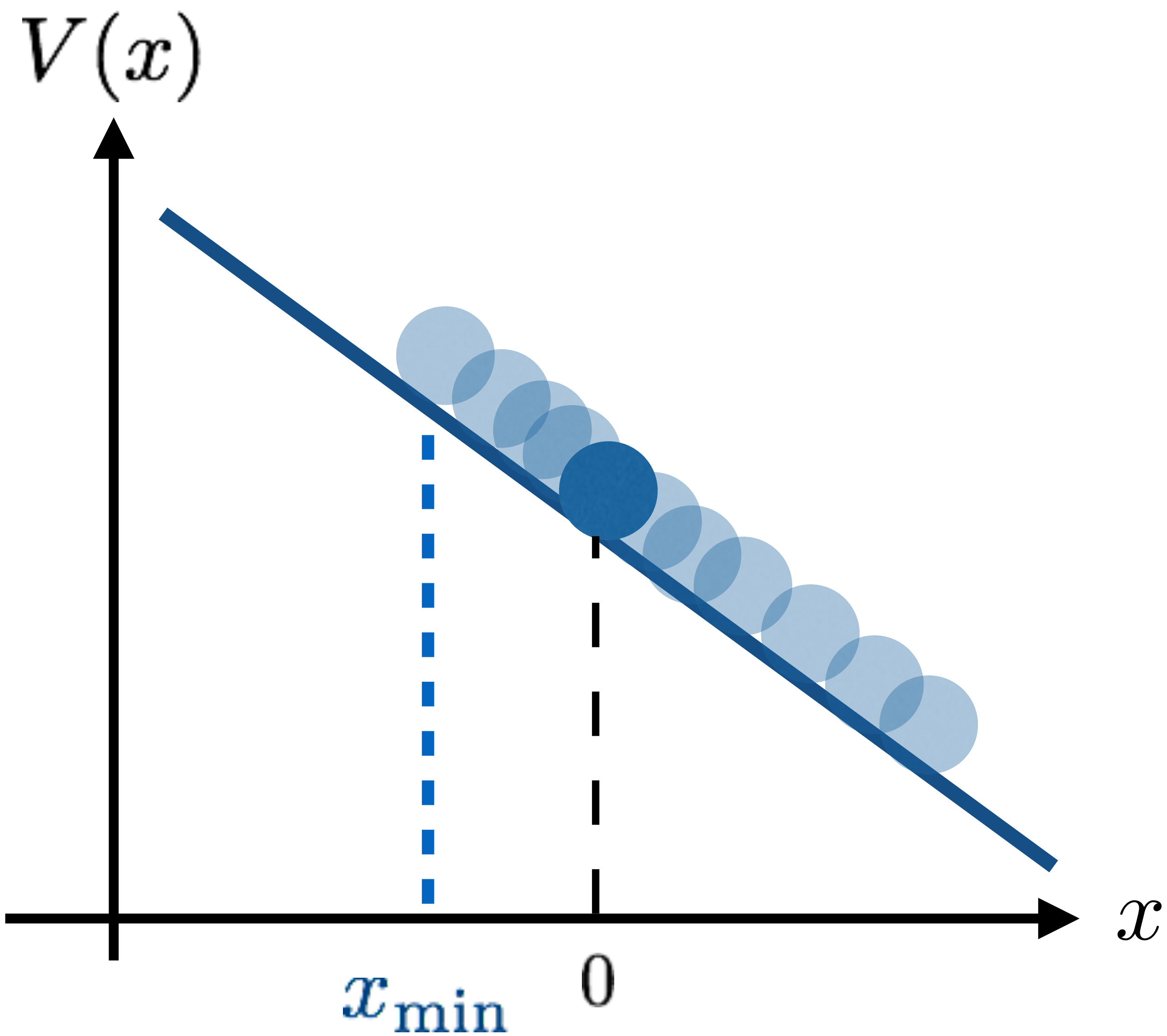
How far can we walk against the stream ?

