Absorption refrigerators based on Coulomb-coupled single-electron systems

Fabio Taddei

NEST, NANO-CNR & Scuola Normale Superiore, Pisa, Italy

Paolo Erdman (SNS & NEST, Pisa, Italy)
Bibek Bhandari (SNS & NEST, Pisa, Italy)
Rosario Fazio (ICTP Trieste & SNS Pisa, Italy)
Jukka Pekola (Aalto University, Finland)

Phys. Rev. B 98, 045433 (2018)

September 19th 2018, Helsinki
QT60 – Workshop on thermodynamics, thermoelectrics and transport in quantum devices

CNRNANO PISA NEST

NEST

SCUOLA NORMALE SUPERIORE PISA

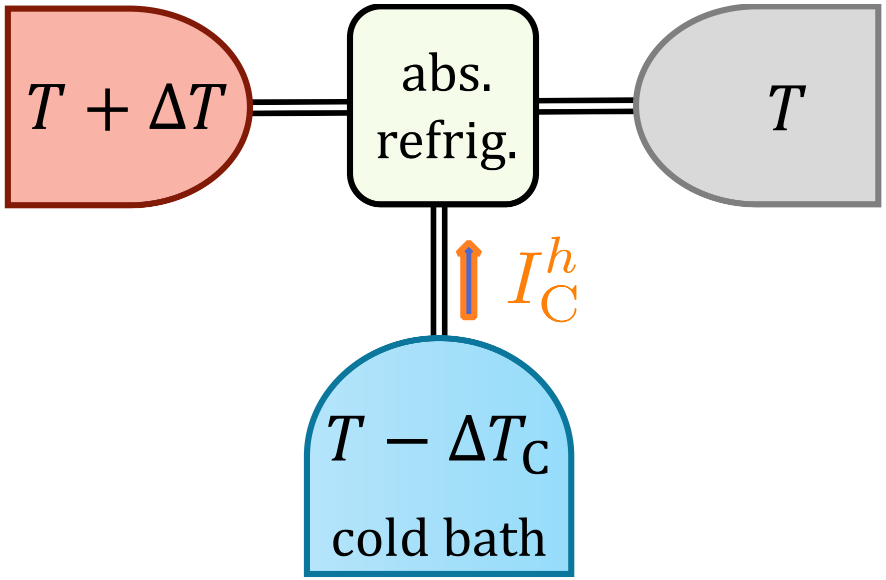
National Enterprise for nanoScience and nanoTechnology

NEST

Absorption refrigerators

➢ Refrigerators that use a **heat source** to drive the cooling process – *cooling is achieved by heating*

✱ **Extract heat from a cold thermal bath** using the interaction with other two thermal baths **at higher temperatures**



✱ **Waste heat** can be used to achieve **cooling** without external control of the system

Absorption refrigerators

✱ Solid state implementations

Pekola, Hekking (2007) - **NS junctions**
Linden, Popescu, Skrzypczyk (2010) - **qubit & qutrits (smallest heat engine?)**
Mari, Eisert & Cleuren, Rutten, Van den Broeck (2012) - **photon bath**
Yi-Xin Chen, Sheng-Wen Li (2012) - **rf-SQUID**
… and many others

→ role of coherence & entanglement

Brunner, Huber, Linden, Popescu, Silva, Skrzypczyk (2014)
Correa, Palao, Alonso, Adesso (2014)
Gelbwaser-Klimovsky, Kurizki (2014)
…

→ single-electron systems

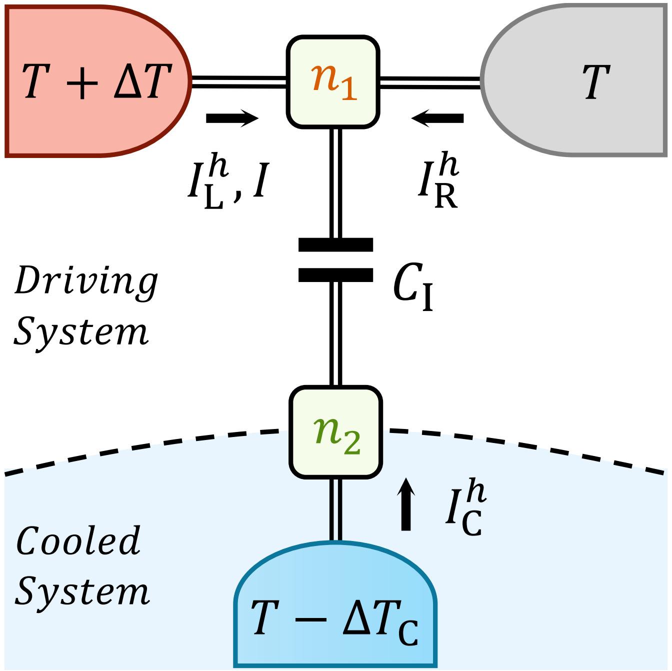
Venturelli, Fazio, Giovannetti (2013)
Sánchez, Thierschmann, Molenkamp (2017)
…

☑ Experiment: trapped atoms

Maslennikov, Ding, Hablutzel, Gan, Roulet, Nimmrichter, Dai, Scarani, Matsukevich (2017)

Two capacitively-coupled single-electron systems

With QDs, first mentioned in: Benenti, Casati, Saito, Whitney (2017)



n_1 $\qquad n_{x1} = V_{g1} \frac{C_g}{e}$

n_2 $\qquad n_{x2} = V_{g2} \frac{C_g}{e}$

* Coulomb-blockade regime

* Electrostatic energy

$$U(n_1, n_2) = E_{C1}(n_1 - n_{x1})^2 + E_{C2}(n_2 - n_{x2})^2 + E_I(n_1 - n_{x1})(n_2 - n_{x2})$$

