# Josephson Photonics -
# Quantum Optics and Cooper Pair Tunneling


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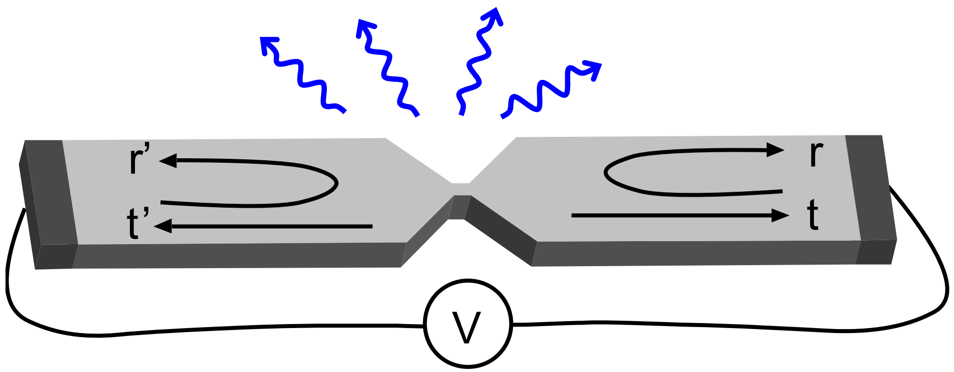




B. Kubala, V. Gramich (Ulm)

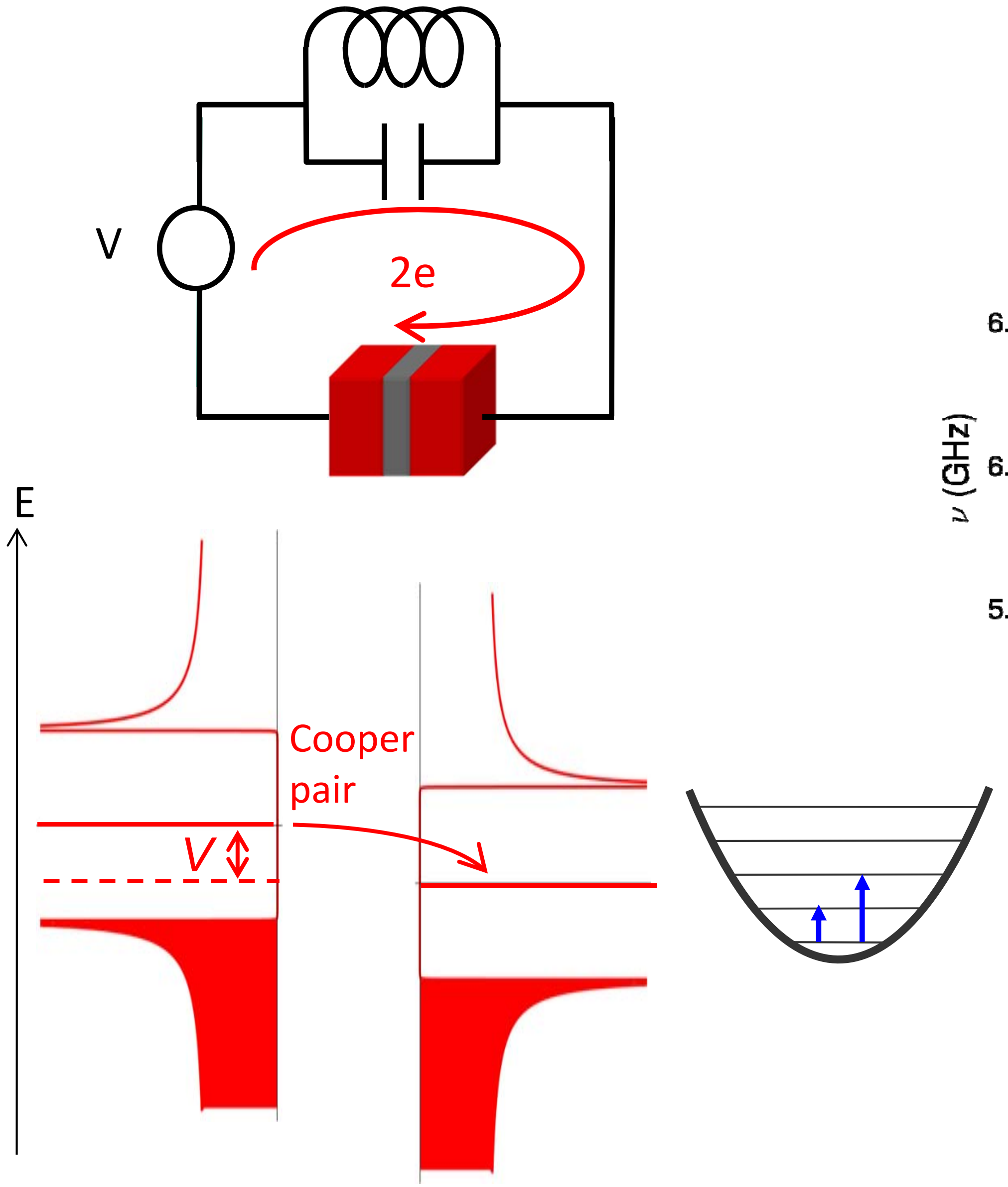
A. Armour (Nottingham)

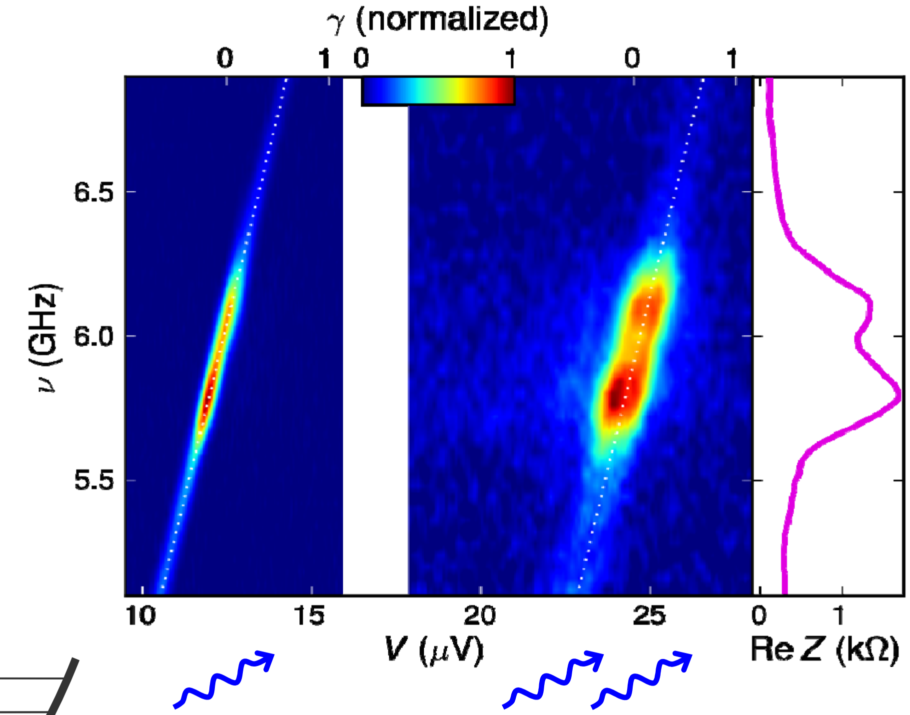
F. Portier, D. Esteve (Saclay)