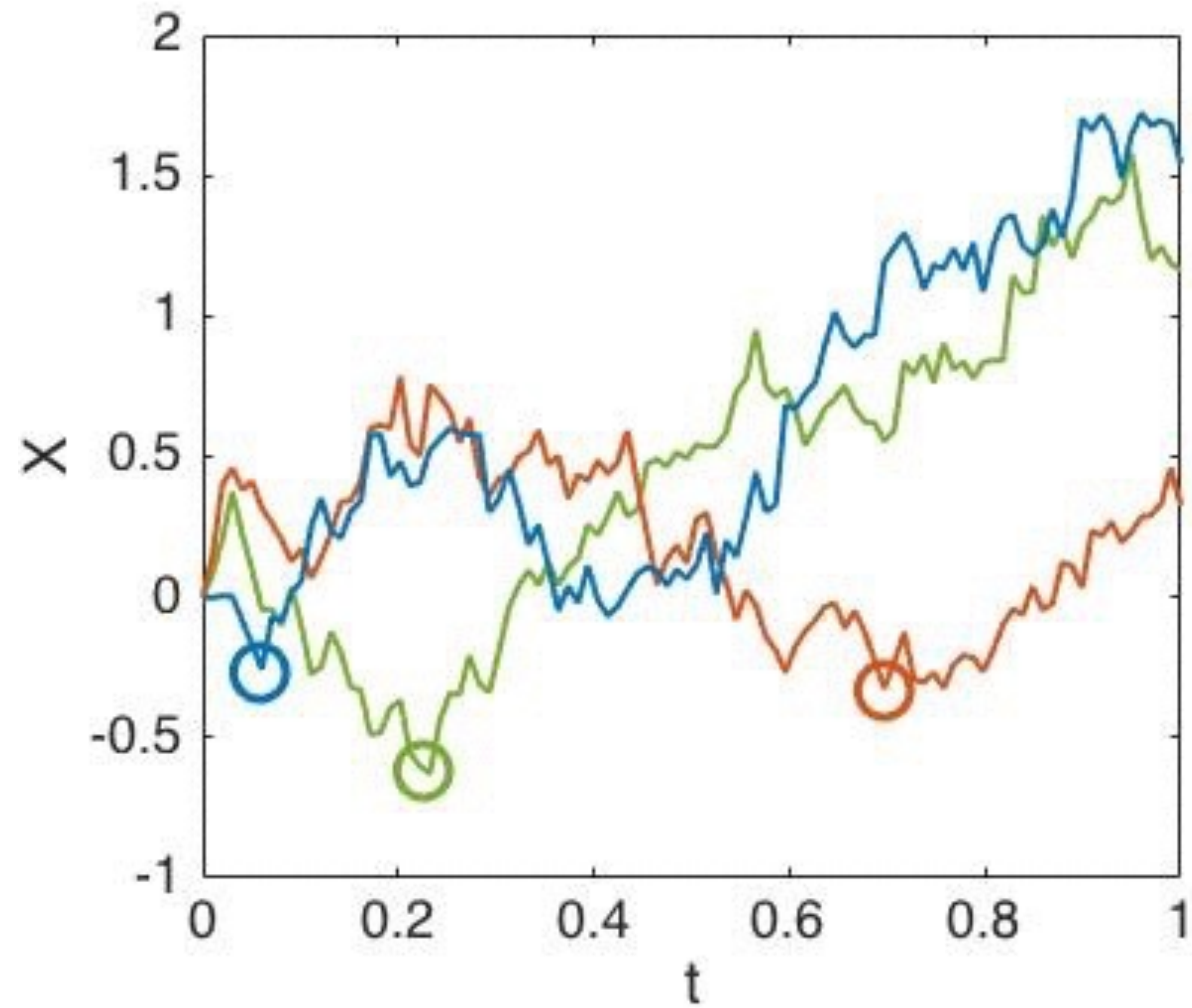
Extreme excursions

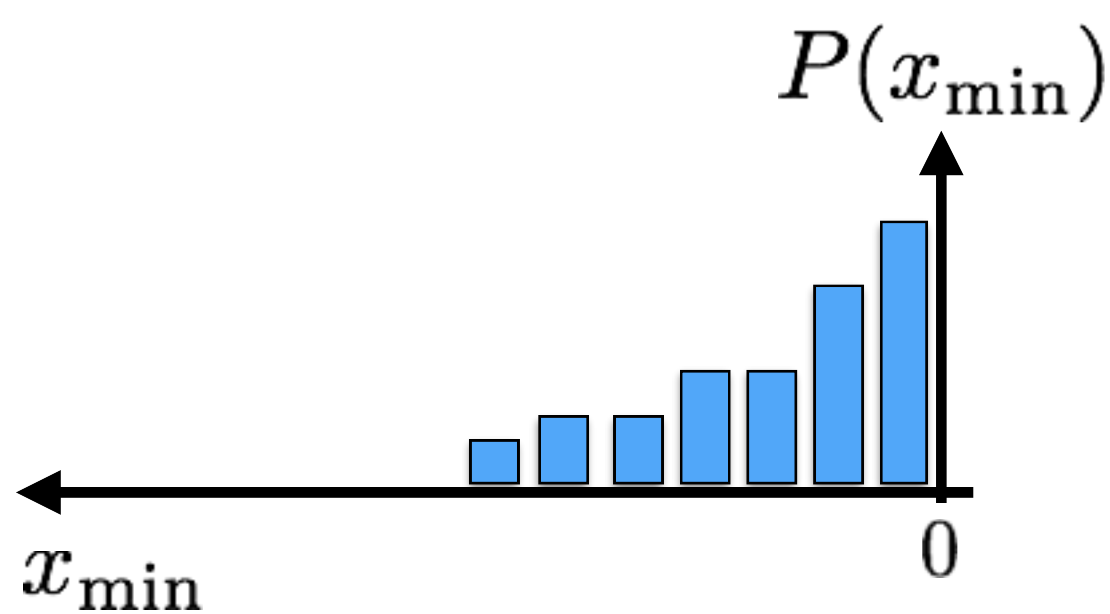


$$\frac{\mathrm{d}x_t}{\mathrm{d}t} = -\mu V'(x_t) + \sqrt{2D}\xi_t$$

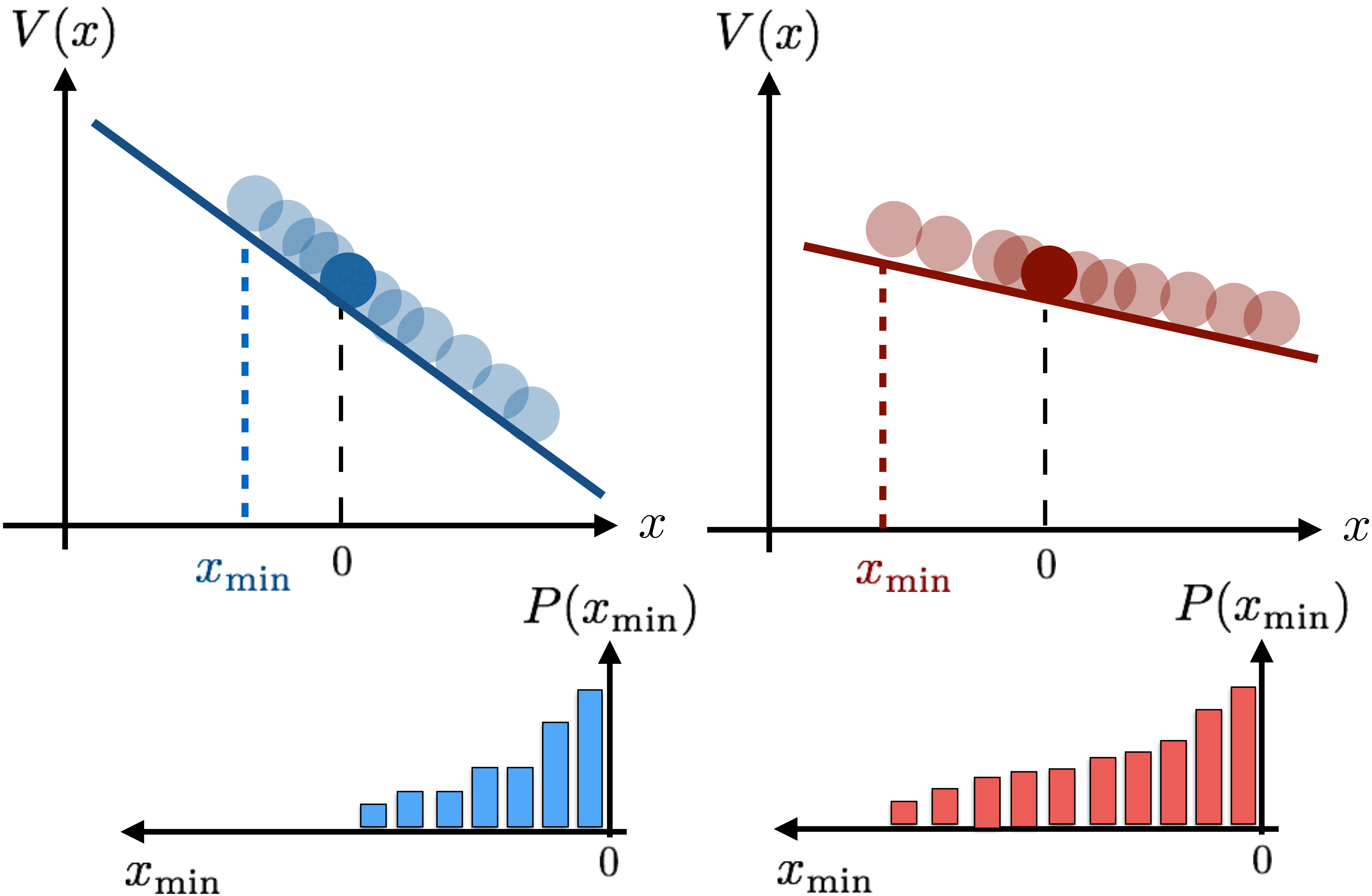
Statistics of minima



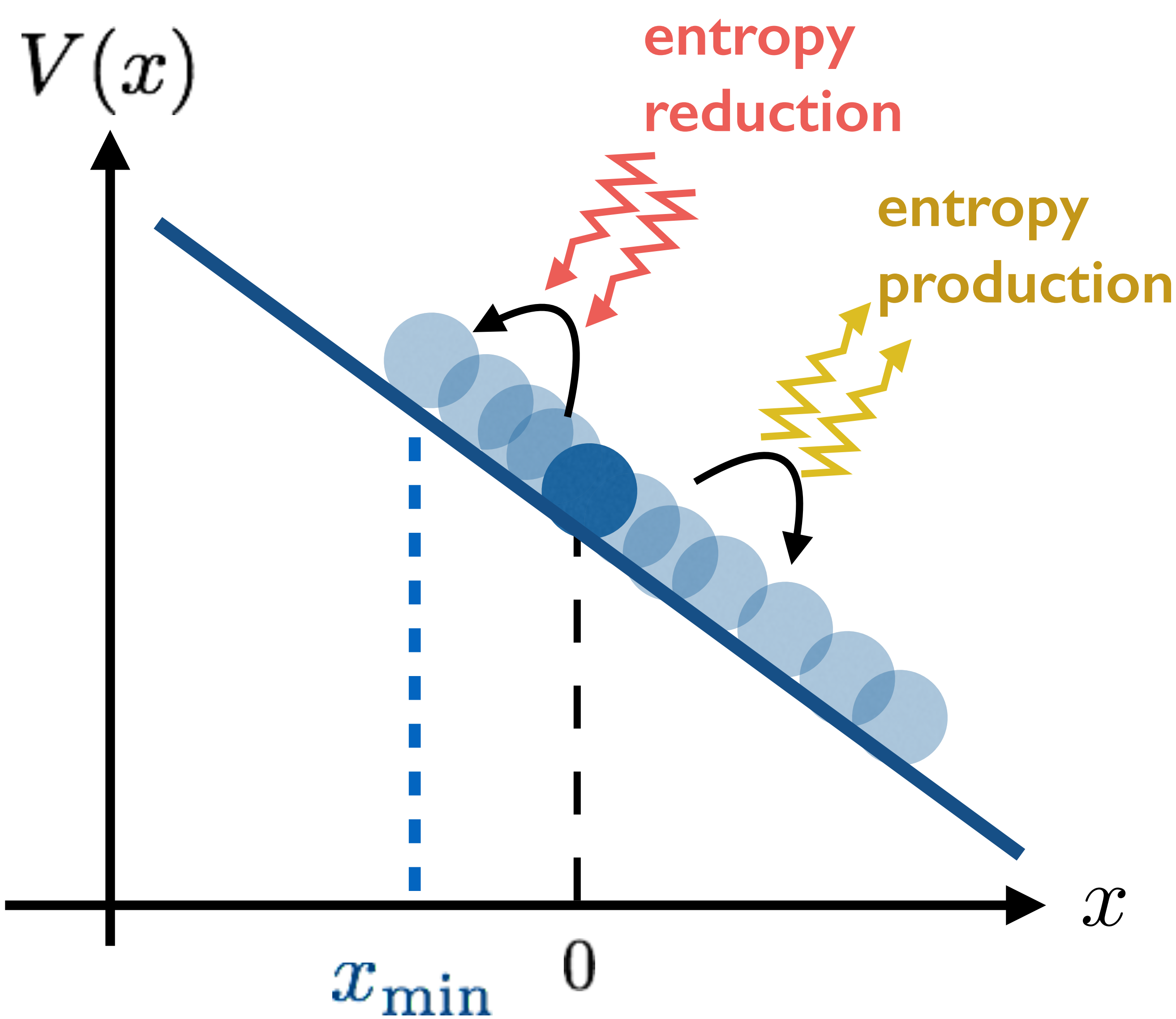


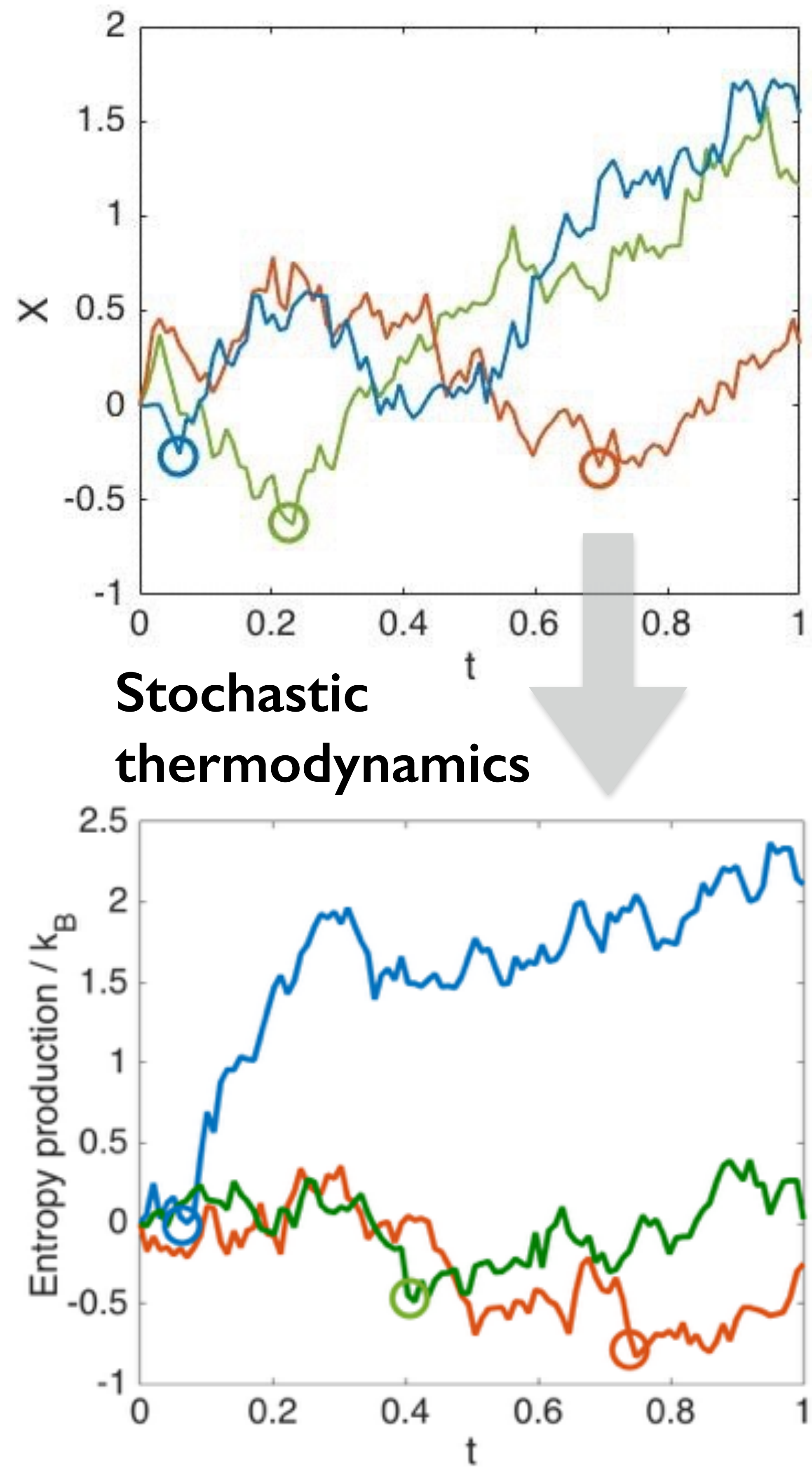


Statistics of minima



Extrema of thermodynamic fluctuations



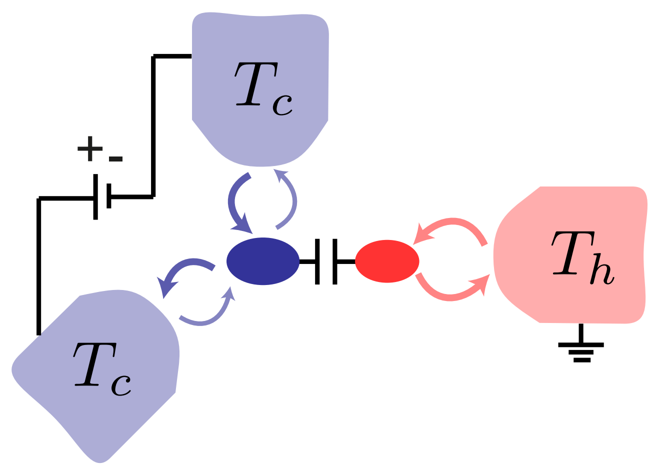


Universal constraints for extreme thermodynamic fluctuations?