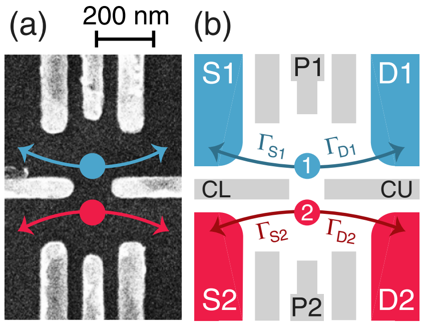
$$E_{Ci} = \frac{e^2}{2C_i}$$

inter-island interaction energy controlled by C_I

* Sequential tunneling

Capacitively-coupled QDs systems

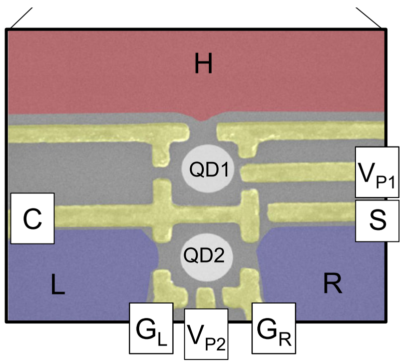
➢ Experimental setups

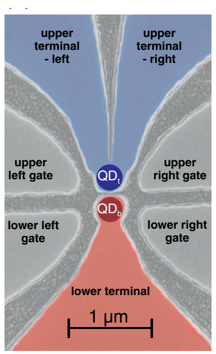


✱ lithographically-patterned QDs

McClure, et al. (2007)
Shinkai, Hayashi, Ota, Muraki, Fujisawa (2009)
Hartmann, Pfeffer, Höfling, Kamp, Worschech (2015)
Thierschmann, Arnold, Mittermüller, Maier, Heyn, Hansen, Buhmann, Molenkamp (2015)

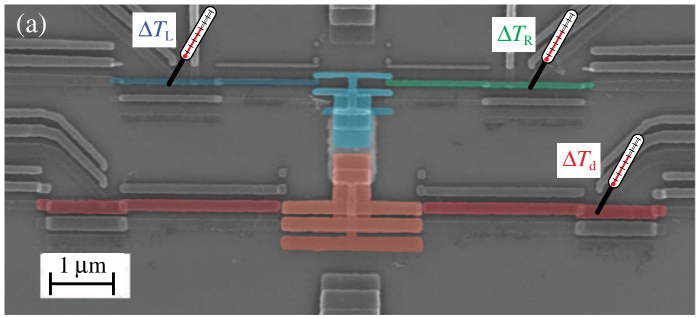
Keller et al. (2016)





✱ graphene-based QDs

Volk et al. (2015)
Bischoff et al. (2016)



✱ metallic islands

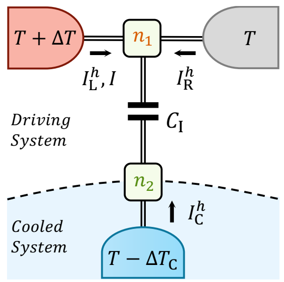
Koski, Kutvonen, Khaymovich, Ala-Nissila, Pekola (2015)
Singh, Roldán, Neri, Khaymovich, Golubev, Maisi, Peltonen, Jülicher, Pekola (2017)

Capacitively-coupled QDs systems

- Consider single occupation (4 charge states) with $n_1, n_2 = 0, 1$
- "Transition energies"

QD1 $\Rightarrow \;\; \Delta U_1(n_2) = U(1, n_2) - U(0, n_2)$

depends on the charge state in QD 2

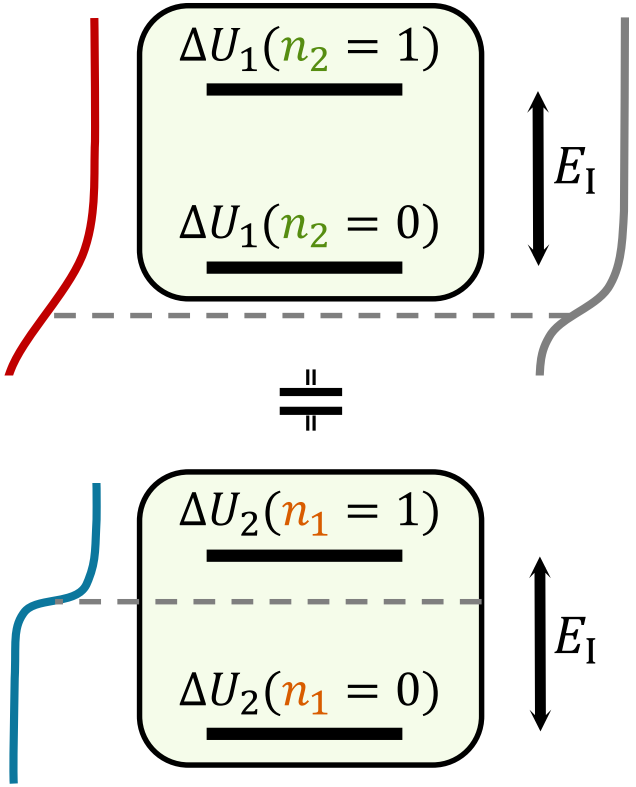


QD2 $\Rightarrow \;\; \Delta U_2(n_1) = U(n_1, 1) - U(n_1, 0)$

depends on the charge state in QD 1

Capacitively-coupled QDs systems

➢ Energy diagram



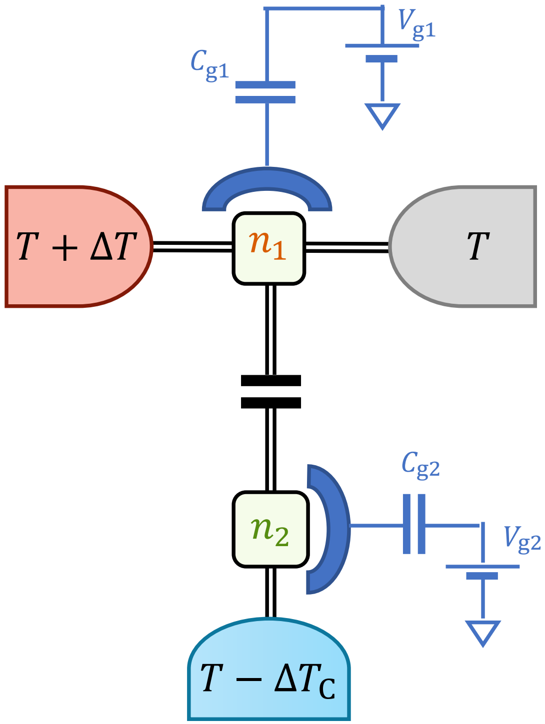
$$\theta_1 = 1 - n_{x2} + \frac{E_{C1}}{E_\mathrm{I}}(1 - 2n_{x1})$$

