Nanomechanical resonators for probing quantum fluids

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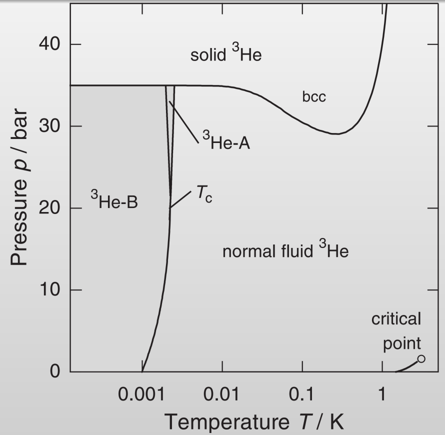
Quantum fluids ^{3}He and ^{4}He

^{3}He is a Fermi system



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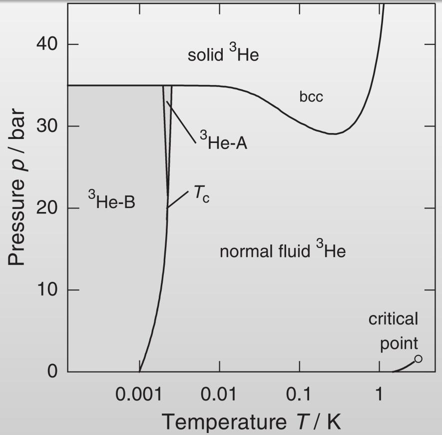
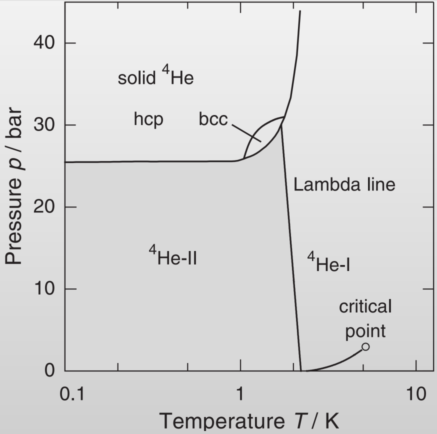


^{4}He is a Bose system



Classical tools for probing $^3\mathrm{He}$ and $^4\mathrm{He}$

Characteristic scales in helium

- Lengths:

	$^3\mathrm{He}$	$^4\mathrm{He}$
Coherence length ξ_0	$50\,\mathrm{nm}$	$0.15\,\mathrm{nm}$

- Velocities:

	$^3\mathrm{He}$	$^4\mathrm{He}$
First sound v_1	$250\,\mathrm{m/s}$	$238\,\mathrm{m/s}$
Landau velocity v_L	$3\,\mathrm{cm/s}$	$60\,\mathrm{m/s}$
Critical velocity v_c	$1\,\mathrm{mm/s}$	$10\,\mathrm{cm/s}$

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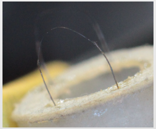
Currently available tools

- Vibrating wires:



  - Dimensions: $\varnothing 3 \div 50\,\mu\mathrm{m} \times 5\,\mathrm{mm}$
  - Operation frequencies: $f = 0.1 \div 10\,\mathrm{kHz}$
  - Amplitudes: $A \sim 10\,\mu\mathrm{m}$

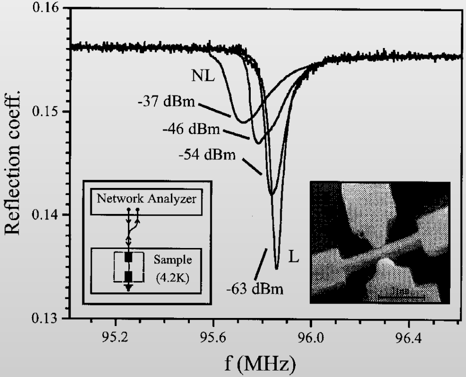
- Tuning forks:

  - Dimensions: $6\,\mathrm{mm} \times 1\,\mathrm{mm} \times 0.2\,\mathrm{mm}$
  - Operation frequencies: $f = 10 \div 100\,\mathrm{kHz}$
  - Amplitudes: $A \sim 1\,\mu\mathrm{m}$

What has been done?