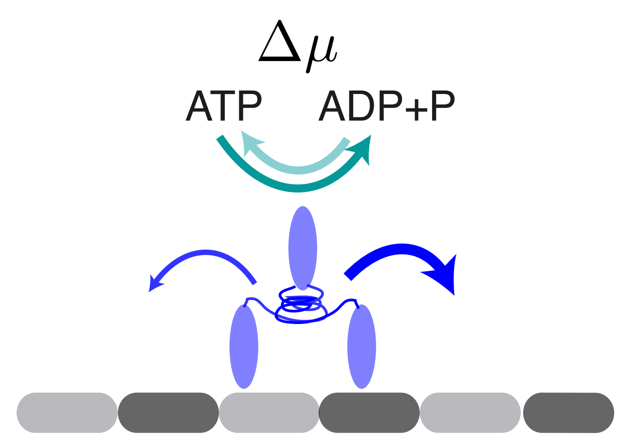
Stochastic entropy production

$$\partial_t P_t = L P_t$$

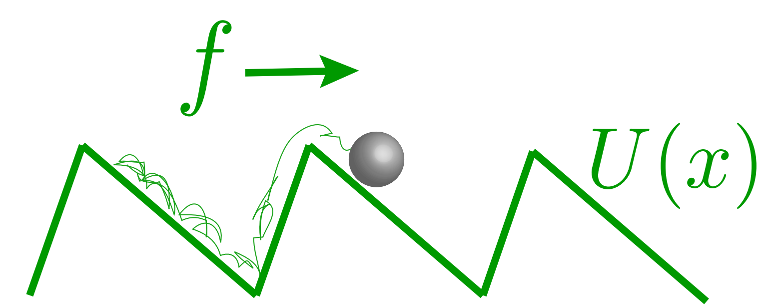
We study nonequilibrium steady states with stochastic dynamics described by Fokker-Planck or Master equations



Quantum dots



Molecular motors



Colloidal systems

Nonequilibrium steady state $P_t(\vec{X}_t) \neq Z^{-1} e^{-V(\vec{X}_t)/k_B T}$

characterized by **currents** J_{ij} , **thermodynamic forces** F_{ij}

and **entropy production** $\sigma = \sum_{i<j} F_{ij} J_{ij} \geq 0$

Stochastic entropy production

$$S(t) = k_{\mathrm{B}} \ln \frac{P\left(X_{[0,t]}\right)}{P\left(X_{[t,0]}\right)}$$

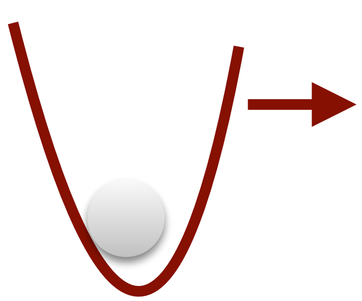
stochastic trajectory

time-reversed trajectory

Equilibrium : reversibility $P\left(X_{[0,t]}\right) = P\left(X_{[t,0]}\right)$ $S(t) = 0$

Non-equilibrium : irreversibility $P\left(X_{[0,t]}\right) \neq P\left(X_{[t,0]}\right)$ $S(t)$ **fluctuates**

Example driven colloidal system



environmental entropy change

$$S(t) = -\frac{Q(t)}{T} + S(X_t) - S(X_0)$$

system's entropy change

$$Q(t) = \int_0^t U'(X_s) \circ \mathrm{d}X_s$$

stochastic heat [Sekimoto 1998]

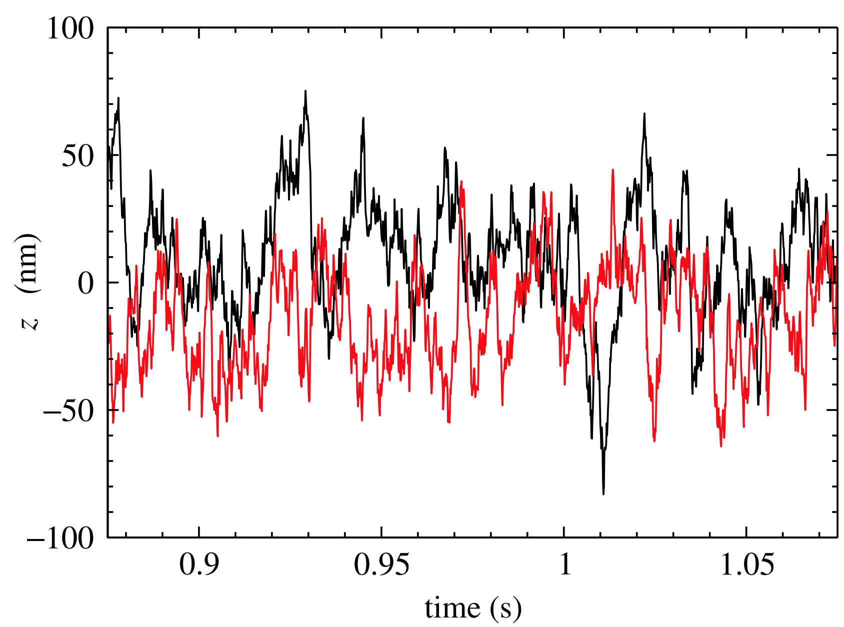
$$S(X) = -k_{\mathrm{B}} \ln P(X)$$

nonequilibrium system entropy

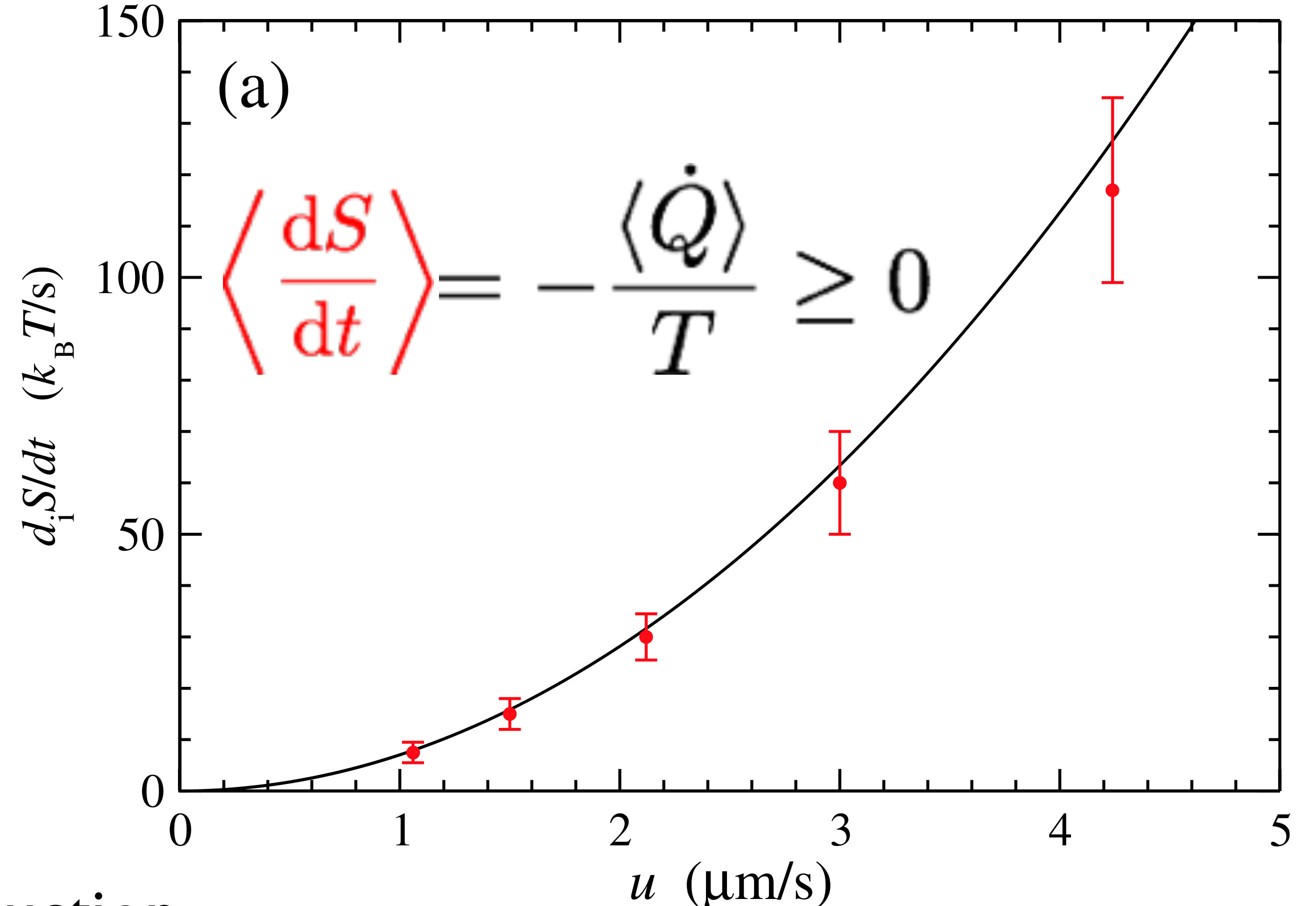
J. Lebowitz, H. Spohn, J. Stat. Mech. 1999; U. Seifert, PRL 2005

Stochastic entropy production

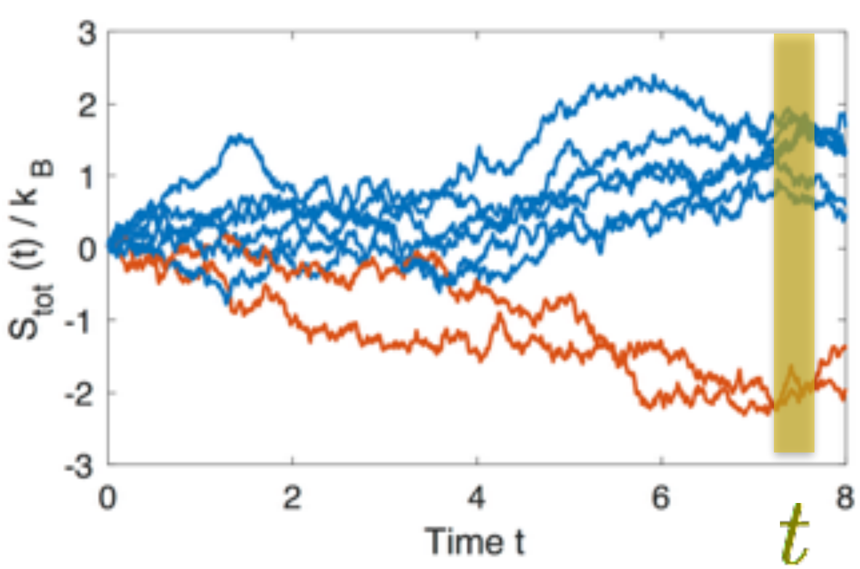
$$S(t) = k_{\mathrm{B}} \ln \frac{P\left(X_{[0,t]}\right)}{P\left(X_{[t,0]}\right)}$$



Andrieux, Gaspard, Ciliberto,
Garnier, Joubaud, Petrosyan, PRL 2007



Universal laws for stochastic entropy production



Detailed Fluctuation theorem

$$\frac{p_S(s;t)}{p_S(-s;t)} = e^{s/k_{\mathrm{B}}}$$

Jarzynski's equality

$$\langle e^{-S(t)/k_{\mathrm{B}}} \rangle = 1$$

Fixed time properties

Martingale theory for entropy production

I. Neri, É. Roldán, F. Jülicher, PRX **7** , 011019 (2017)

In steady state $e^{-S(t)/k_{\mathrm{B}}}$ is a Martingale process:

$$\langle e^{-S_{\mathrm{tot}}(t)/k_{\mathrm{B}}} | X_{[0,\tau]} \rangle = e^{-S_{\mathrm{tot}}(\tau)/k_{\mathrm{B}}}$$