$$\theta_2 = 1 - n_{x1} + \frac{E_{C2}}{E_\mathrm{I}}(1 - 2n_{x2})$$

☑ Transition energies can be written as

$$\Delta U_1(n_2) = \theta_1 E_\mathrm{I} + (n_2 - 1)E_\mathrm{I}$$

$$\Delta U_2(n_1) = \theta_2 E_\mathrm{I} + (n_1 - 1)E_\mathrm{I}$$



Capacitively-coupled QDs systems

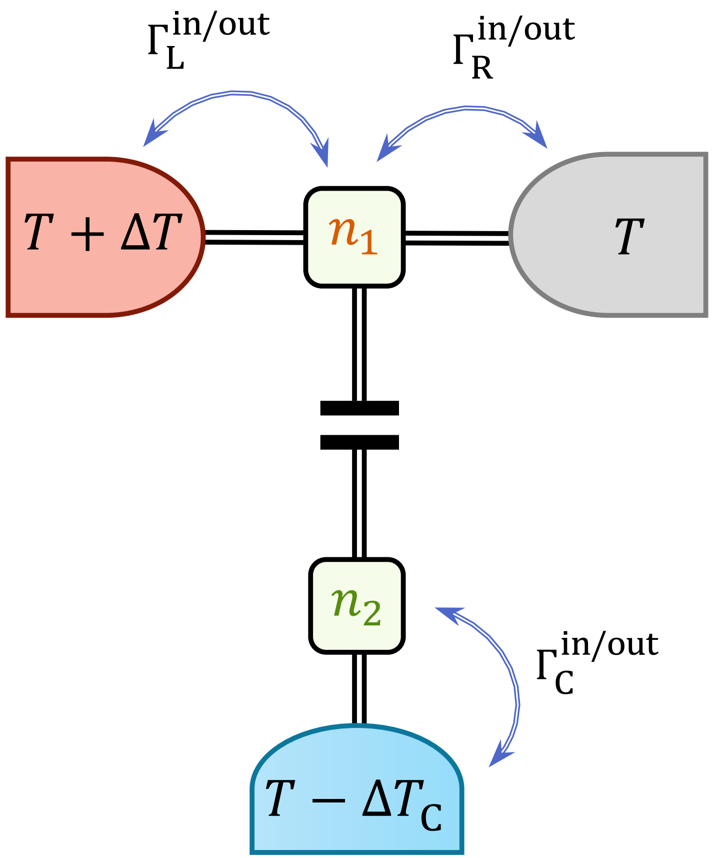
➢ Heat currents

→ occupation probability $P(n_1, n_2)$

→ tunneling rates

$$\Gamma_{\mathrm{L}}^{\mathrm{in/out}}(n_2) \qquad \Gamma_{\mathrm{R}}^{\mathrm{in/out}}(n_2)$$

$$\Gamma_{\mathrm{C}}^{\mathrm{in/out}}(n_1)$$



☑ Tunneling rates **depend on the occupation of the other QD**

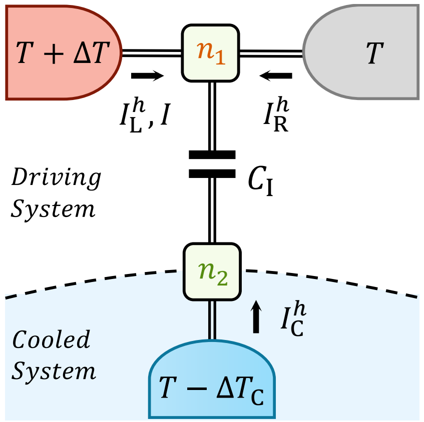
Capacitively-coupled QDs systems

➢ Tunneling rates

✱ Example, sequential tunneling rates for **QD 1** (and electrode L)

coupling strength

$$\Gamma_{\mathrm{L}}^{\mathrm{in}} = \frac{\gamma}{\hbar} f_{\mathrm{L}}[\Delta U_1]$$

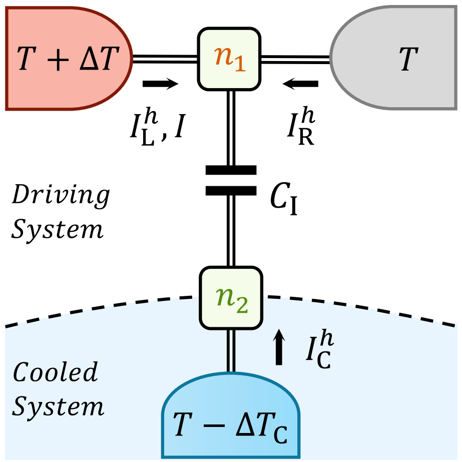


☑ $\Delta U_1 = \Delta U_1(n_2)$

$$\Rightarrow \quad \Gamma_{\mathrm{L}}^{\mathrm{in}}(n_2) = \frac{\gamma}{\hbar} f_{\mathrm{L}}[\Delta U_1(n_2)]$$