A. Kraus, et.al Nanotechnology 11, 165 (2000)

Nanomechanical vibrating wire resonator for phonon spectroscopy in liquid helium.



Sample

- Material:
  - Silicon+Metallisation (Ti/Au)
  - Density: $\rho_{\mathrm{Si}} = 2.3\,\mathrm{g}\cdot\mathrm{cm}^{-3}$
  - Youngs modulus: $E = 47\,\mathrm{GPa}$
- Dimensions:
  - Length: $L \sim 1.2\,\mu\mathrm{m}$
  - Thickness: $H \approx (\mathrm{Si})\,400\,\mathrm{nm} + (\mathrm{Au})\,50\,\mathrm{nm}$
  - Width: $W \approx 200\,\mathrm{nm}$
  - Linear mass density: $\varrho \approx 3 \times 10^{-10}\,\mathrm{kg}\cdot\mathrm{m}^{-1}$

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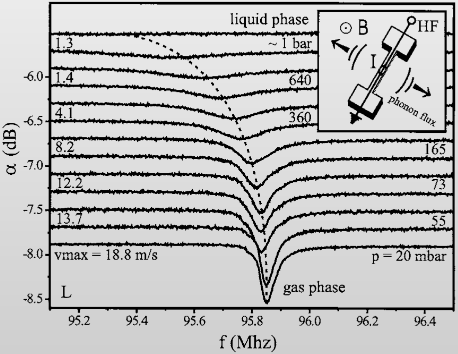


Measurements

- Magneto-driving mode
  - Applied power: $-65 \div -30\,\mathrm{dBm}$
  - Magnetic field: $1\,\mathrm{T}$

What can be improved?

- Make beam sizes comparable with a coherence length;
- Decrease linear mass density – this will increase sensitivity;

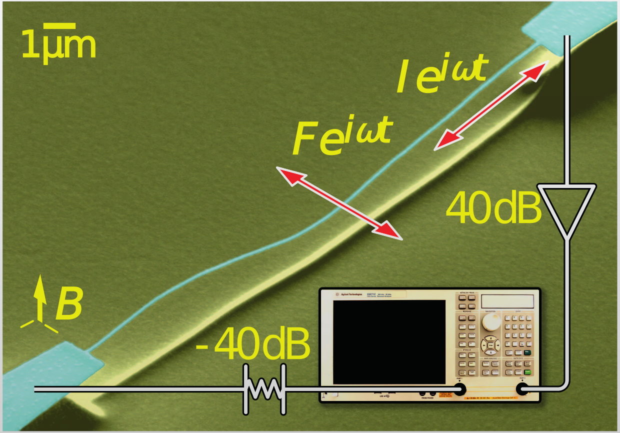
Our experiments

Sample

- Material: Aluminium
  - Superconductor with $T_c \approx 1.2\,\mathrm{K}$
  - Density: $\rho_{\mathrm{Al}} = 2.7\,\mathrm{g}\cdot\mathrm{cm}^{-3}$
  - Youngs modulus: $E = 70\,\mathrm{GPa}$
- Dimensions:
  - Length: $L \in (1 \div 500)\,\mu\mathrm{m}$
    Allows to cover the broad frequency range from $1\,\mathrm{kHz}$ *to* $100\,\mathrm{MHz}$

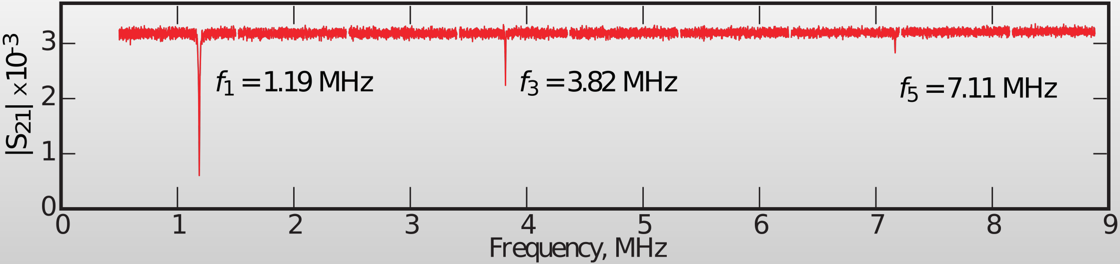
    $$\left.\begin{array}{l}\text{Width: } W \approx 0.1\,\mu\mathrm{m} \\ \text{Thickness: } H \approx 0.1\,\mu\mathrm{m}\end{array}\right\} \sim \xi_0$$
  - Linear mass density:
    $\varrho \approx 2.5 \times 10^{-11}\,\mathrm{kg}\cdot\mathrm{m}^{-1}$