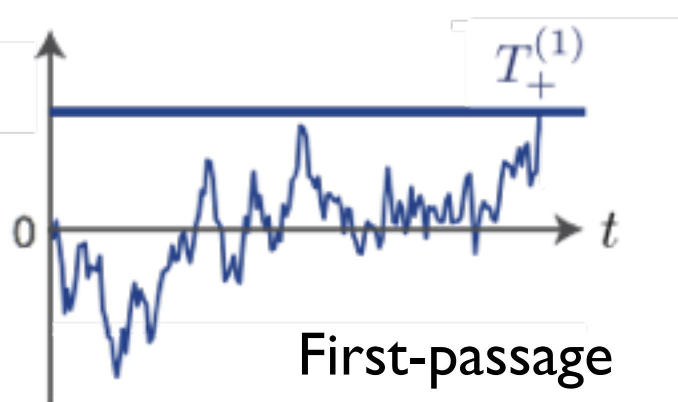
for any future time $t \geq \tau$

"Its expected value in the future (conditioned on a past history) equals to the last known value"

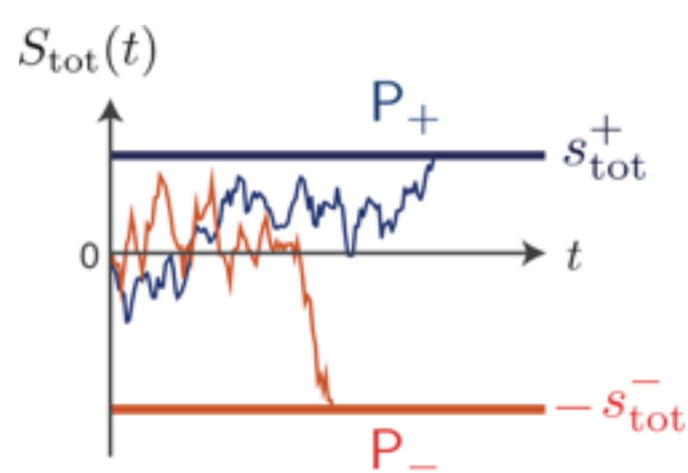
The martingale property <u>generalizes the Integral Fluctuation Theorem</u>

$$\tau = 0 \qquad \langle e^{-S_{\mathrm{tot}}(t)/k_{\mathrm{B}}} \rangle = e^{-S_{\mathrm{tot}}(0)/k_{\mathrm{B}}} = 1$$

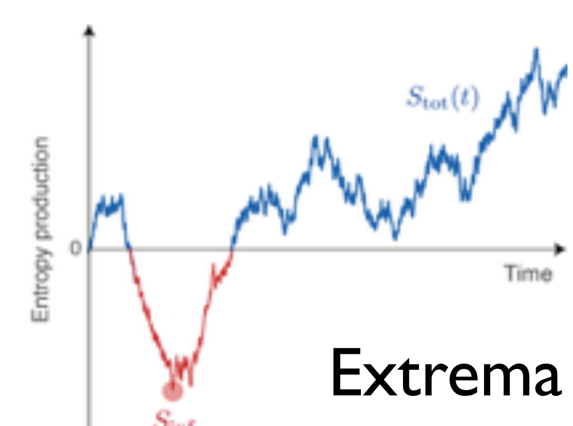
…and implies new universal properties of entropy production



First-passage times



Splitting probabilities



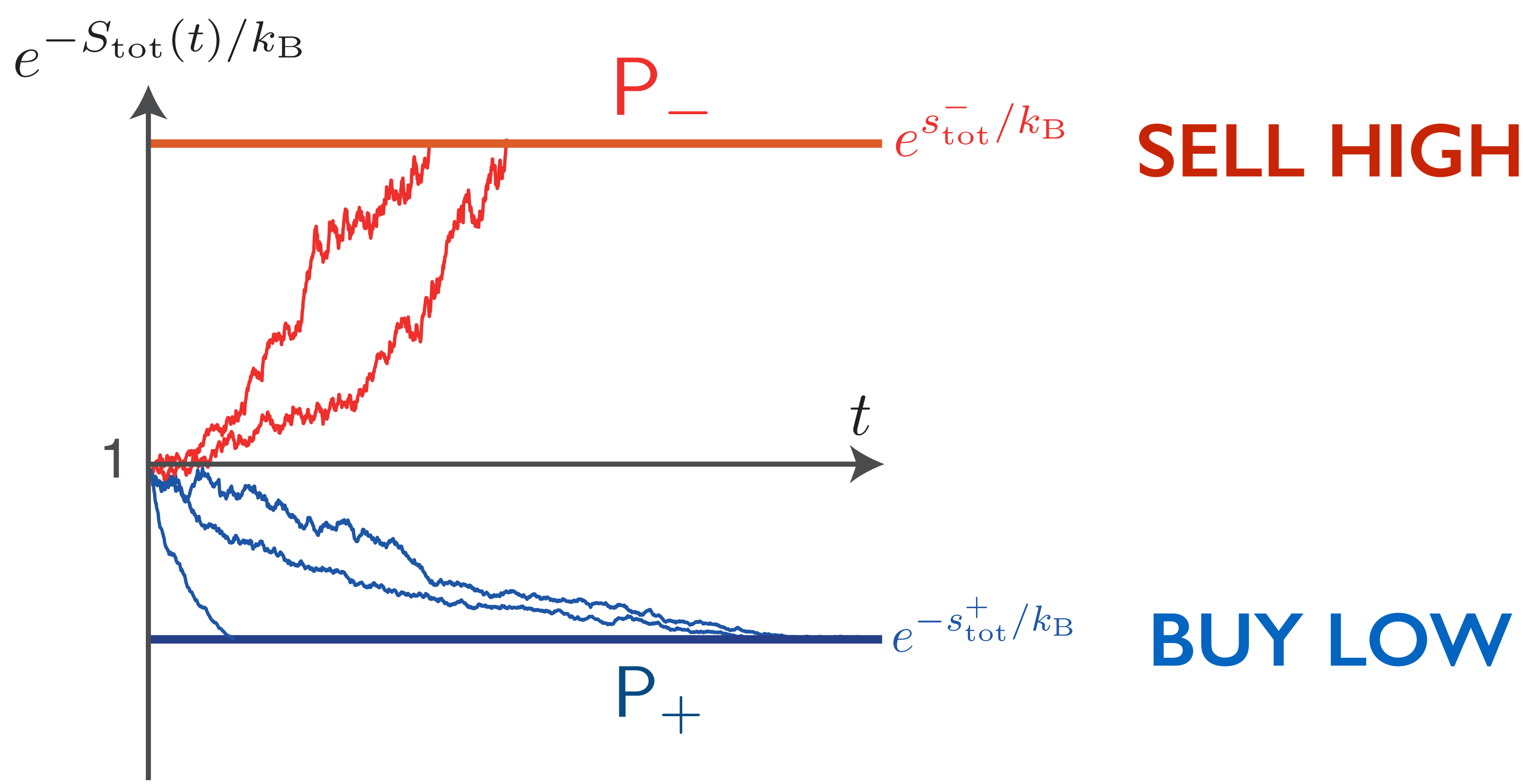
Extrema

Martingale theory for entropy production

I. Neri, É. Roldán, F. Jülicher, PRX **7** , 011019 (2017)

Martingales are often used to represent fair games or **risk-free markets**.

Doob's optional stopping theorem for Martingales

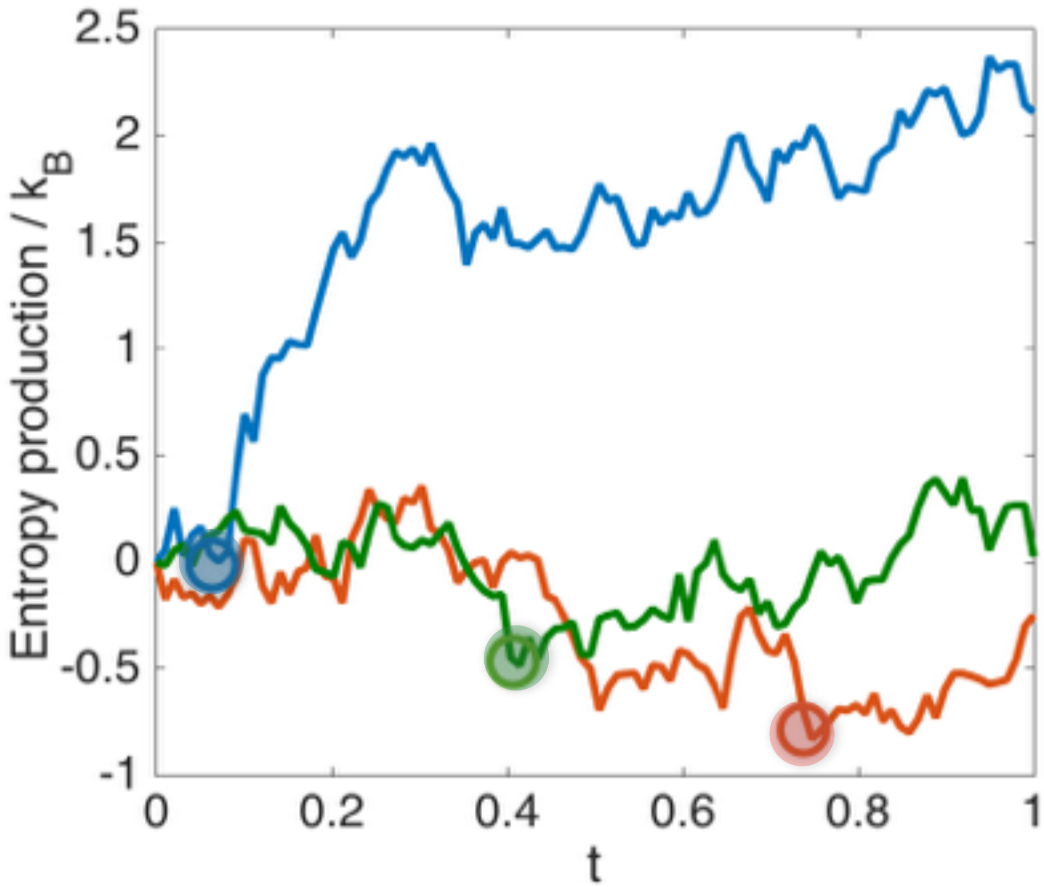


$$\langle e^{-S_{\rm tot}(T)/k_{\rm B}} \rangle = 1$$

You can't get profit in a fair game !

random stopping time

Statistics of infima of entropy production



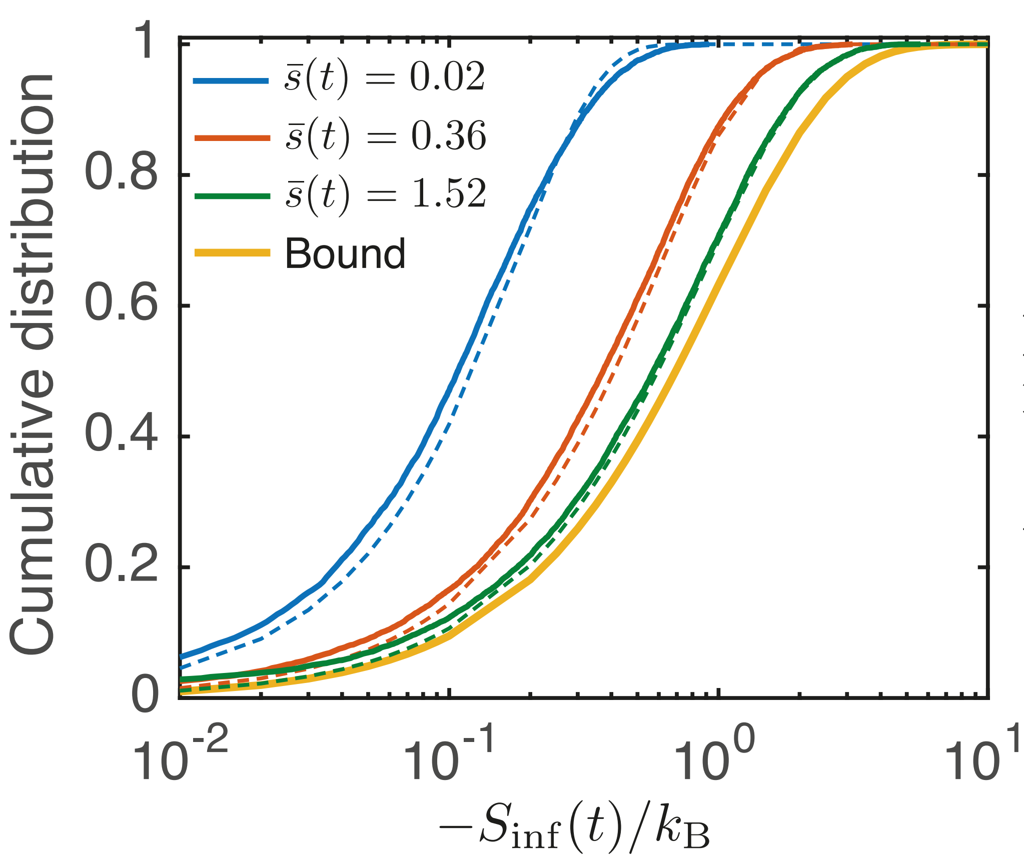
What is the minimum value (infimum) of stochastic entropy production in a time interval [0,t]?