☑ Tunneling rates depend on the occupation of the other QD

Capacitively-coupled QDs systems



- Cooling power I_{C}^{h}
- COP (coefficient of performance) for refrigeration

$$\eta = \frac{I_{\mathrm{C}}^{h}}{I_{\mathrm{L}}^{h}}$$

cooling power

input heat

Can be calculated only assuming **detail balance condition**

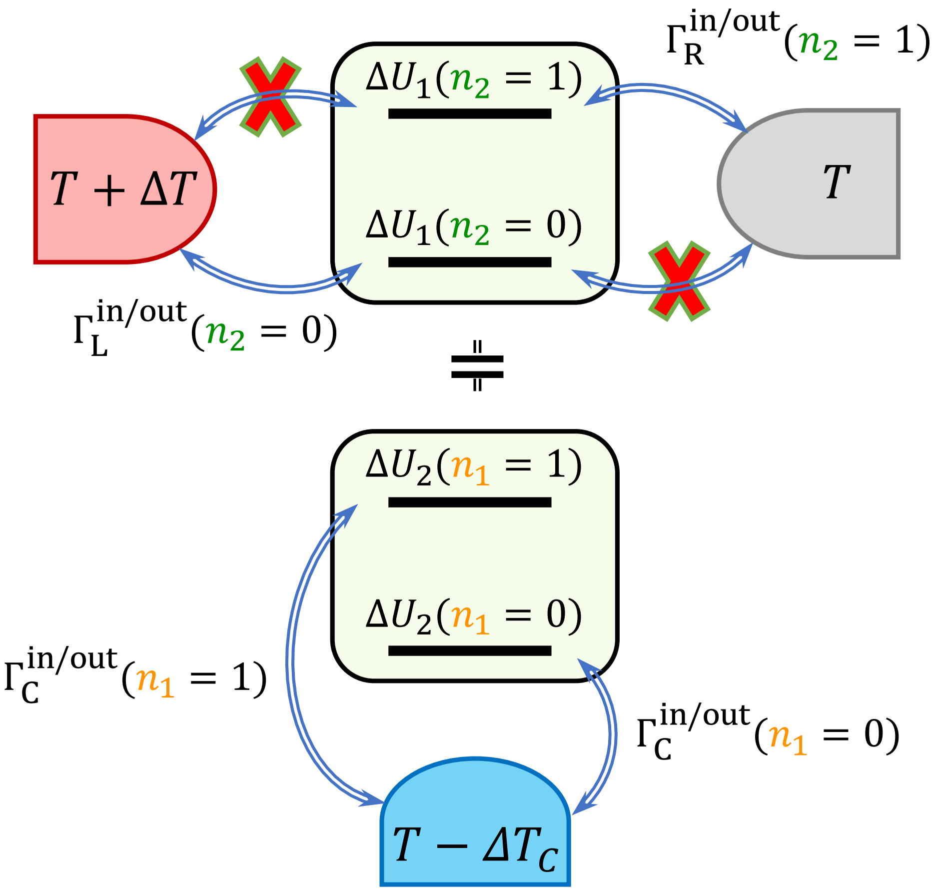
$$\Gamma_{\mathrm{L/R}}^{\mathrm{out}}(n_2) = \exp\left[\frac{\Delta U_1(n_2)}{k_B T_{\mathrm{L/R}}}\right]\Gamma_{\mathrm{L/R}}^{\mathrm{in}}(n_2)$$

$$\Gamma_{\mathrm{C}}^{\mathrm{out}}(n_1) = \exp\left[\frac{\Delta U_2(n_1)}{k_B T_{\mathrm{C}}}\right]\Gamma_{\mathrm{C}}^{\mathrm{in}}(n_1)$$

Capacitively-coupled QDs systems

☑ Cooling power is maximized under the conditions

$$\Gamma_{\mathrm{L}}^{\mathrm{in/out}}(n_2 = 1) = 0 \qquad \Gamma_{\mathrm{R}}^{\mathrm{in/out}}(n_2 = 0) = 0$$



Capacitively-coupled QDs systems

CNRNANO PISA NEST

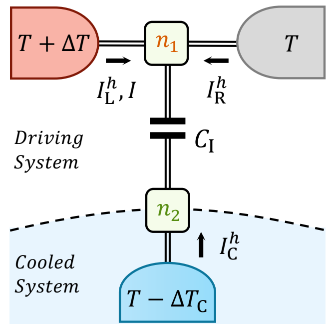
- ☑ COP takes simple form

$$\eta = \frac{1}{\theta_1 - 1}$$

$$\theta_1 > \theta_1^* \equiv 1 + \frac{1}{\eta_{\mathrm{C}}^{\mathrm{h}} \eta_{\mathrm{C}}^{\mathrm{r}}}$$

$$\theta_1 = 1 - n_{x2} + \frac{E_{C1}}{E_{\mathrm{I}}}(1 - 2n_{x1})$$

$$\eta_{\mathrm{C}}^{\mathrm{h}} = 1 - \frac{T}{T_{\mathrm{L}}} \qquad \eta_{\mathrm{C}}^{\mathrm{r}} = \frac{T_{\mathrm{C}}}{T - T_{\mathrm{C}}}$$



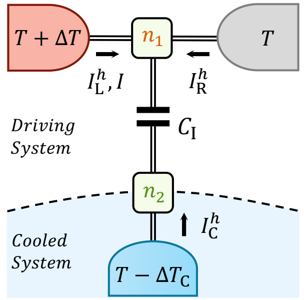
- ☑ COP is maximum when $\theta_1 = \theta_1^*$

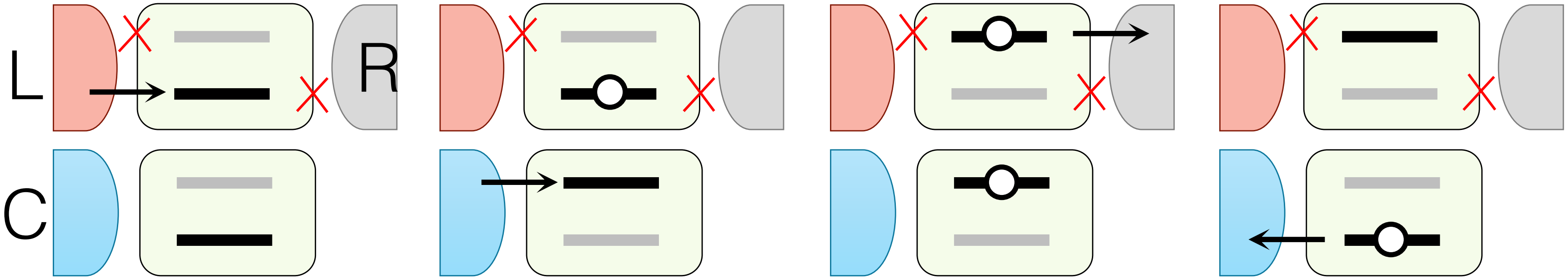
$$\Rightarrow \quad \eta_{\max} \equiv \eta_{\mathrm{C}}^{\mathrm{h}} \eta_{\mathrm{C}}^{\mathrm{r}}$$