S. Dambach (Ulm)


Helsinki, 19-09-2018

# Photon emission from a dc-voltage biased Josephson junction




$$2eV = \hbar\omega \qquad 2eV = 2\hbar\omega$$

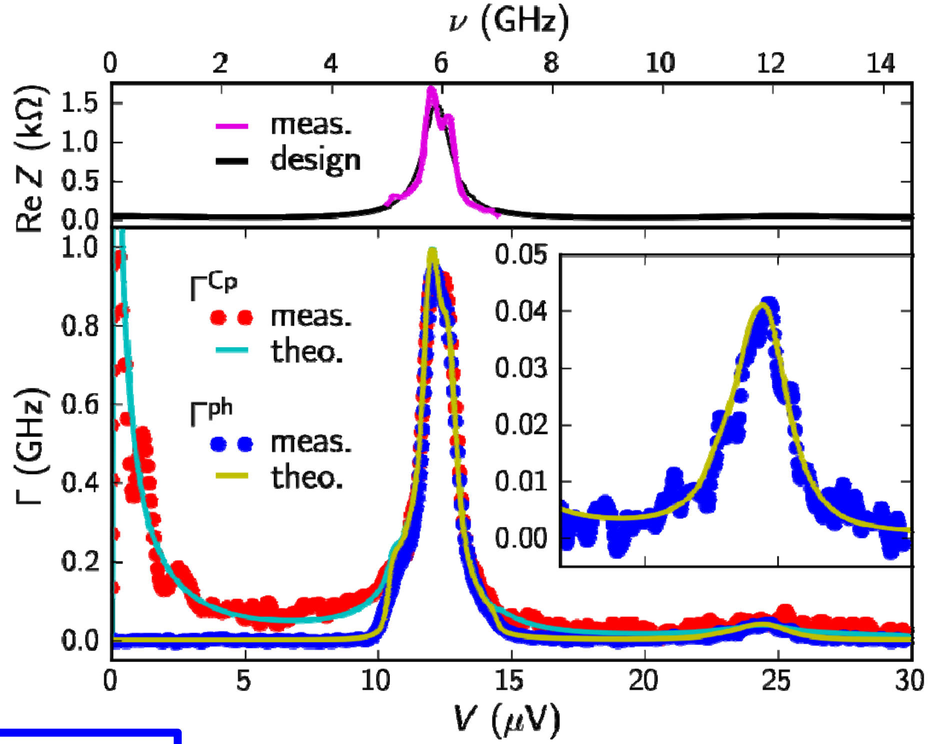
# Dynamical Coulomb blockade

$$H = \hbar\omega\, a^\dagger a - E_J \cos[2eVt/\hbar + \varphi]$$

$$\varphi = \sqrt{r}\,(a^\dagger + a)$$

Charge-light coupling:
„fine structure constant“

$$r = \pi \frac{Z_{\text{res}}}{R_Q}$$




$$\Gamma^{2e}_{\rightarrow}(V) = \frac{\pi E_J^2}{2\hbar} \sum_p \left|\langle p|e^{i\varphi}|0\rangle\right|^2 \delta(2eV - p\hbar\omega)$$

$$= \frac{\pi E_J^2}{2h} \sum_p \frac{\exp(-r)\, r^p}{p!} \delta(2eV - p\hbar\omega)$$

$$\Gamma^{h\nu}_{\rightarrow}(V = p\hbar\omega/2e) = p\,\Gamma^{2e}_{\rightarrow}$$

$$\Gamma^{2e}_{\rightarrow} \ll \gamma$$

H. Pothier, PhD thesis, 1991
Hofheinz et al, PRL 106, 217005 (`11)

# From Coulomb blockade to nonlinear quantum oscillator

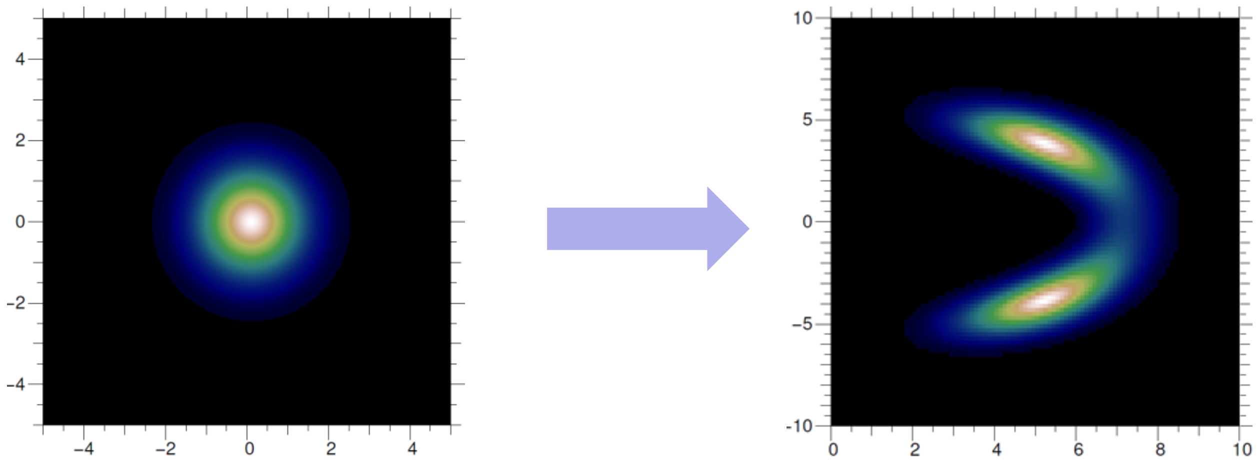
Rotating frame $\omega_J \equiv 2eV/\hbar \approx p\,\omega$

$$H_p = \hbar\Delta_p n + (-i)^p \frac{E_J^*}{2} : [(a^\dagger)^p + (-1)^p a^p] \, \frac{J_p(2\sqrt{r\,n})}{n^{p/2}} : \quad , \quad E_J^* = E_J \mathrm{e}^{-r/2}$$

$$\dot\rho(t) = -\frac{i}{\hbar}[H_p, \rho(t)] + \frac{\gamma}{2}\left[2a\rho(t)a^\dagger - a^\dagger a \rho(t) - \rho(t) a^\dagger a\right] \longrightarrow \dot\rho(t) = 0$$