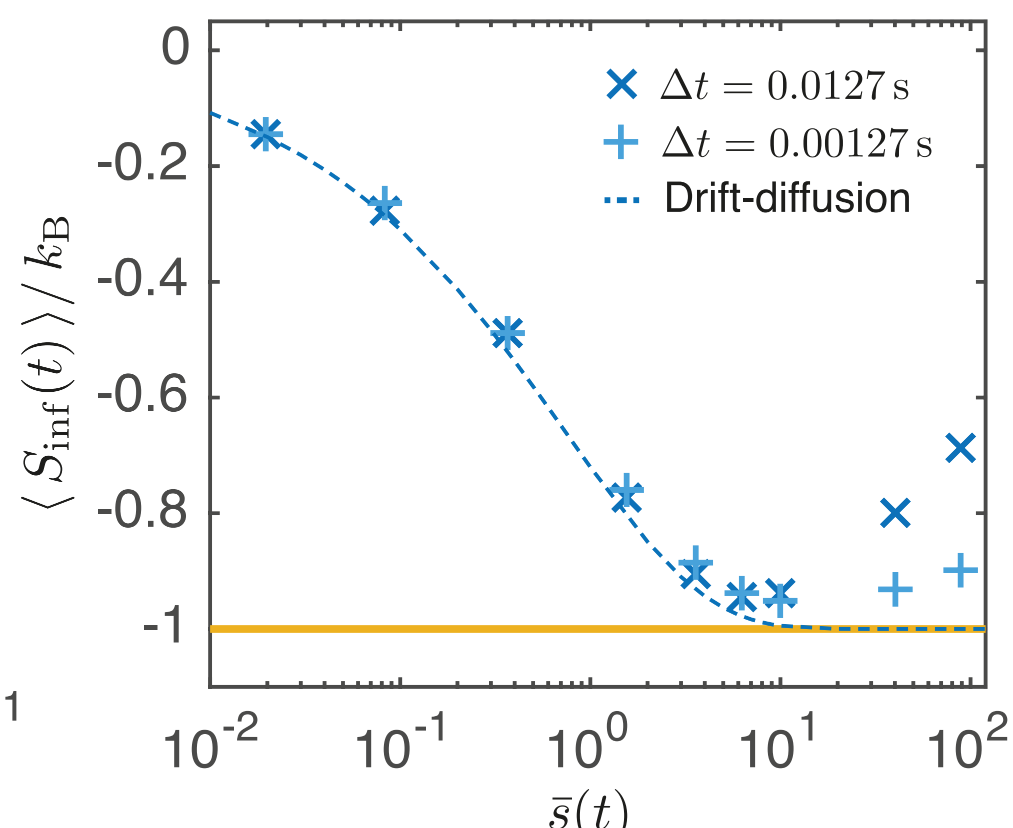
$$\Pr\left(S_{\inf}(t) \geq -s\right) \geq 1 - e^{-s/k_{\mathrm{B}}}$$

CDF of an exponential r.v. with mean $-k_{\mathrm{B}}$

“Infimum Law”

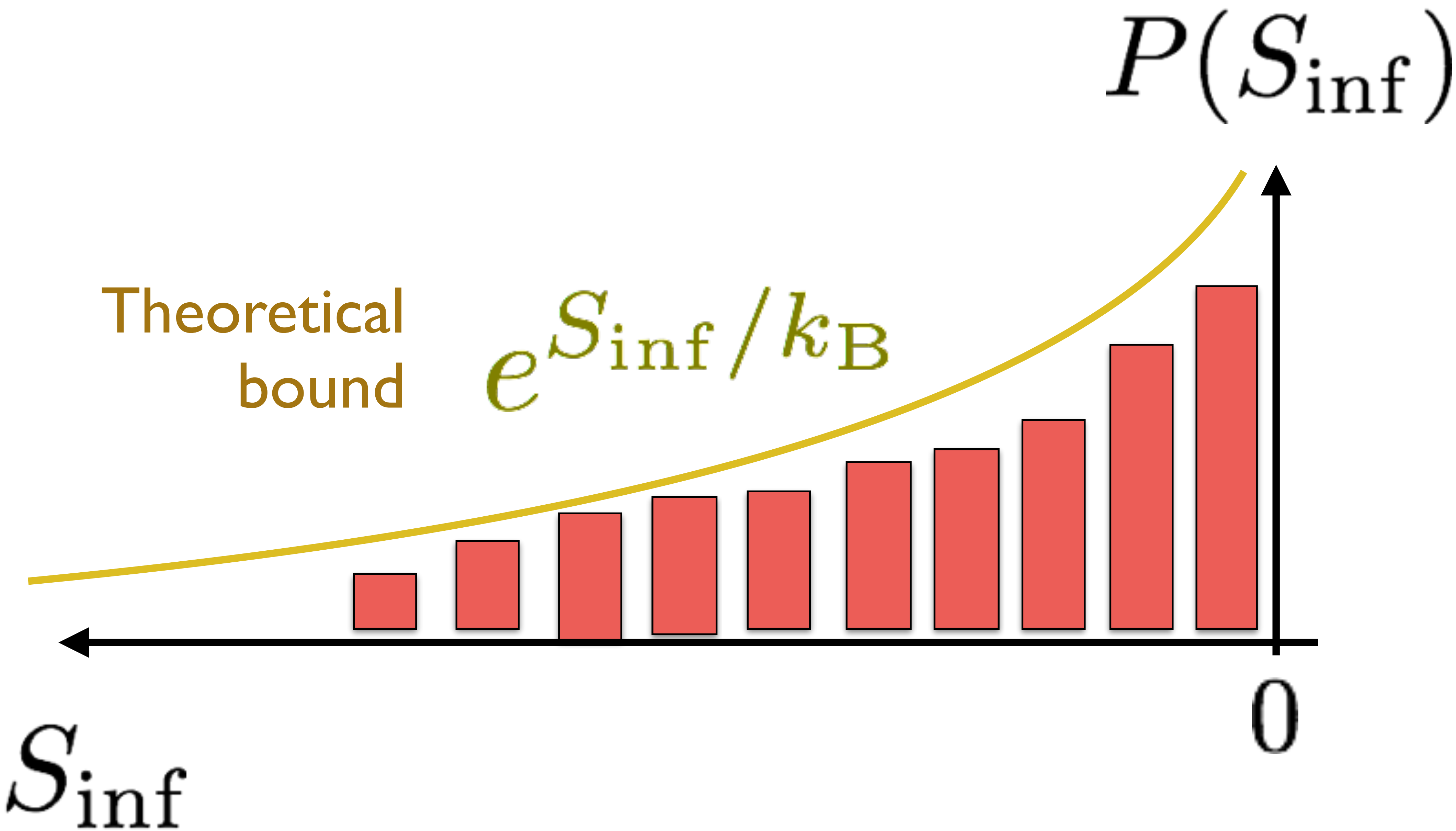
$$\langle S_{\inf}(t)\rangle \geq -k_{\mathrm{B}}$$





PRX **7** , 011019 (2017)

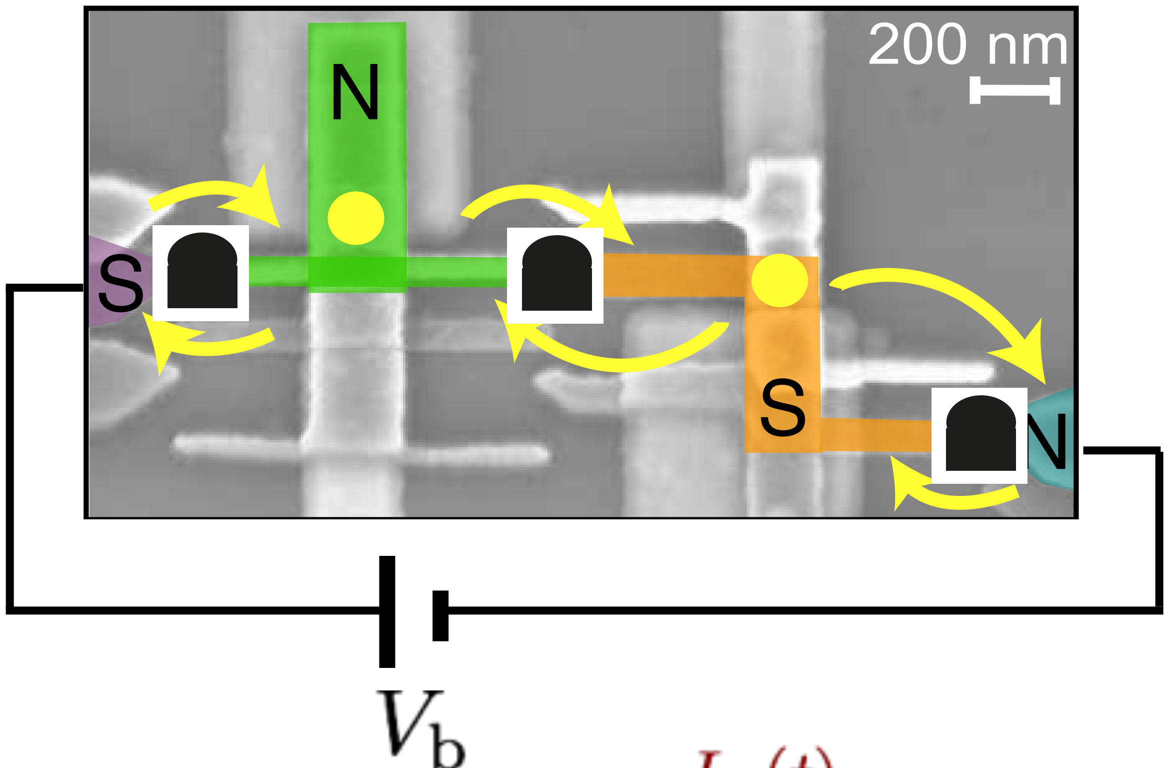
Statistics of infima of entropy production



Experimental test?