→ **combination of two two-terminal reversible machines each operating at Carnot's efficiency**

Capacitively-coupled QDs systems

✱ **Simple picture** (sequence of electron transitions that leads to the removal of heat from reservoir C)





$\delta Q_L = (\theta_1 - 1)E_I$ $\quad$ $\delta Q_C = \theta_2 E_I$ $\quad$ $\delta Q_R = -\theta_1 E_I$ $\quad$ $\delta Q_C = (1-\theta_2)E_I$

$\Delta U_1(0)$ $\quad$ $\Delta U_2(1)$ $\quad$ $-\Delta U_1(1)$ $\quad$ $-\Delta U_2(0)$

$n_1 = 0$	$n_1 = 1$	$n_1 = 1$	$n_1 = 0$
$n_2 = 0$	$n_2 = 0$	$n_2 = 1$	$n_2 = 1$

Capacitively-coupled QDs systems

- In one cycle:

  the amount of **heat extracted from C**

$$\delta Q_{\mathrm{C}}^{\mathrm{tot}} = E_{\mathrm{I}}$$

  the amount of **input heat provided by L**

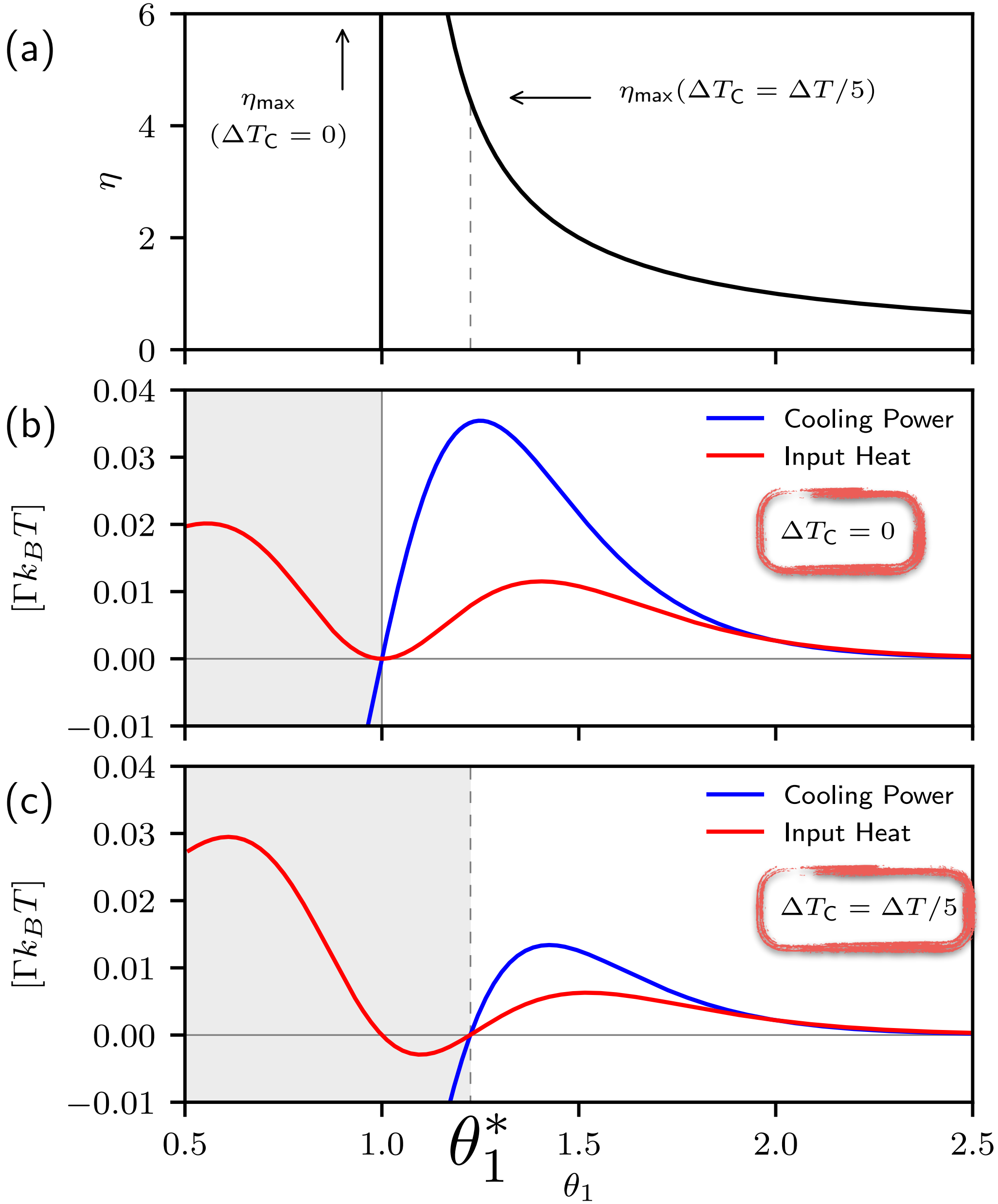
$$\delta Q_{\mathrm{L}}^{\mathrm{tot}} = (\theta_1 - 1) E_{\mathrm{I}}$$

COP

$$\eta = \frac{\delta Q_{\mathrm{C}}^{\mathrm{tot}}}{\delta Q_{\mathrm{L}}^{\mathrm{tot}}} = \frac{1}{\theta_1 - 1}$$