Cavity field: $\Gamma^{2e}_{\rightarrow} \ll \gamma$ $\quad\quad$ $\Gamma^{2e}_{\rightarrow} \gg \gamma$

$p = 1$




Gramich, Kubala, JA et al, PRL 111, 247002 (`13)

# Josephson photonics

ARTICLE
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**Photon-assisted tunnelling with nonclassical light**

J.-R. Souquet[1,2], M.J. Woolley[3], J. Gabelli[1], P. Simon[1] & A.A. Clerk[2]

PHYSICAL REVIEW B 90, 020506(R) (2014)

**Realization of a single-Cooper-pair Josephson laser**

Fei Chen,[1] Juliang Li,[1] A. D. Armour,[2] E. Brahimi,[1] Joel Stettenheim,[1] A. J. Sirois,[3] R. W. Simmonds,[4] M. P. Blencowe,[1] and A. J. Rimberg[1,*]

PRL 119, 137001 (2017) PHYSICAL REVIEW LETTERS week ending 29 SEPTEMBER 2017

**Emission of Nonclassical Radiation by Inelastic Cooper Pair Tunneling**

M. Westig,[1] B. Kubala,[2] O. Parlavecchio,[1] Y. Mukharsky,[1] C. Altimiras,[1] P. Joyez,[1] D. Vion,[1] P. Roche,[1] D. Esteve,[1] M. Hofheinz,[1,*] M. Trif,[3] P. Simon,[3] J. Ankerhold,[2,†] and F. Portier[1,‡]
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[2]*Institute for Complex Quantum Systems and IQST, University of Ulm, 89069 Ulm, Germany*
[3]*Laboratoire de Physique des Solides, Université Paris-Sud, 91405 Orsay, France*

REPORT Science 355, 939 (`17)

OPTOELECTRONICS

## Demonstration of an ac Josephson junction laser

**M. C. Cassidy,[1] A. Bruno,[1] S. Rubbert,[2] M. Irfan,[2] J. Kammhuber,[1] R. N. Schouten,[1,2] A. R. Akhmerov,[2] L. P. Kouwenhoven[1,2*]**

PHYSICAL REVIEW B 96, 214509 (2017)

**Noise switching at a dynamical critical point in a cavity-conductor hybrid**

Andrew D. Armour,[1] Björn Kubala,[2] and Joachim Ankerhold[2]
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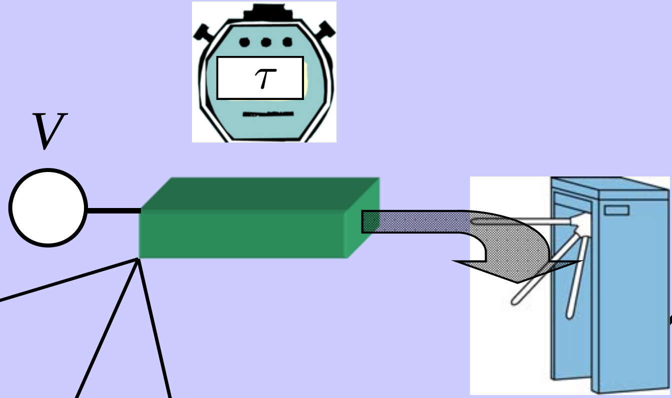
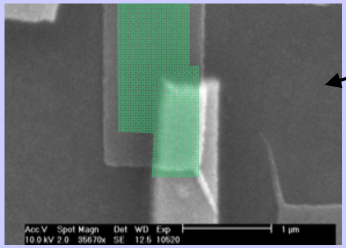
Leppäkangas et al, PRL 110, 267004 (`13)
Armour et al, PRL 111, 247001 (`13)
Altimiras et al, PRL 112, 236803 (`14)
S. Meister et al, PRB 92, 174532 (`15)

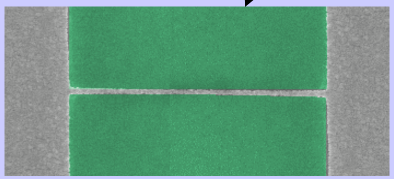
Gramich et al, PRL 113, 027001 (`14)
Armour et al, PRB 91, 184508 (`15)
Ast et al, Nature Comm. 7, 13009 (`16)
Dykman et al, arXiv: 1702.07931 (`17)