Electronic double dot

Experimental setup



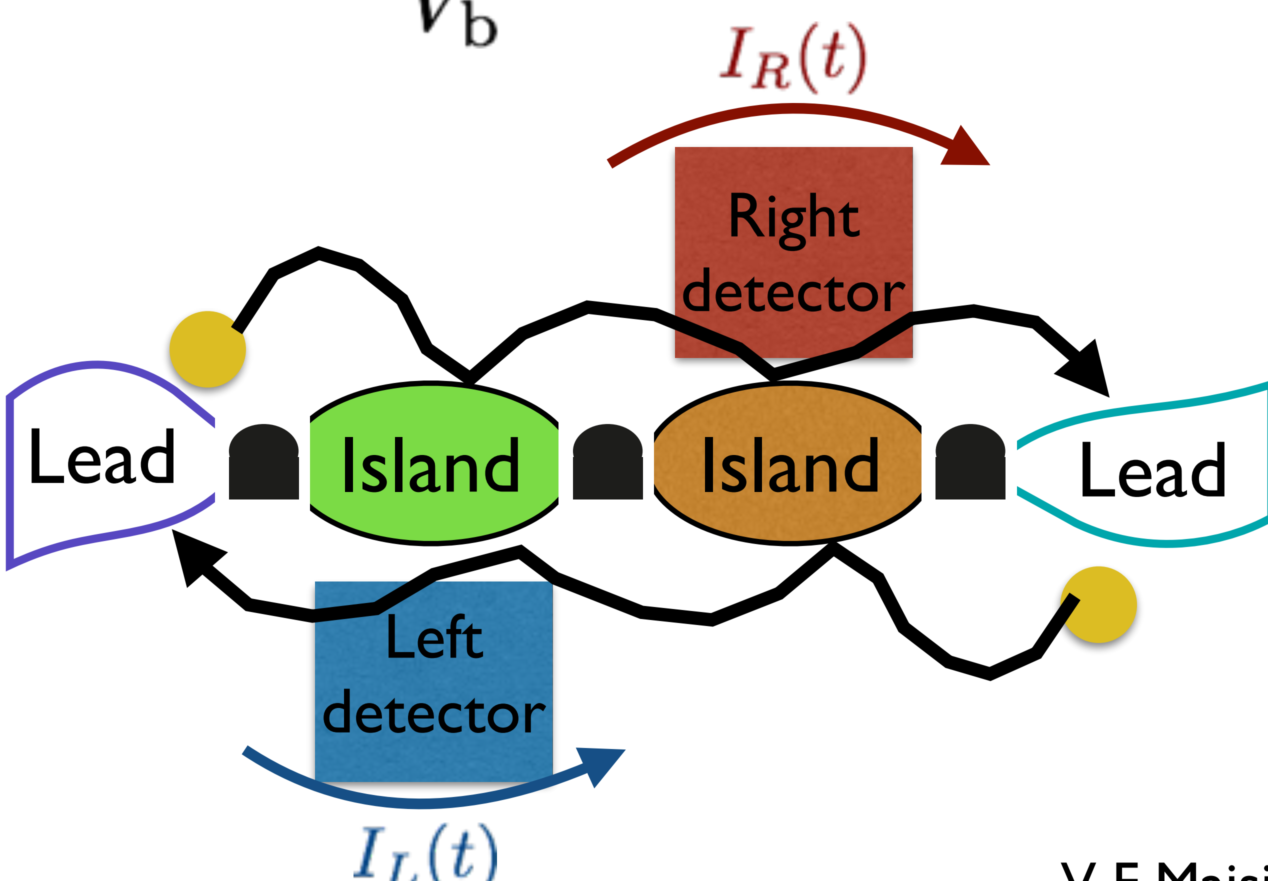
Normal metal (N): Copper
Superconductor (S): Aluminium

Base temperature 50mK

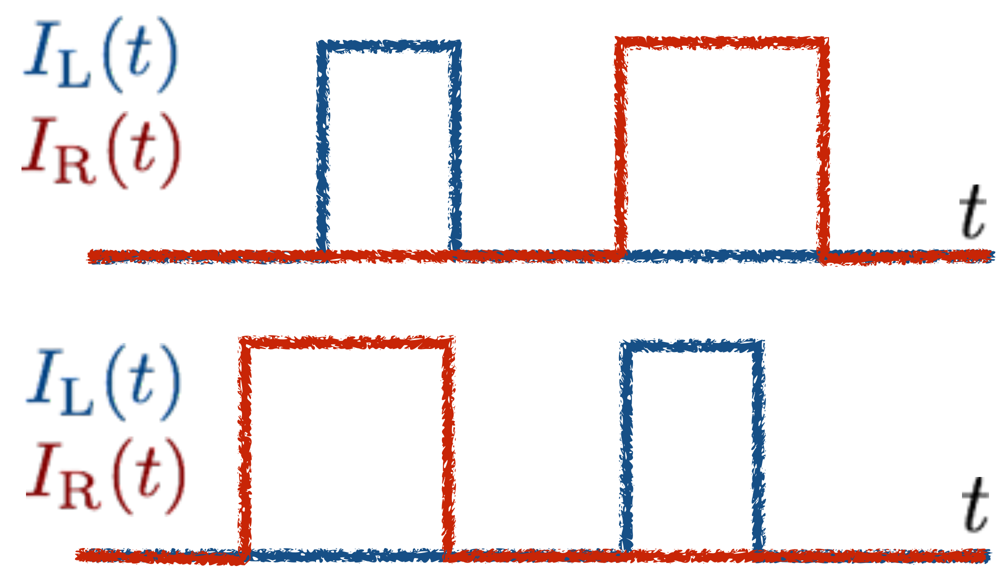
Tunnel junctions (Aluminium oxide)
R ~ MΩ , C ~ pF

External bias voltage ~ μV

Coulomb-blockade

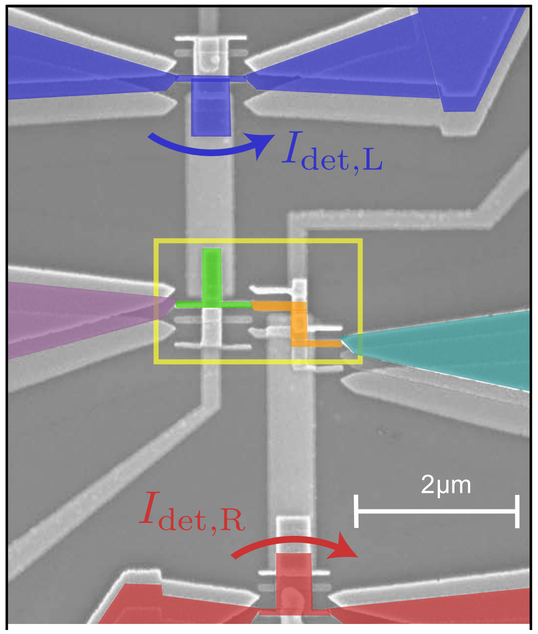


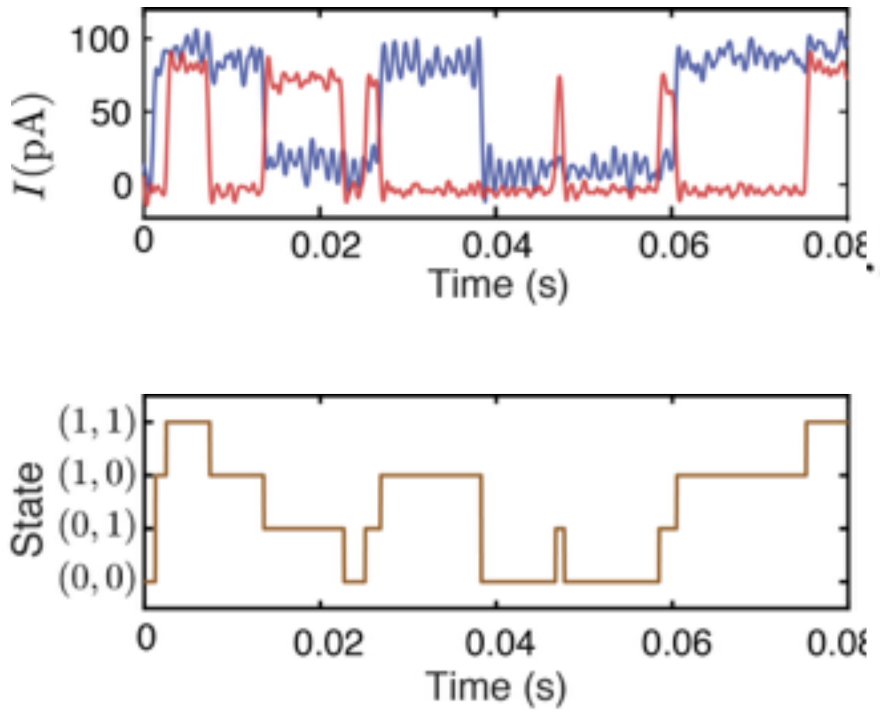
Detector currents

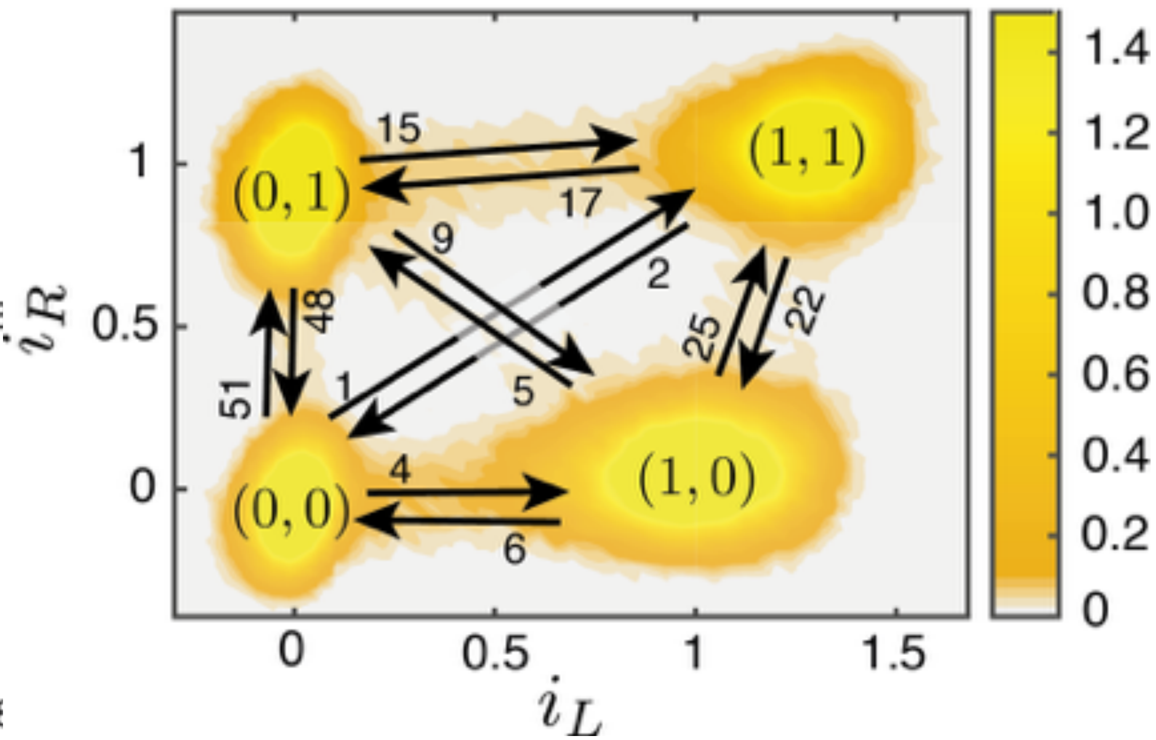


S. Singh, É. Roldán, I. Neri, I. M. Khaymovich, D. S. Golubev,
V. F. Maisi, J. T. Peltonen, F. Jülicher, J. P. Pekola, arXiv 1712.01693 (2017)