Experimental setup



Aluminium beams in vacuum

Harmonics



Fundamental frequency

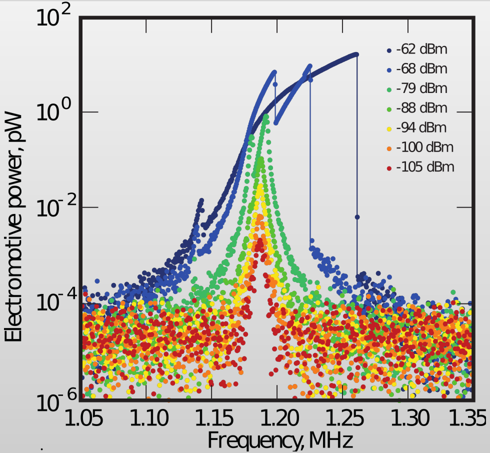
$$f_1 = \frac{\pi W}{3L^2}\sqrt{\frac{E}{\rho}}\sqrt{1 + \frac{3}{\pi^2}\left(\frac{L}{W}\right)^2 \frac{\Delta L}{L}}$$

Stress analysis

- Compressive stress at $300\,\mathrm{K}$ with $f_1 \approx 230\,\mathrm{kHz}$
- Tensile stress at $4.2\,\mathrm{K}$ with $\dfrac{\Delta L}{L} \approx 4 \times 10^{-4}$

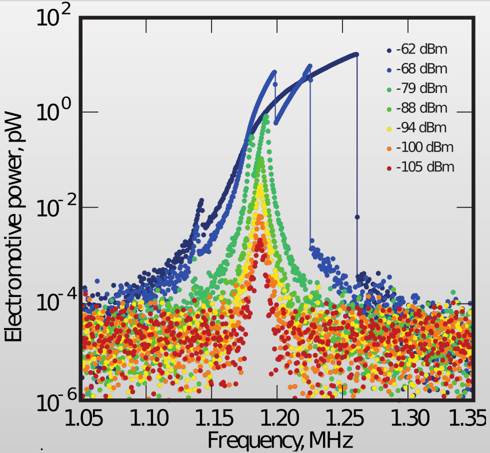
Power dependence in vacuum

Duffing oscillator

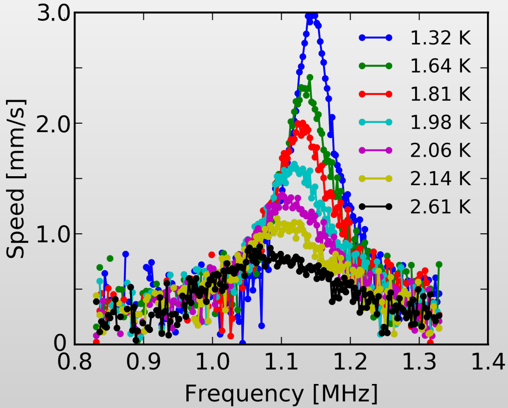


Measurements in liquid helium

Duffing oscillator

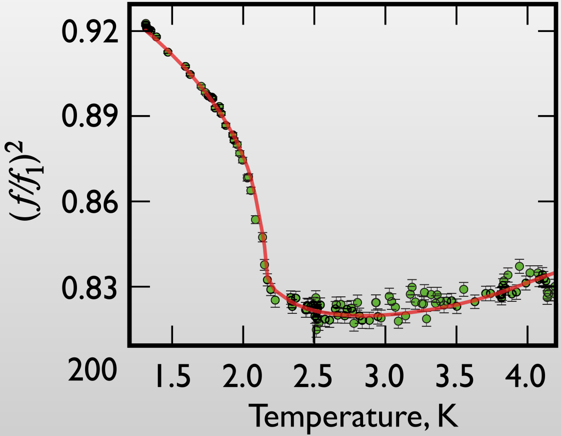


Resonance in liquid $^4\mathrm{He}$



Measurements in liquid helium

The temperature dependence of the resonance frequency in liquid $^4\mathrm{He}$



$$\left(\frac{f_1}{f}\right)^2 = 1 + \overbrace{\beta \frac{\rho_{\mathrm{He}}}{\rho_{\mathrm{Al}}}}^{\text{fluid back-flow}} + \underbrace{\mathcal{B} \frac{\rho_n}{\rho_{\mathrm{Al}}} \frac{S}{V} \sqrt{\frac{\eta}{\pi \rho_n f_1}}}$$