Capacitively-coupled QDs systems

$$\Delta T = 1/10\ T; \theta_2 = 1; E_{\rm I} = 6k_BT$$



☑ Cooling power is **zero** and efficiency is **max** when

$$\theta_1 = \theta_1^*$$

$$\theta_1^* = 1 + \frac{1}{\eta_{\rm C}^{\rm h}\eta_{\rm C}^{\rm r}}$$

☑ Cooling power weakly depends on θ_2

☑ $\eta_{\rm max}$ diverges

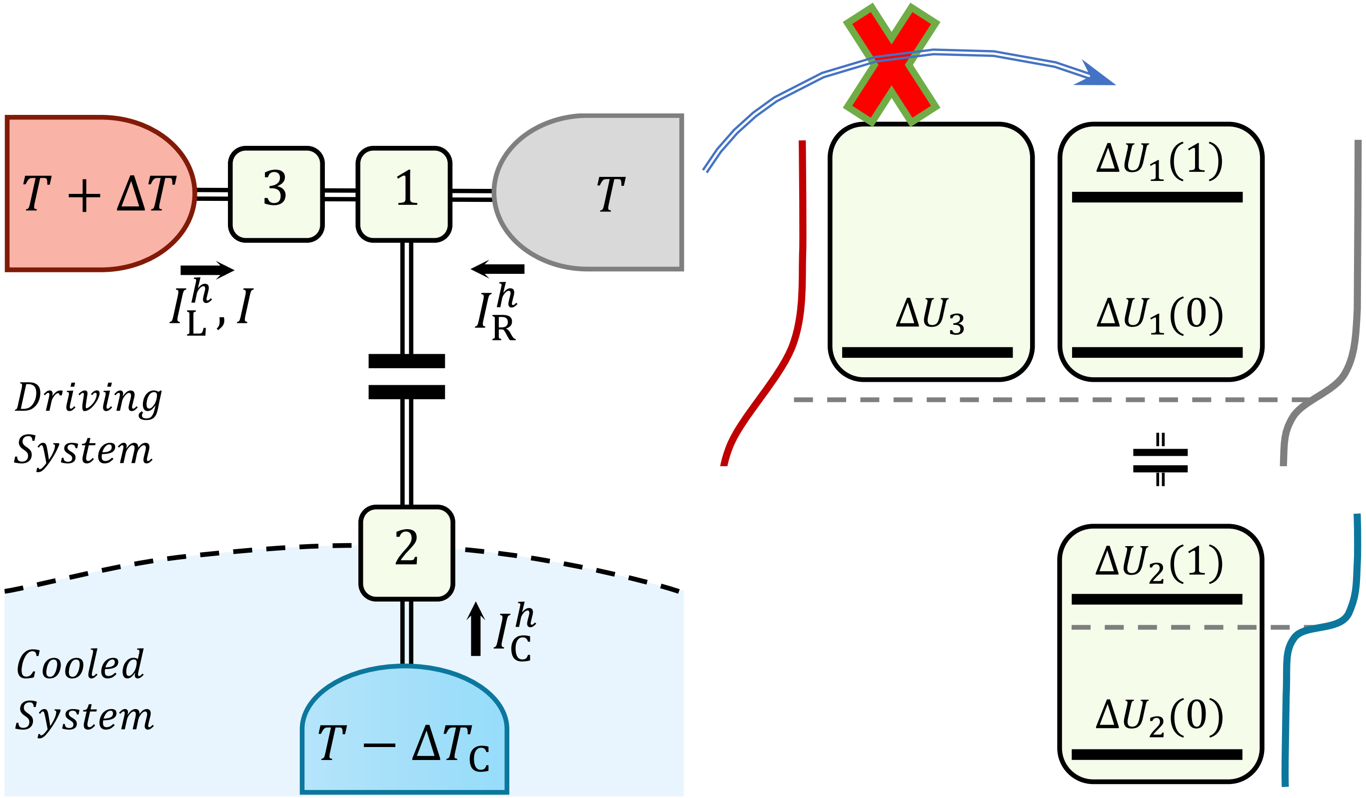
☑ $\eta_{\rm max}$ finite

☑ max cooling power and corresponding efficiency decrease

National Enterprise for nanoScience and nanoTechnology

Experimental proposal 1: quantum dots

- Two capacitively-coupled QDs + one additional QD



- "Energy filtering" effect used to suppress $\Gamma_L^{in/out}(1)$ by aligning

$$\Delta U_3 = \Delta U_1(0)$$

Experimental proposal 1

CNRNANO PISA NEST

- Electrostatic energy

$$U(n_1, n_2, n_3) = E_{C1}(n_1 - n_{x1})^2 + E_{C2}(n_2 - n_{x2})^2 + E_{C3}(n_3 - n_{x3})^2 + E_{\mathrm{I}}(n_1 - n_{x1})(n_2 - n_{x2}),$$

- Assume that: QD3 and QD1 are weakly coupled
  each QD can be only singly-occupied
  sequential tunneling regime

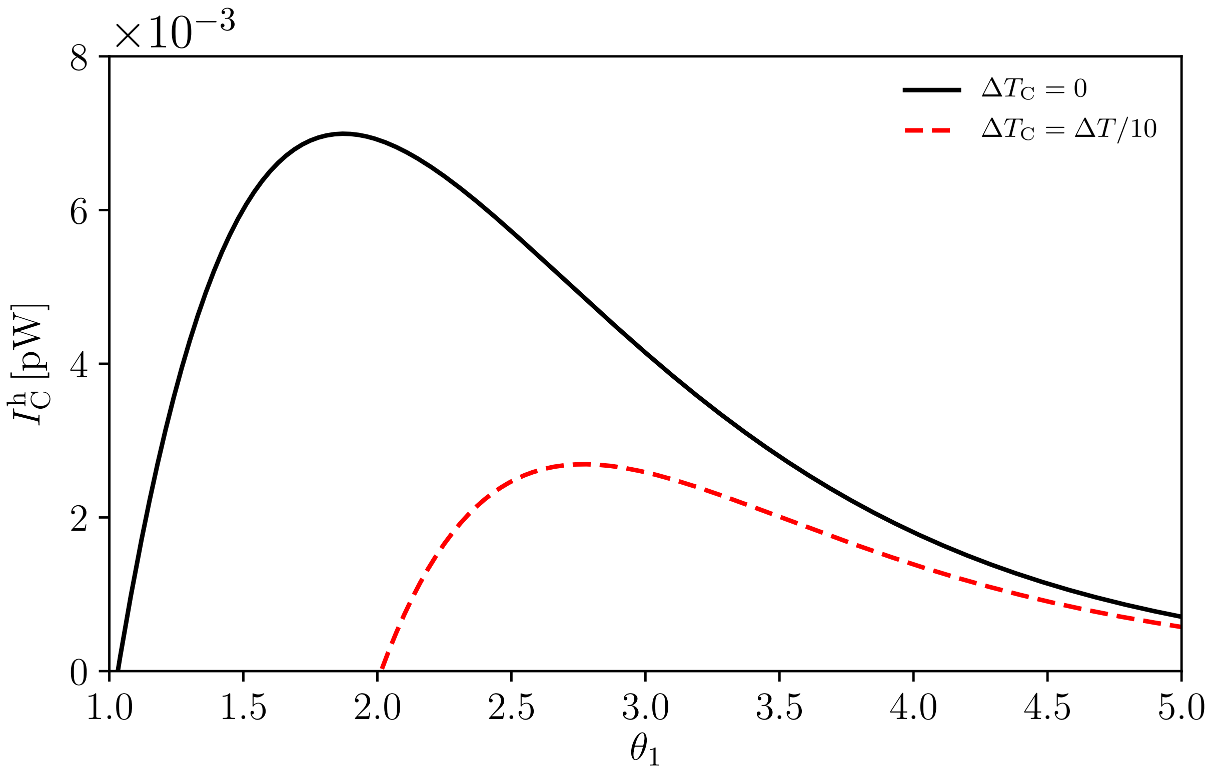
- Master equation to calculate probabilities for different occupations