# Counting charges

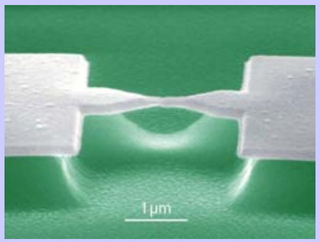


Tunnel contact



Diffusive wire

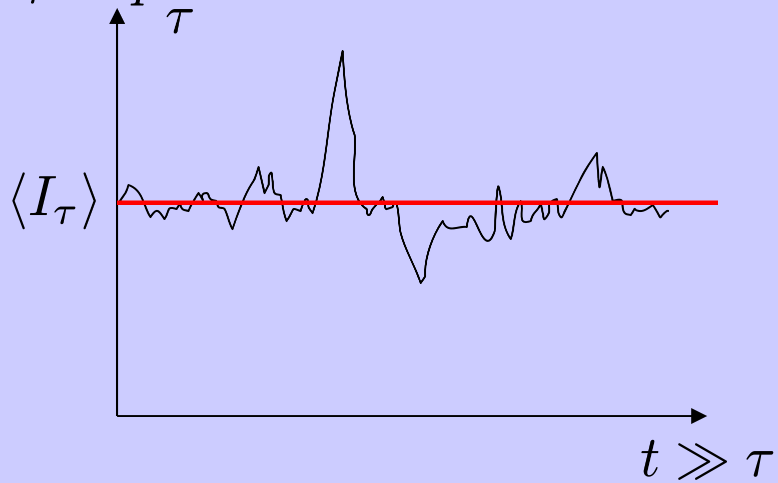


Atomic point contact

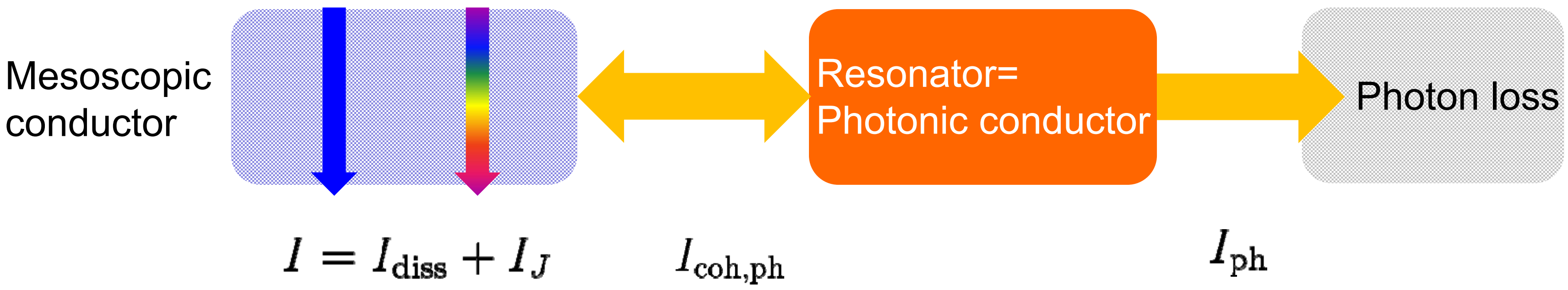
R. Landauer:

The noise is the signal

$$I_\tau = q\,\frac{n}{\tau}$$



# Counting charges via photonic conductor




Steady state $\langle I_J \rangle / 2e = \langle I_{\rm coh,ph} \rangle = \langle I_{\rm ph} \rangle$

Josephson @ low voltage $I_{\rm diss} = 0$

Cooper pair noise from photon noise ?

# Photon counting statistics

Generating function $\exp[\mathcal{F}_{ph}(\chi,t)] = \mathrm{Tr}\{\exp[\mathcal{L}_\chi t]\,\rho(0)\}$

$$\mathcal{L}_\chi = \mathcal{L}_p + \frac{\gamma}{2}(2\mathrm{e}^{-i\chi}a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

Unitary transformation in Liouville space $\mathcal{L}_\chi \to \mathcal{L}_{p\chi}^{\mathrm{coh,ph}}$

$$\mathcal{L}_{p\chi}^{\mathrm{coh,ph}}\rho = \frac{i}{\hbar}(H_\chi\rho - \rho H_{-\chi}) + \mathcal{L}_\gamma$$

$$H_\chi = H_p(a \to \mathrm{e}^{i\chi/2}a)$$

New generating function $\exp[\mathcal{F}_{\mathrm{coh,ph}}(\chi,t)] = \mathrm{Tr}\left\{\exp[\mathcal{L}_{p\chi}^{\mathrm{coh,ph}}\,t]\,\rho(0)\right\}$

Coherent photon transfer – quasi-probability $\tilde{P}(N,t)$

Kubala, Armour, JA, arXiv: 1606.02200

# Photon counting statistics

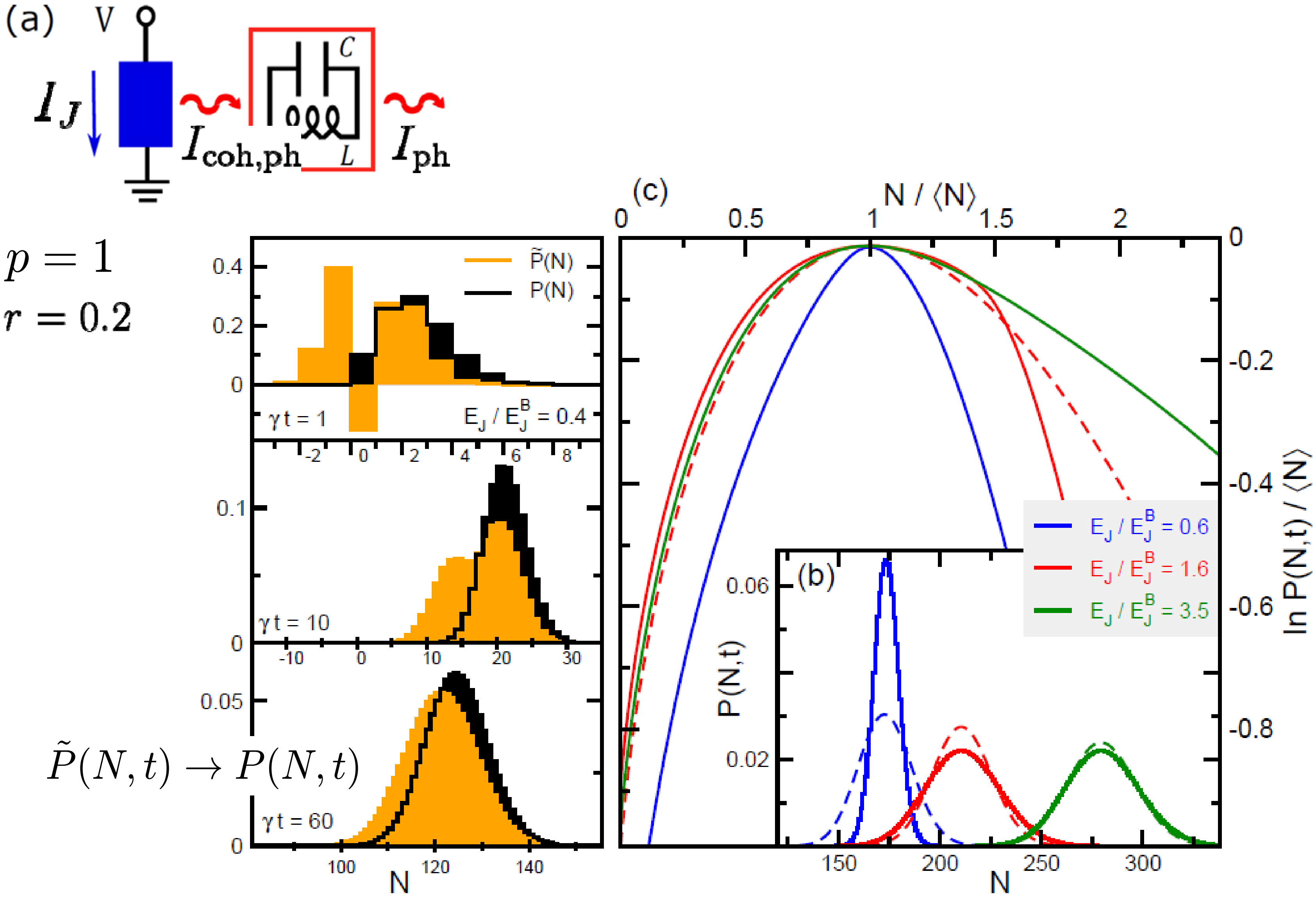
$$\mathcal{F}_{\mathrm{coh,ph}}(p\chi, \gamma t \gg 1) = \mathcal{F}_{\mathrm{ph}}(\chi, \gamma t \gg 1)$$