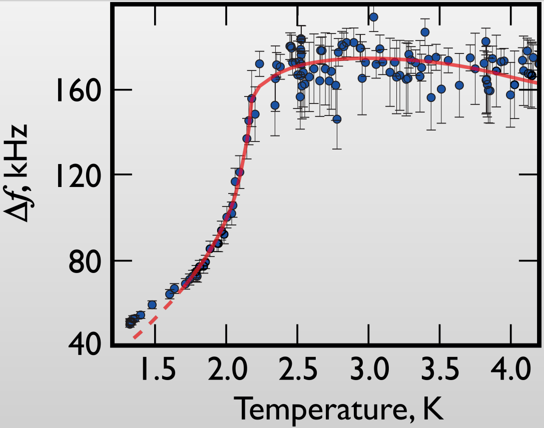
Normal fluid dragged in a layer of thickness the viscous penetration depth $\delta = \sqrt{\frac{\eta}{\pi \rho f}} \approx 100\,\mathrm{nm}$

- There are two fitting parameters:

Experiment	Theory
$\beta = 1.18 \pm 0.02$	$\beta = \frac{\pi}{4} \frac{h}{w}$
$\mathcal{B} = 1.19 \pm 0.01$	$\mathcal{B} = 1$

Measurements in liquid helium

The temperature dependence of the resonance width in liquid $^4\mathrm{He}$



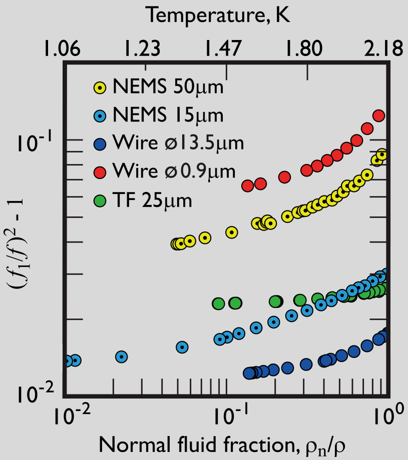
$$\Delta f = \mathcal{C}\frac{1}{2}\underbrace{\frac{\rho_n}{\rho_{\mathrm{Al}}}\frac{S}{V}\sqrt{\frac{\eta}{\pi\rho_n f_1}}}\left(\frac{f}{f_1}\right)^2 f$$

Normal fluid dragged in a layer of thickness the viscous penetration depth $\delta = \sqrt{\dfrac{\eta}{\pi\rho f}} \approx 100\,\mathrm{nm}$

- There is one fitting parameter:

Experiment	Theory
• $\mathcal{C} = 2.62 \pm 0.06$	• $\mathcal{C} = 2$

Comparison with other devices



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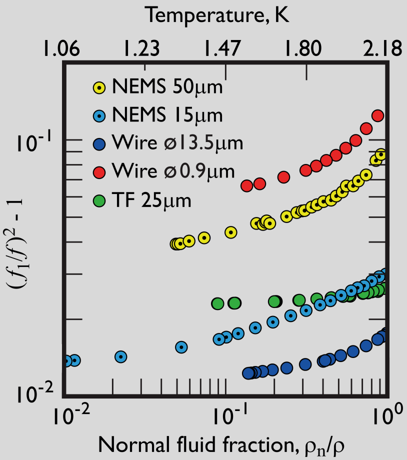
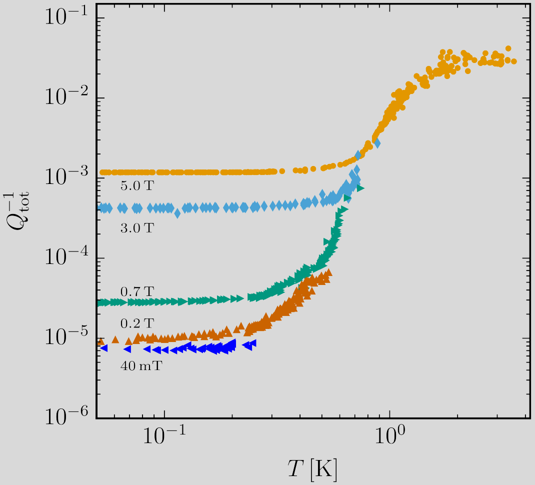