Entropy fluctuations in the double dot







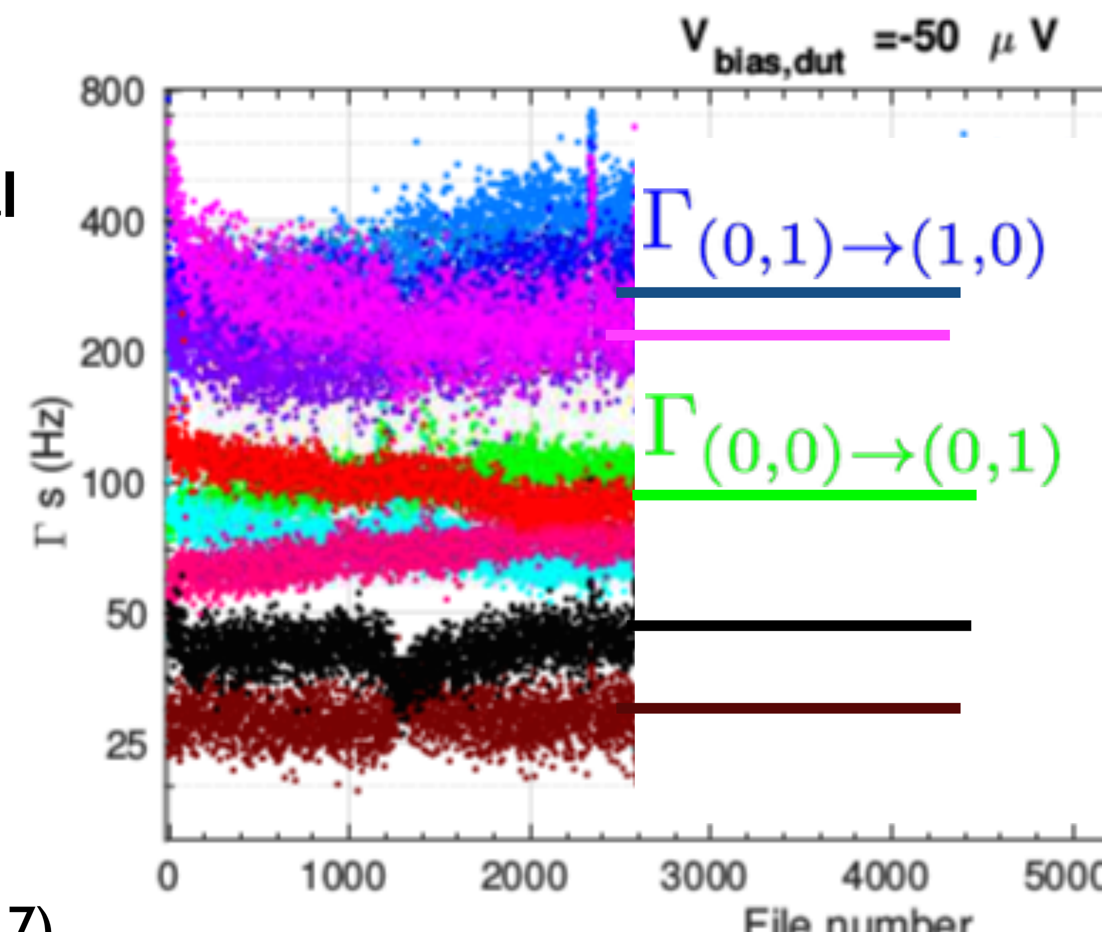
Stochastic entropy production

$$S_{\rm tot}(t) = \sum_{\alpha} s_\alpha X_\alpha(t)$$

net current along α

$$s_\alpha = k_{\rm B} \ln \frac{\Gamma(n_\alpha \to n'_\alpha)}{\Gamma(n'_\alpha \to n_\alpha)} + k_{\rm B} \ln \frac{P^{\rm st}(n_\alpha)}{P^{\rm st}(n'_\alpha)}$$

Experimental steady state



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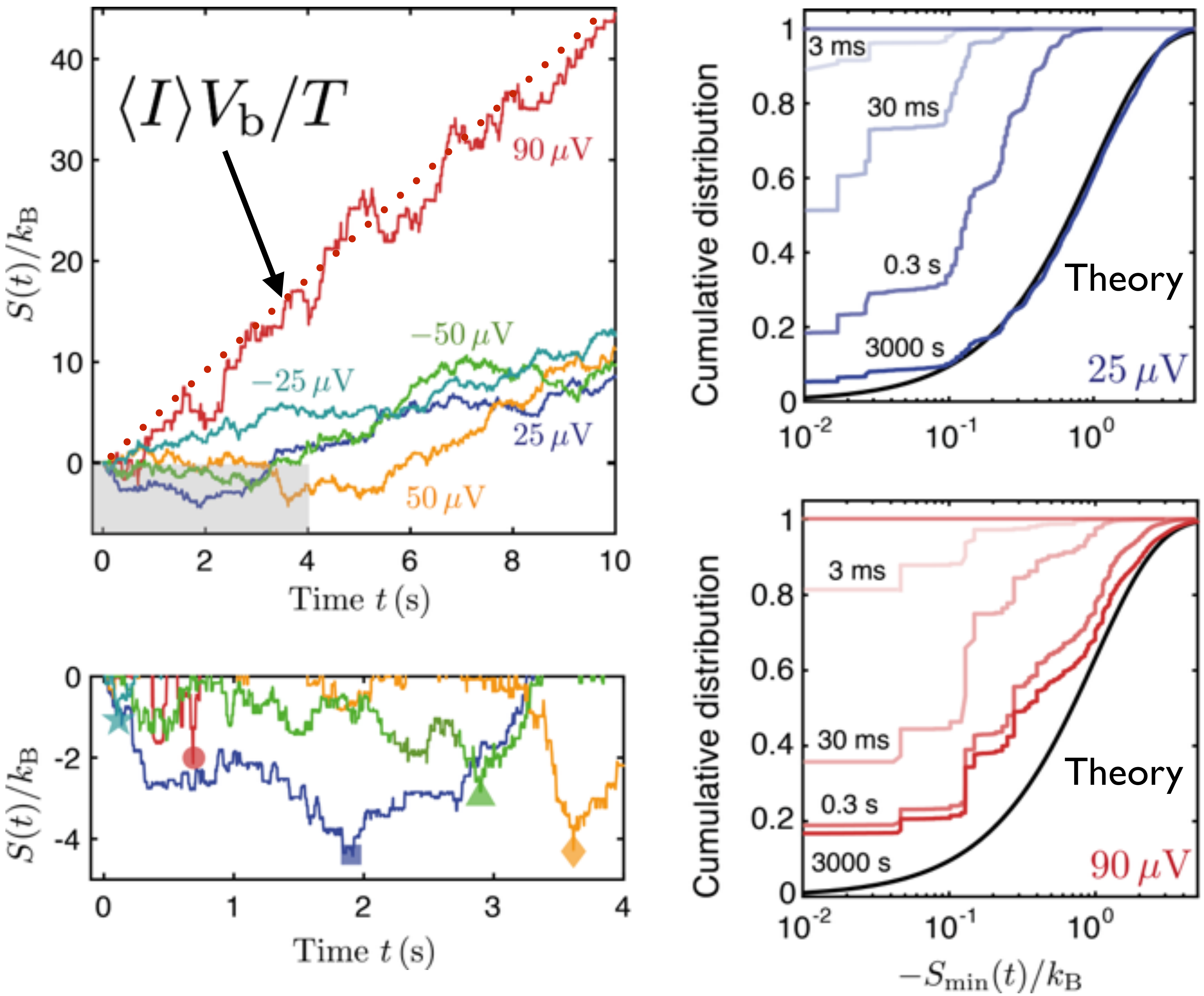
Extreme values of stochastic entropy production



Close to equilibrium

$$\langle \dot{Q}_{\text{Joule}} \rangle < 1 k_B T/\text{s}$$

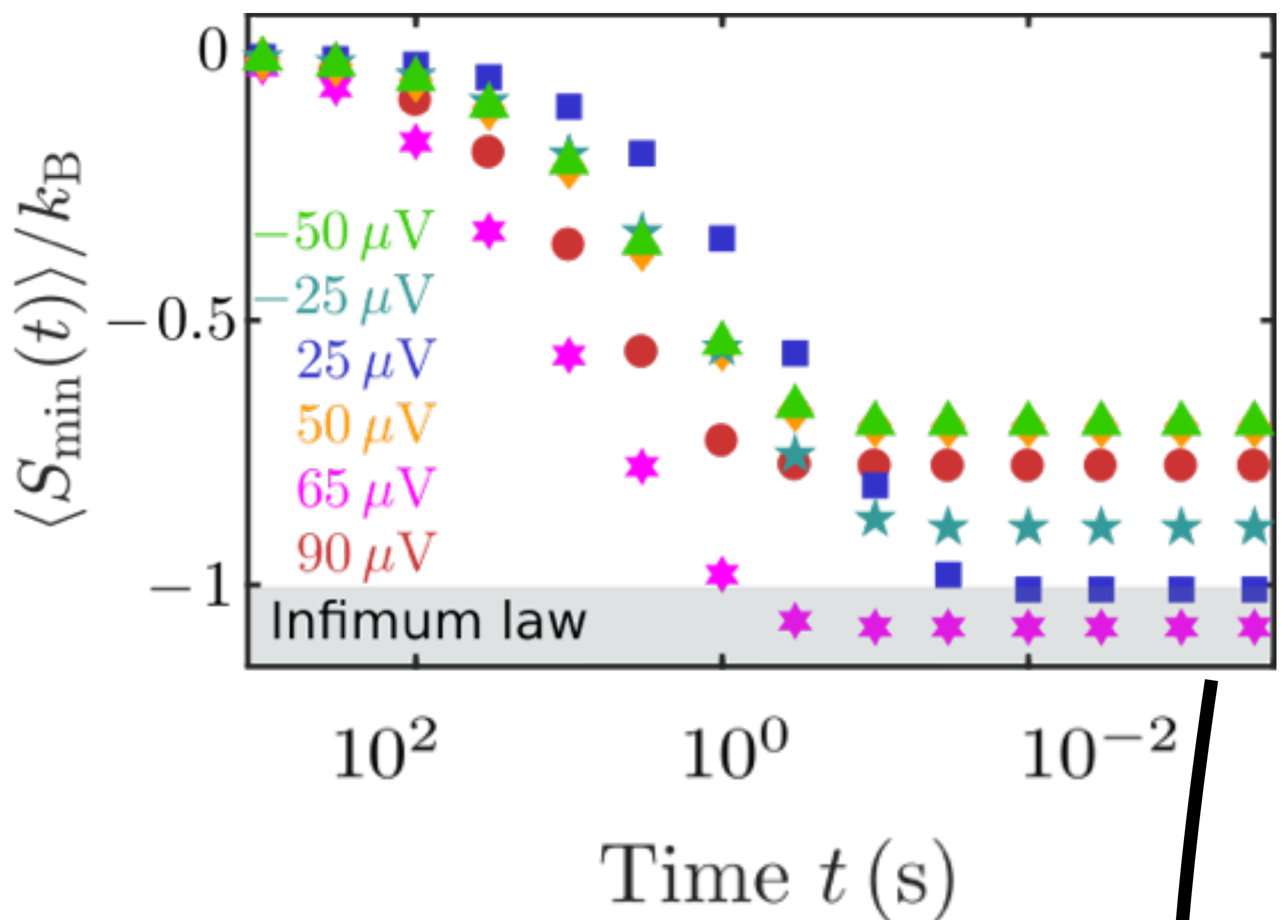
Far from equilibrium

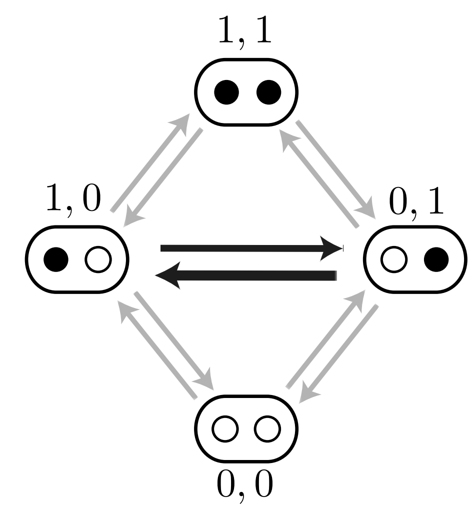
$$\langle \dot{Q}_{\text{Joule}} \rangle \sim 5 k_B T/\text{s}$$