Cumulants photon numbers $$\lim_{\gamma t \to \infty} \langle\langle N^k \rangle\rangle_{\mathrm{coh,ph}} = \langle\langle N^k \rangle\rangle_{\mathrm{ph}} / p^k$$

Long times:

Statistics of emitted photons = statistics of transferred Cooper pairs

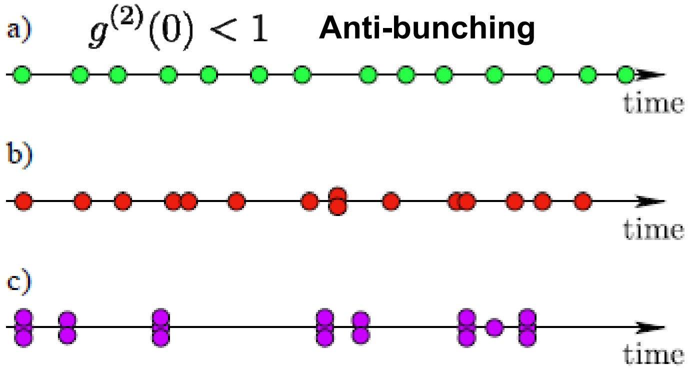
# Full counting statistics



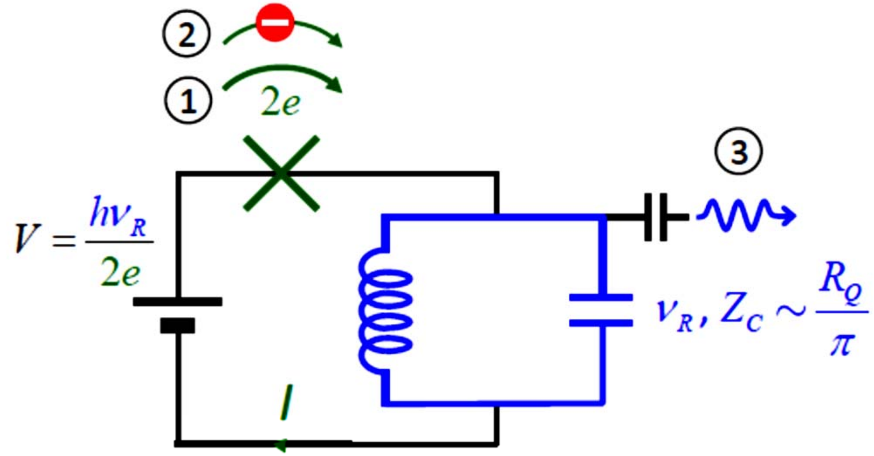

Kubala, Armour, JA, arXiv: 1606.02200

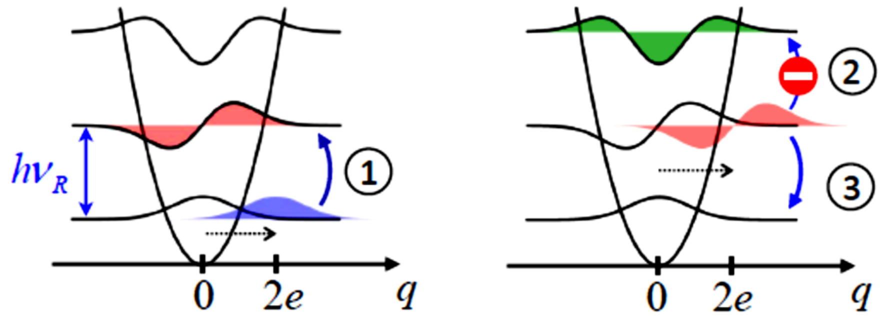
# Designing photon statistics: Towards single photon emission

$$g^{(2)}(\tau) = \frac{\langle a^\dagger(\tau) a^\dagger a\, a(\tau)\rangle_{\rm st}}{\langle n\rangle_{\rm st}^2}$$