Summary I

- The damping mechanics of NEMS in liquid helium at the temperatures spanning the superfluid transitions is well described by the hydrodynamic model in the framework of the two fluid model;
- The demonstrated sensitivity to the normal fluid density of NEMS is better than sensitivity of traditional instruments: quartz tuning forks or vibrating wires.

Measurements in liquid at temperatures below $1\,\mathrm{K}$



- The total damping:

$$Q_{\mathrm{tot}}^{-1} = Q_{\mathrm{md}}^{-1} + Q_{\mathrm{int}}^{-1} + Q_{\mathrm{ph}}^{-1} + Q_{\mathrm{rot}}^{-1} + Q_{\mathrm{ac}}^{-1}$$

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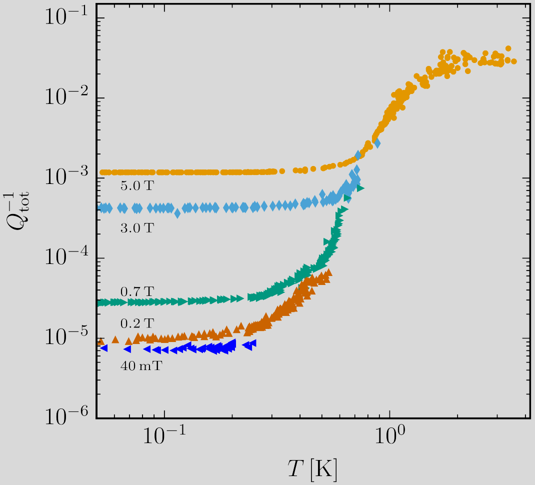

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- Magnetomotive losses:

$$Q_{\mathrm{md}}^{-1} \propto B^2$$

- Internal losses:

Superconducting	Normal
$Q_{\mathrm{int}}^{-1} \approx 2 \times 10^{-7}$	$Q_{\mathrm{int}}^{-1} \approx 1 \times 10^{-6}$

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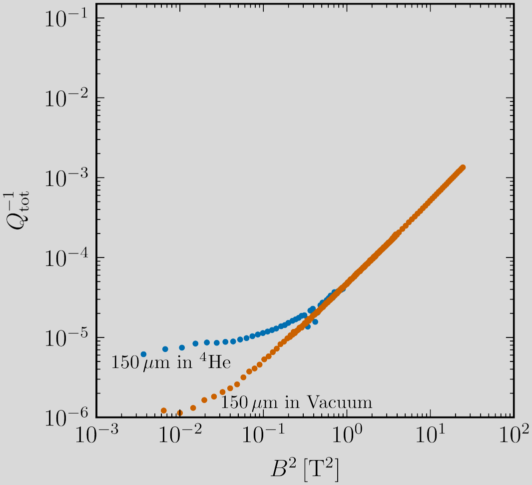

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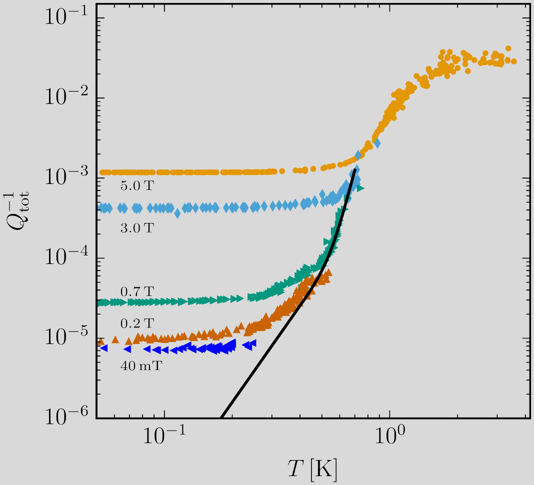