Experimental proposal 1

CNRNANO PISA NEST

✱ Cooling power, using realistic parameters

Keller, Lim, Sánchez, López, Amasha, Katine, Shtrikman, Goldhaber-Gordon (2016)



$$\Delta U_3 = \Delta U_1(0); \theta_2 = 1/2$$
$$E_{\mathrm{I}} = 0.72 \text{ meV}$$
$$T = \Delta T = 4.17 \text{ K}$$

- ☑ Heat extraction obtained even with **one condition** only: $\Gamma_{\mathrm{L}}^{\mathrm{in/out}}(1) \simeq 0$
- ☑ $I_{\mathrm{C}}^{h} \sim 10^{-2}$ pW

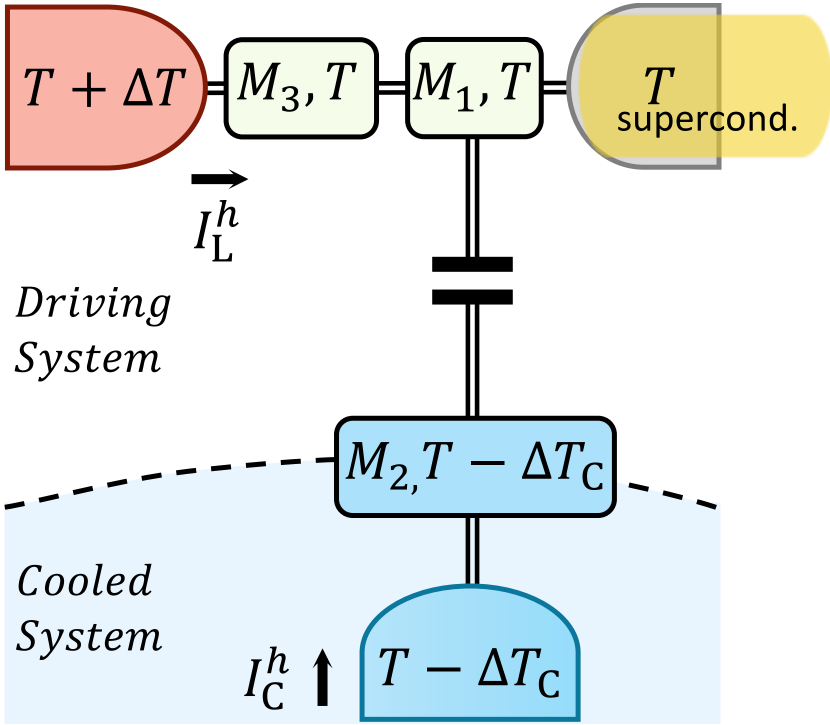
- ☑ Cooling power weakly dependent on θ_2
- ☑ Cooling power could be increased with an additional QD which implements $\Gamma_{\mathrm{R}}^{\mathrm{out}}(0) \simeq 0$

National Enterprise for nanoScience and nanoTechnology

NEST

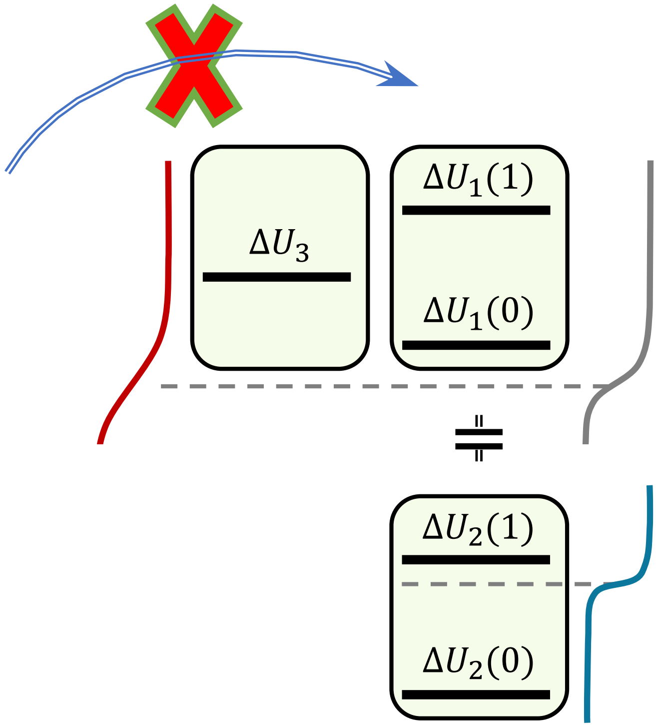
Experimental proposal 2: metallic islands