$g^{(2)}(0) > 1$ Bunching



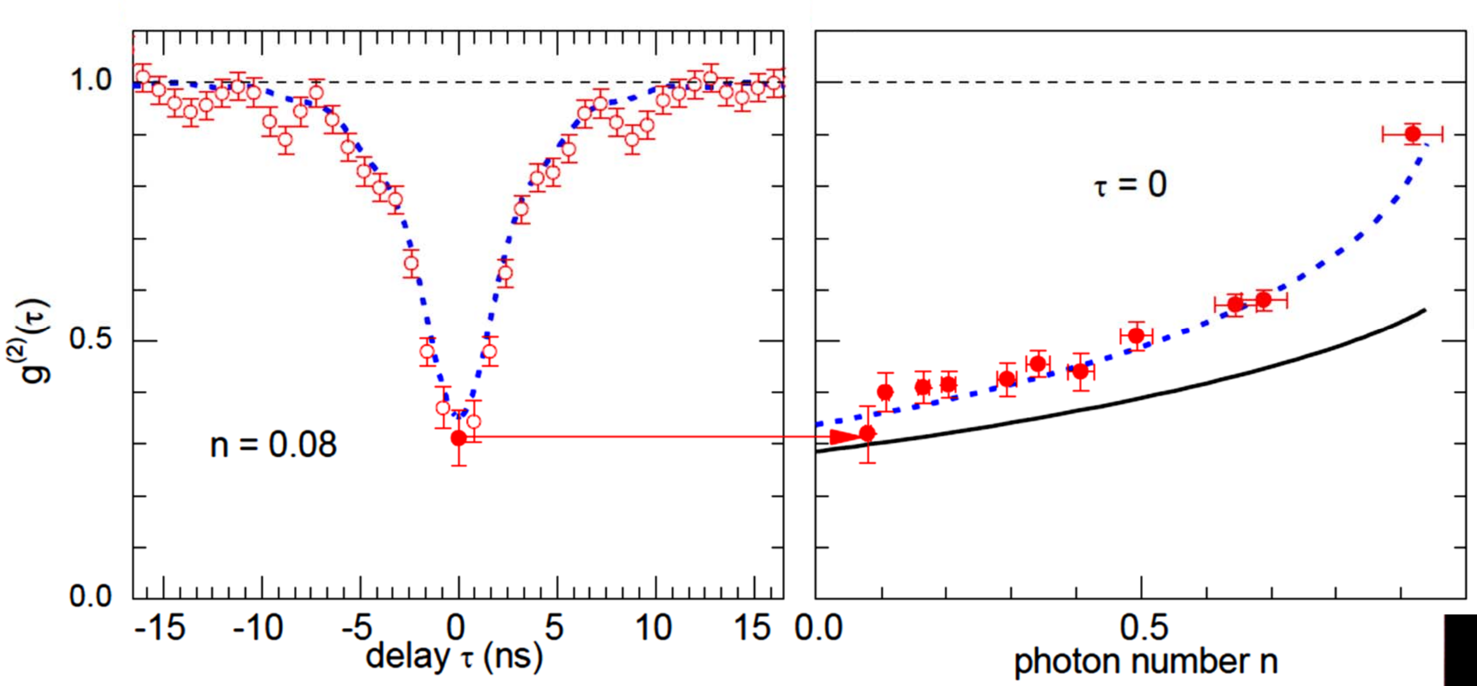




Weak driving

$$g^{(2)}(0) = (1 - r/2)^2$$

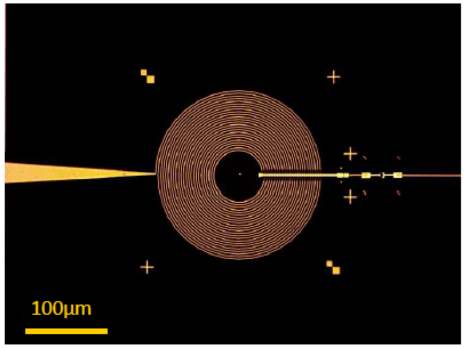

Gramich, JA et al, PRL 113, 027001 (`14)
Dambach et al, PRB 92, 054508 ('15)

# Towards single photon sources




$r \sim 1$

$Z_{res} = \sqrt{L/C}$

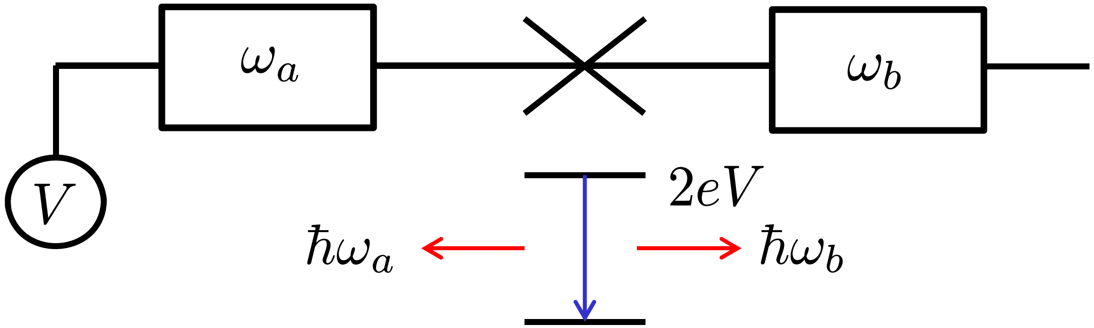



$\sim 10^7$ photons/s

Rolland, Portier, JA et al, to be submitted this week

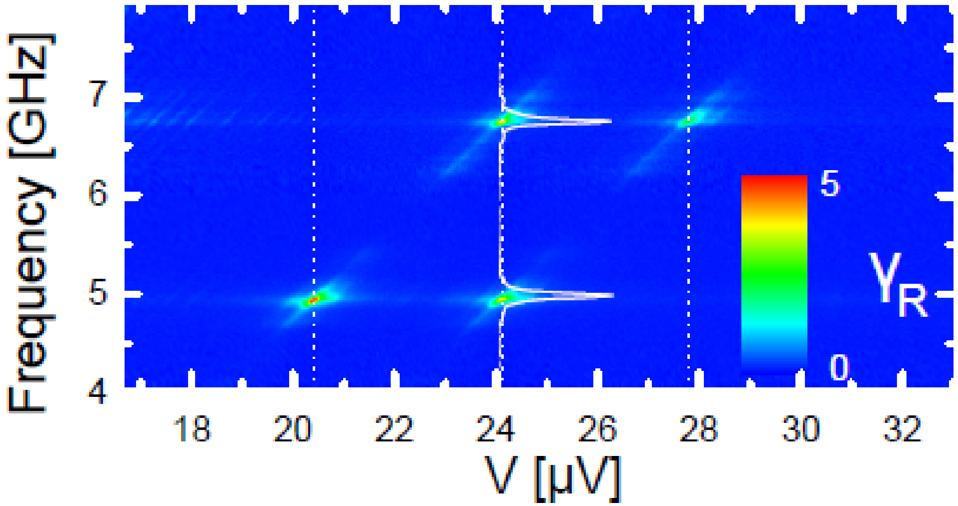
# Designing photon statistics: Towards entangled photons