- The total damping:

$$Q^{-1}_{\mathrm{tot}} = Q^{-1}_{\mathrm{md}} + Q^{-1}_{\mathrm{int}} + Q^{-1}_{\mathrm{ph}} + Q^{-1}_{\mathrm{rot}} + Q^{-1}_{\mathrm{ac}}$$

- Phonons scattering:

$$Q^{-1}_{\mathrm{ph}} \propto T^{4}$$

- Rotons scattering:

$$Q^{-1}_{\mathrm{rot}} \propto \exp\left(-\frac{\Delta}{k_{\mathrm{B}}T}\right)$$

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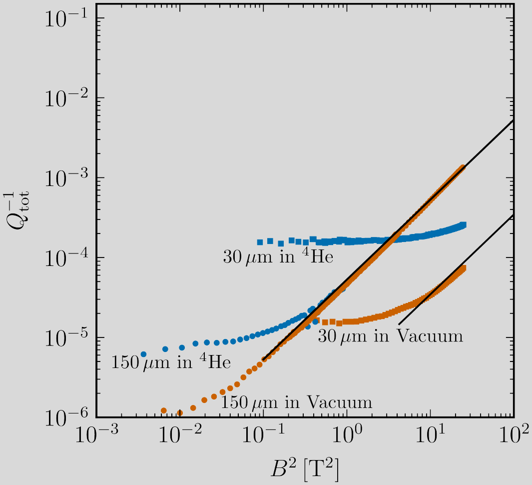

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- Acoustic losses (dipole emission):

$$Q_{\mathrm{ac}}^{-1} = f_0^2$$

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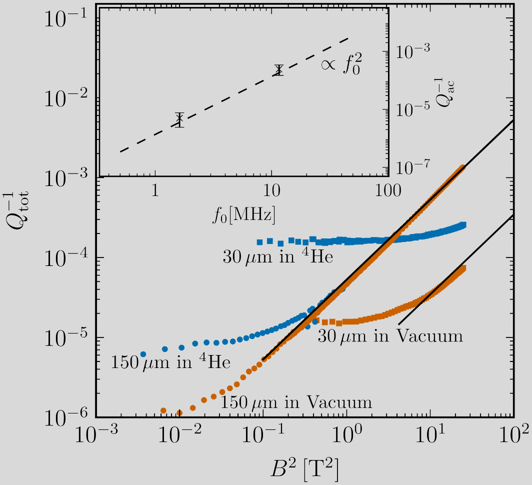

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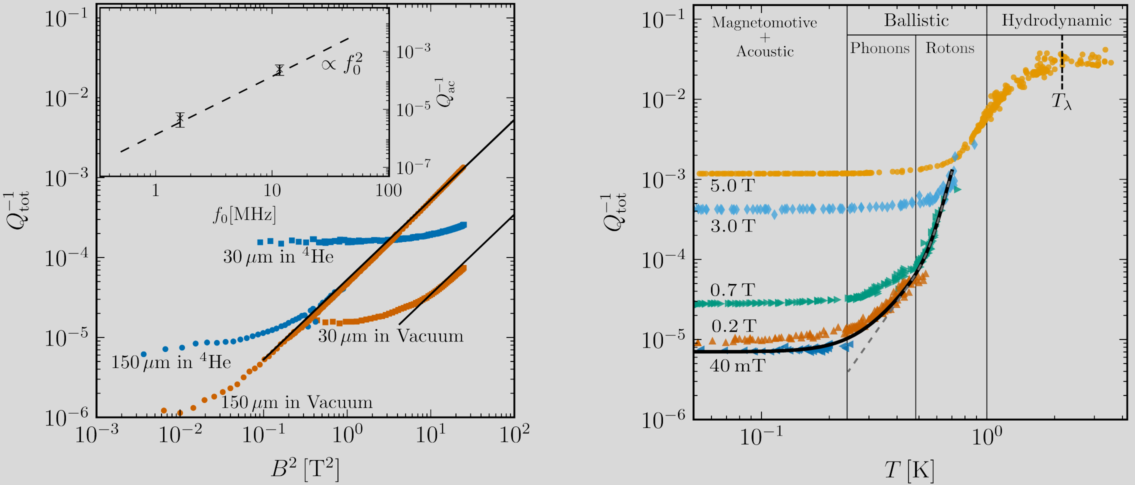
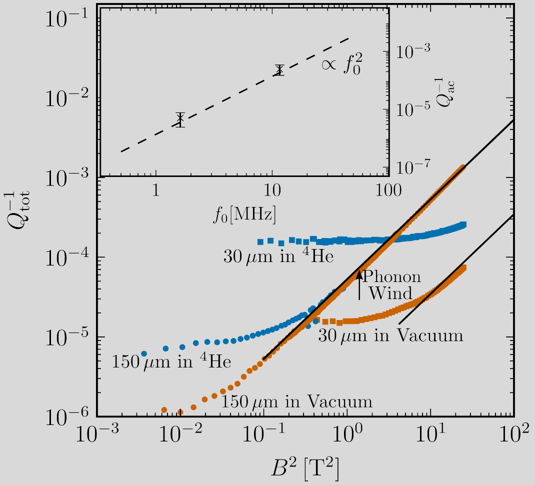
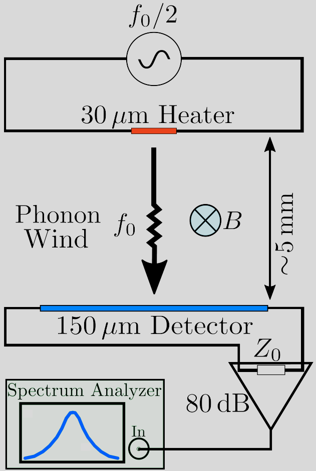