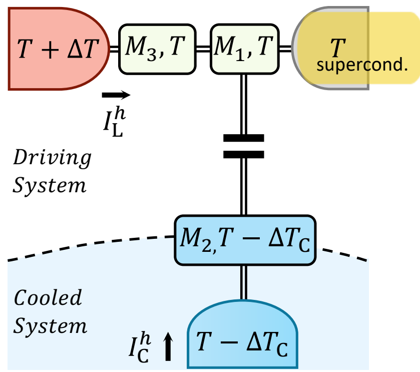
- Level spacing $\ll k_B T$
- Electrons in a MI are thermalized
- Two capacitively-coupled MIs
  + one additional MI
  + a superconducting electrode



Experimental proposal 2: metallic islands

* In order to suppress $\Gamma_{\mathrm{L}}^{\mathrm{in}}(1)$ one needs to tune ΔU_3





* One superconducting electrode (gap in the DOS) used to suppress

$$\Gamma_{\mathrm{R}}^{\mathrm{in/out}}(0)$$

Experimental proposal 2: metallic islands

- Assume that: each MI can be only singly-occupied
- Superconductor normalised DOS

$$\mathcal{N}_{\mathrm{R}}(\epsilon) = \left| Re\left(\frac{\epsilon + i\gamma}{\sqrt{(\epsilon+i\gamma)^2 - \Delta^2}}\right)\right|$$

- Tunneling rates

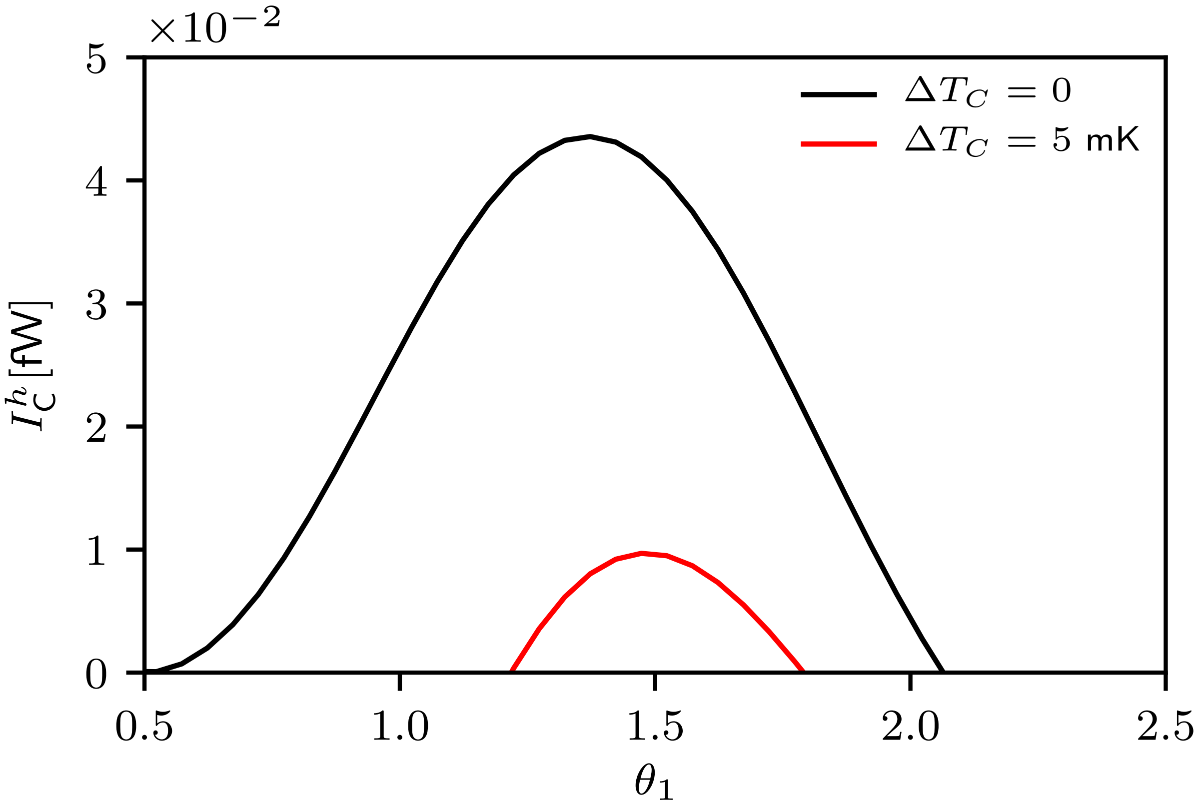
$$\Gamma_{\alpha\nu} = \frac{1}{e^2 R_{\alpha\nu}} \int d\epsilon \mathcal{N}_{\lambda}(\epsilon)\mathcal{N}_{\mu}(\epsilon - \Delta\tilde{U}_{\nu\alpha}) f_{\lambda}(\epsilon) \left[1 - f_{\mu}(\epsilon - \Delta\tilde{U}_{\nu\alpha})\right]$$

Experimental proposal 2: metallic islands

CNRNANO PISA NEST

- Cooling power, using realistic parameters

Koski, Kutvonen, Khaymovich, Ala-Nissila, Pekola (2015)
Dutta, Peltonen, Antonenko, Meschke, Skvortsov, Kubala, König, Winkelmann, Courtois, Pekola (2017)



$$T = 100 \text{ mK}; \Delta T = 200 \text{ mK}$$
$$\theta_2 = 1/2; \theta_3 = \theta_1 + 1/2$$
$$E_{\mathrm{I}} = 25\ \mu\text{eV}; \Delta = 35\ \mu\text{eV}; \gamma = 10^{-3}\ \mu\text{eV}$$

☑ Heat extraction obtained approx. satisfing:

$$\Gamma_{\mathrm{L}}^{\mathrm{in}}(1) \simeq 0$$

$$\Gamma_{\mathrm{R}}^{\mathrm{in/out}}(1) \simeq 0$$

$$I_{\mathrm{C}}^{h} \sim 10^{-2}\ \text{fW}$$

☑ Heat extraction occurs also for $\theta_1 < 1$

☑ Cooling power increases ~ linearly with ΔT

National Enterprise for nanoScience and nanoTechnology

NEST

Conclusions

CNRNANO PISA NEST

- Absorption refrigerator with two capacitively-coupled QDs

  * **Derived optimal condition (which maximize cooling power and COP)**
  * **Implementation with an additional QD**
  * **Implementation with metallic islands**

Thank you!