$$2eV = \hbar(\omega_a + \omega_b)$$



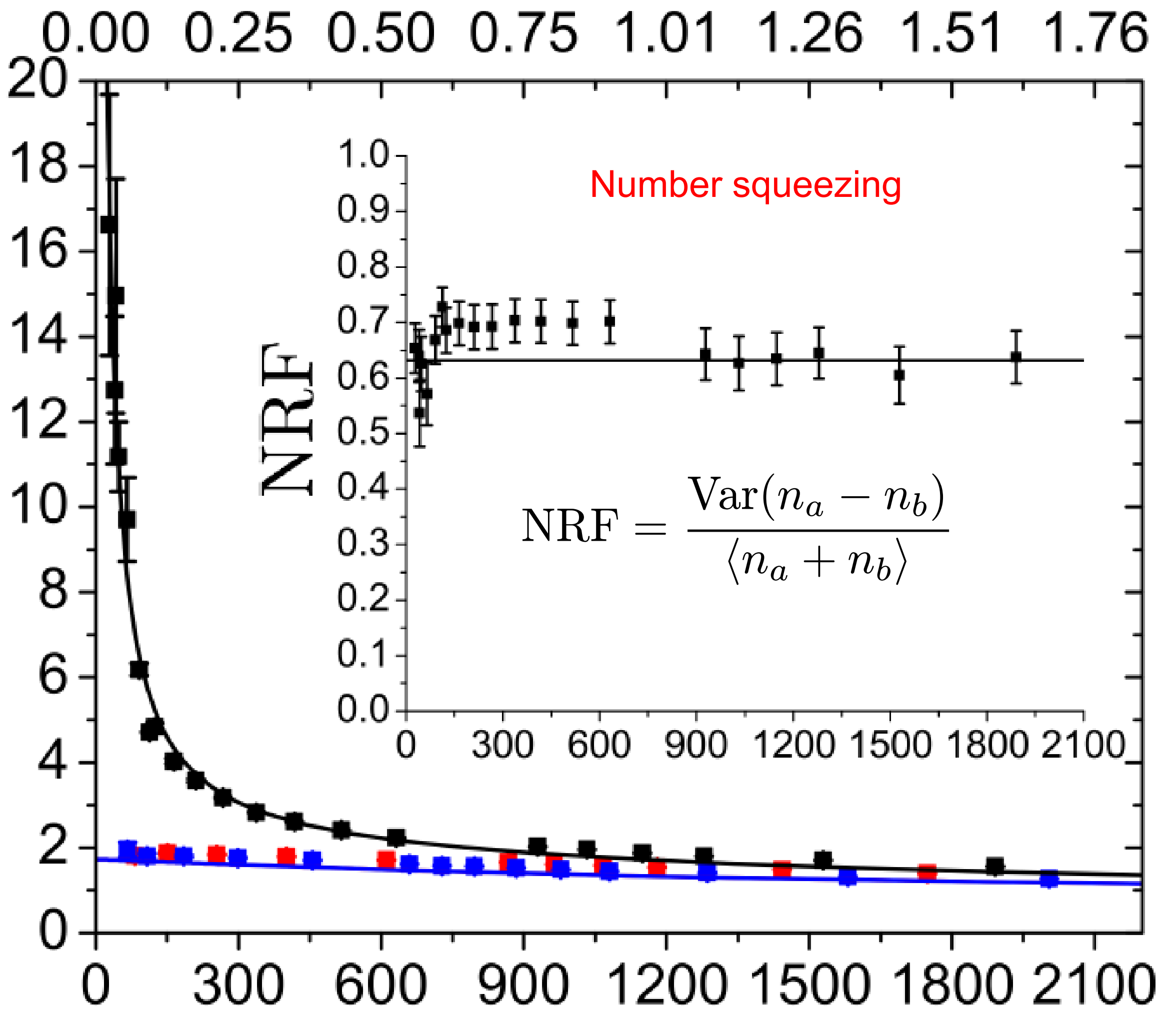

Non-classical light:
violation of Cauchy-Schwartz

$$g^{(2)}_{ab}(0) \not\leq \sqrt{g^{(2)}_{aa}(0)\, g^{(2)}_{bb}(0)}$$




Leppäkangas et al, NJP 2014
Armour, Kubala, JA, PRB (2015)

# Two cavities: correlated photons



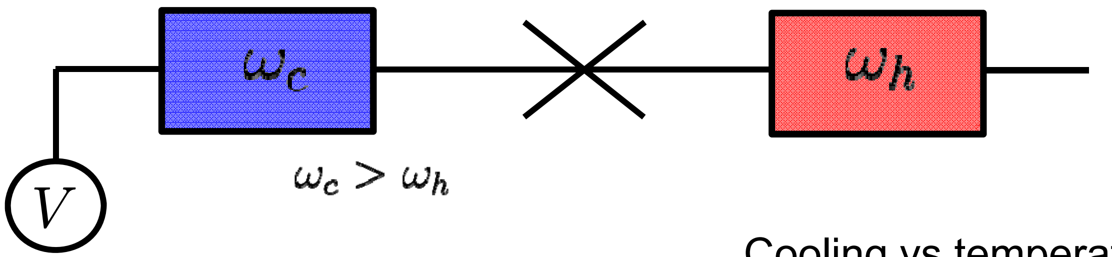

- $g^{(2)}_{ab}(0)$
- $g^{(2)}_{aa}(0)$
- $g^{(2)}_{bb}(0)$

$$g^{(2)}_{ab}(0) \not\leq \sqrt{g^{(2)}_{aa}(0)\, g^{(2)}_{bb}(0)}$$

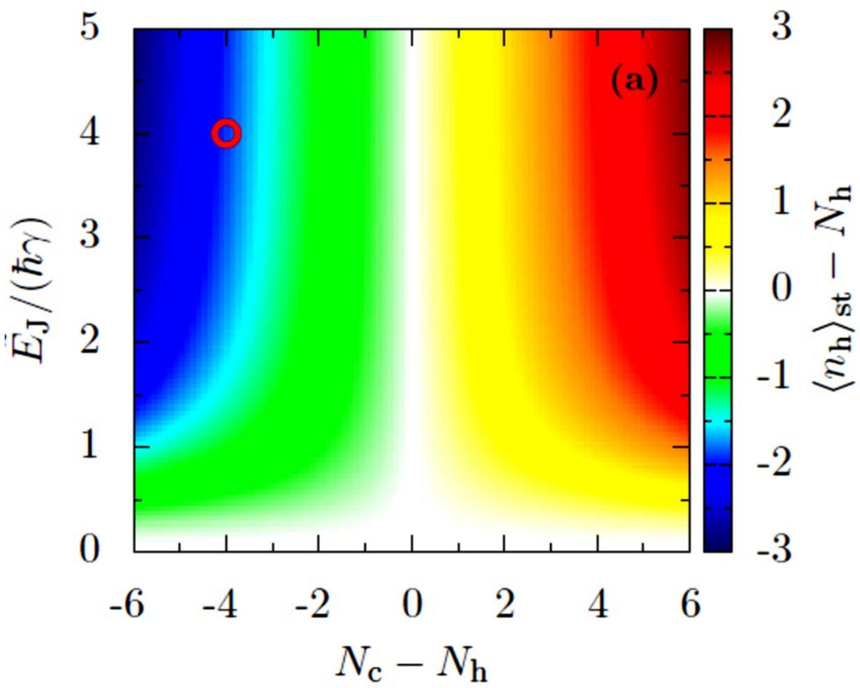
Portier, Esteve, Kubala, JA et al, PRL 137001 (2017)