$$\Pr\left(S_{\min}(t) \geq -s\right) \leq 1 - e^{-s/k_B}$$

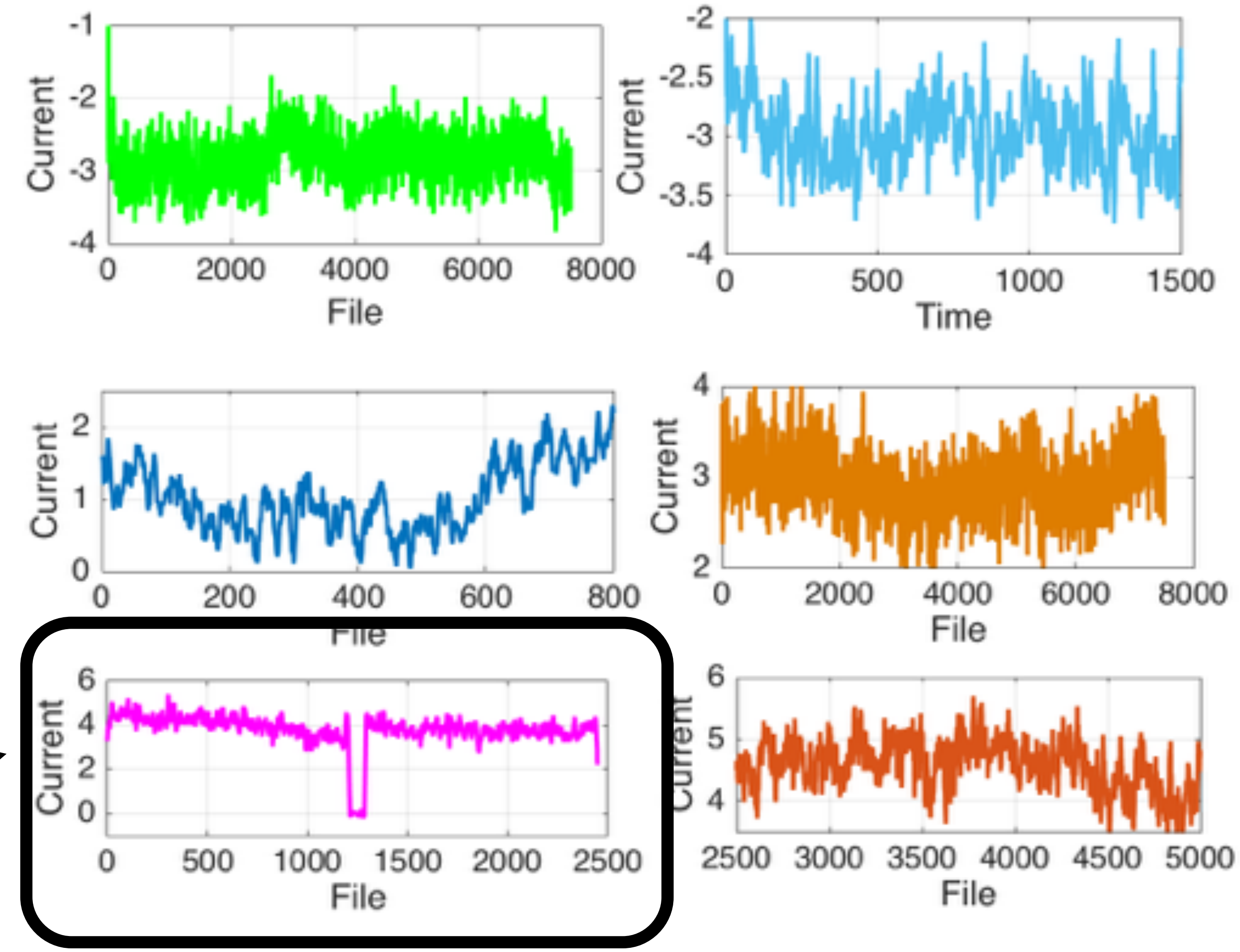
S. Singh, É. Roldán, I. Neri, I. M. Khaymovich, D. S. Golubev, V. F. Maisi, J. T. Peltonen, F. Jülicher, J. P. Pekola, arXiv 1712.01693 (2017)

Testing "infimum law" $\langle S_{\inf}(t)\rangle \geq -k_{\mathrm{B}}$





non-stationary conditions



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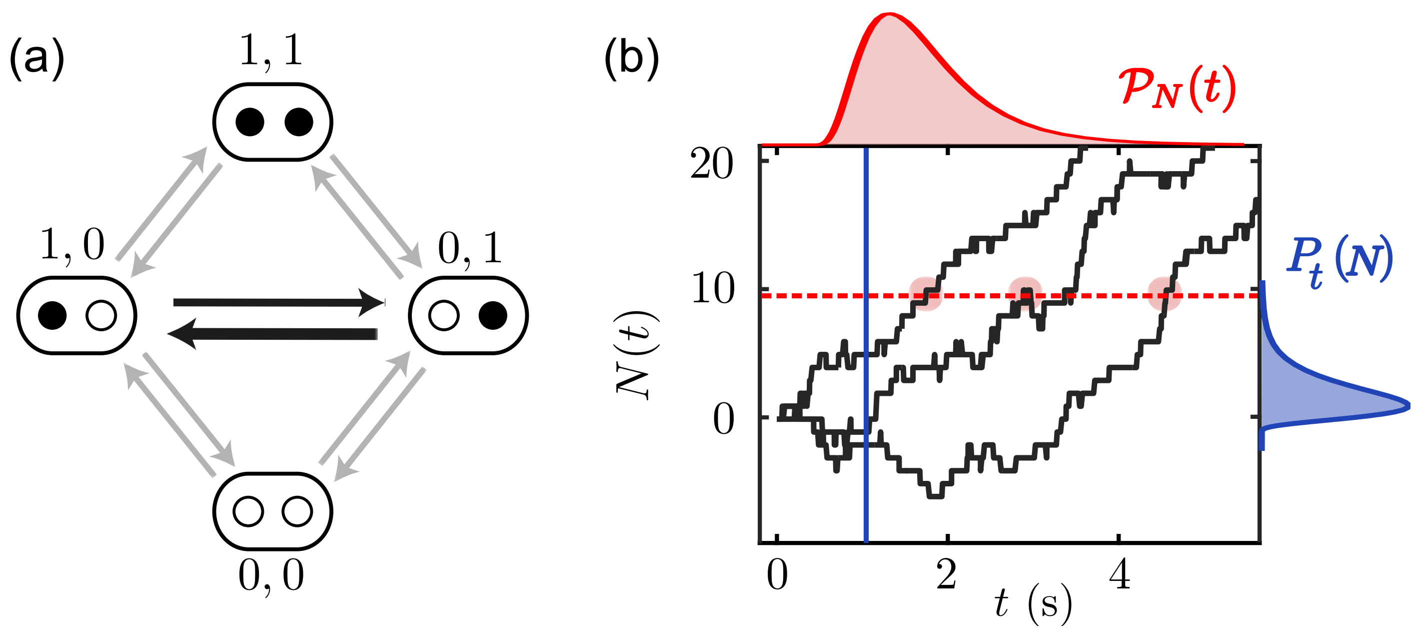
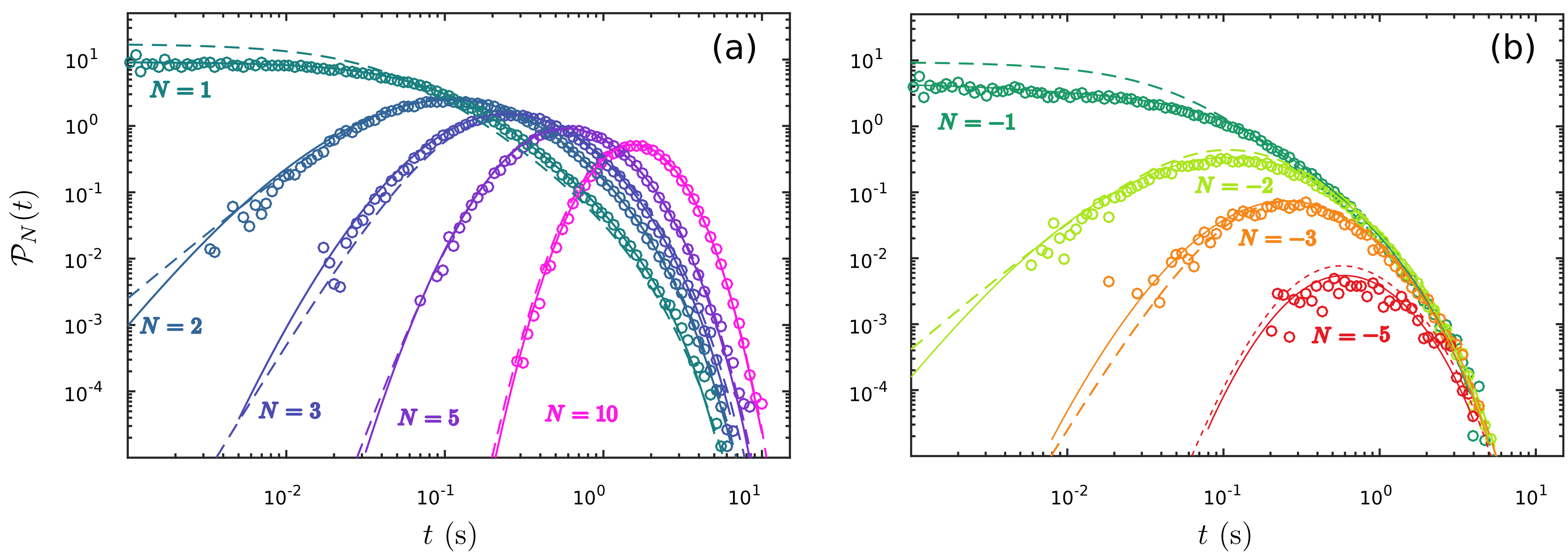
Universal first-passage-time distribution

S. Singh et. al, arXiv: **TODAY** or **TOMORROW**



$$\mathcal{P}_N(t) = \frac{|N^*| e^{-\frac{\mathcal{C}_1 \mathcal{C}_2}{\mathcal{C}_3} t}}{t} \left(\frac{\mathcal{C}_2 + \sqrt{\mathcal{C}_1 \mathcal{C}_3}}{\mathcal{C}_2 - \sqrt{\mathcal{C}_1 \mathcal{C}_3}} \right)^{\frac{N^*}{2}} \times I_{|N^*|} \left(\frac{\mathcal{C}_1 \sqrt{\mathcal{C}_2^2 - \mathcal{C}_1 \mathcal{C}_3}}{\mathcal{C}_3} t \right).$$

non-Gaussianity



Thanks!

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Joonas Peltonen
Aalto University

Frank Jülicher
MPIPKS Dresden

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Jukka Pekola
Aalto University

Happy birthday!