# Thermodynamic device: refrigerator

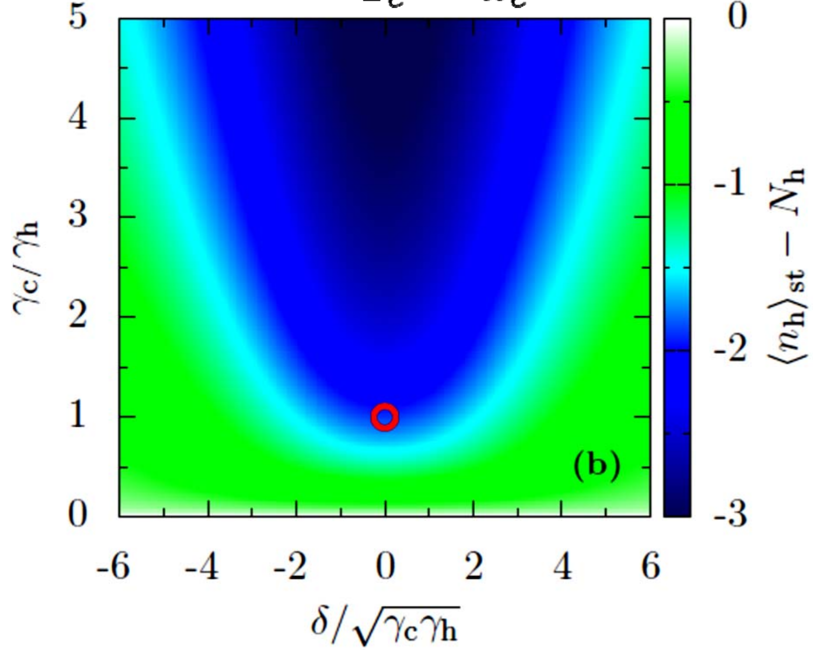
$$2eV + \hbar\omega_h = \hbar\omega_c$$




Cooling vs temperature gradient



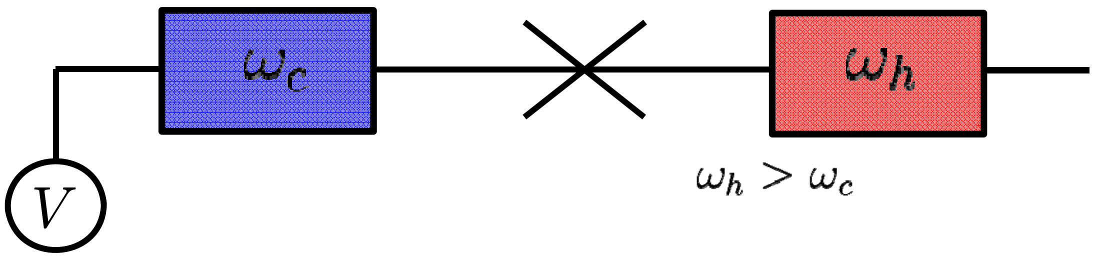

Cooling vs de-tuning $\frac{T_h}{T_c} < \frac{\omega_h}{\omega_c}$ , $(\hbar\omega_c < \Delta)$

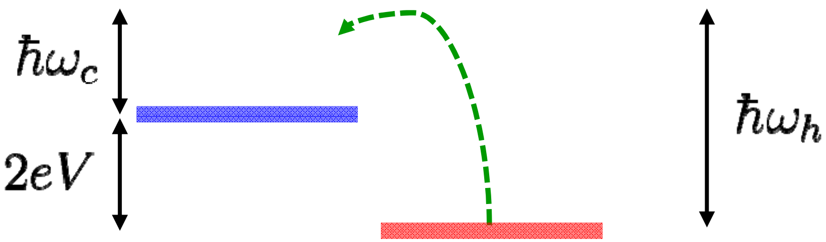



Dambach, Kubala, JA, arXiv: 1806.07700

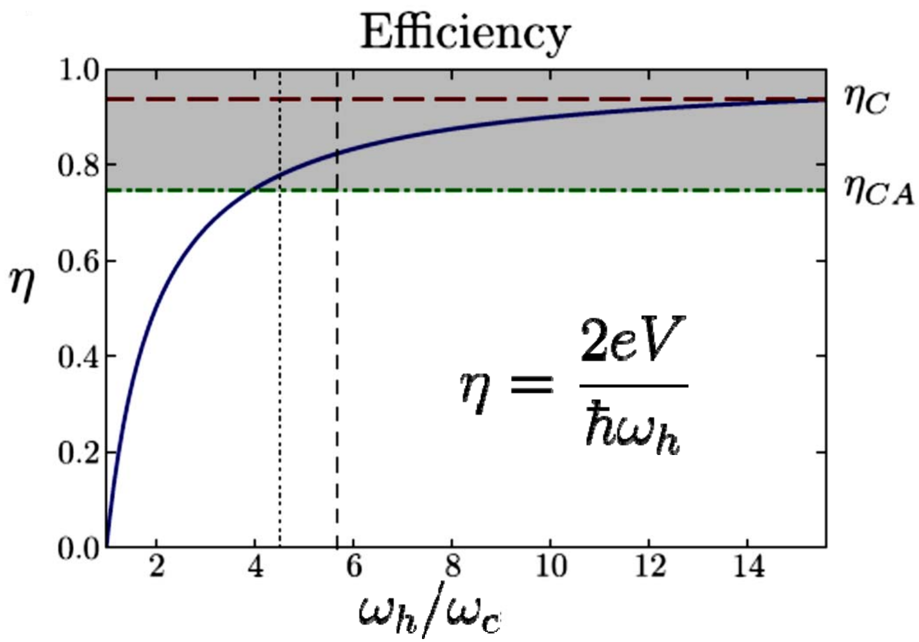
# Thermodynamic device: Heat engine

$$2eV + \hbar\omega_c = \hbar\omega_h$$




Net current against bias $\quad \dfrac{\omega_c}{\omega_h} > \dfrac{T_c}{T_h}$




Hofer, Souquet, Clerk, PRB 041418 (2016)

- Quantum optics of mesoscopic conductors: Josephson photonics
- Cooper pair noise ⟷ emitted photon statistics
- Single photon emission
- Correlated photon pairs
- (Quantum) Thermodynamics devices