Driving a nanomechanical resonator with *"phonon wind"* in superfluid ^{4}He





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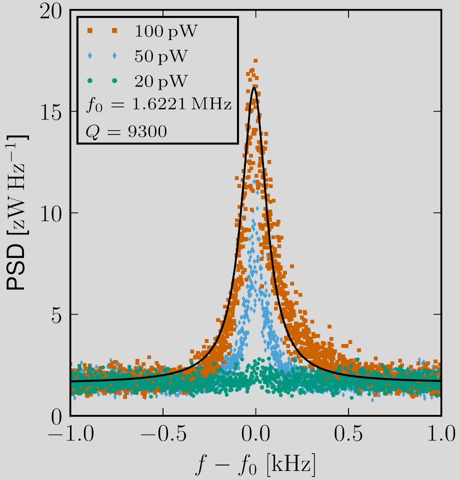

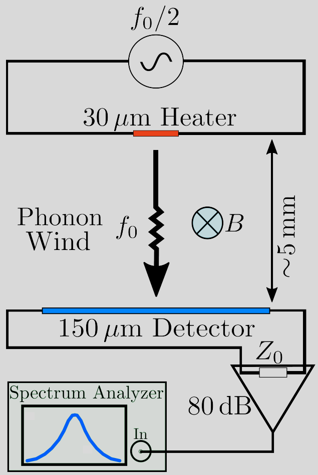

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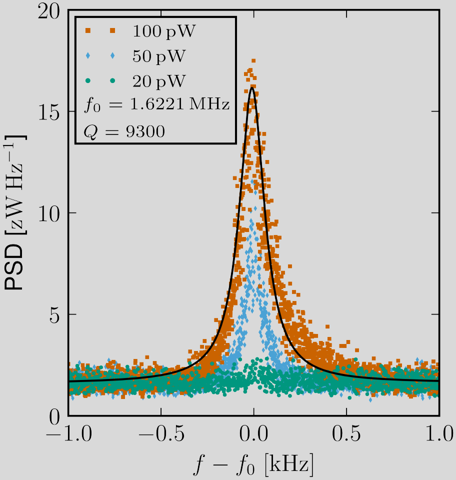
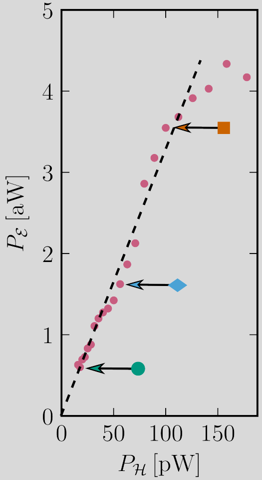




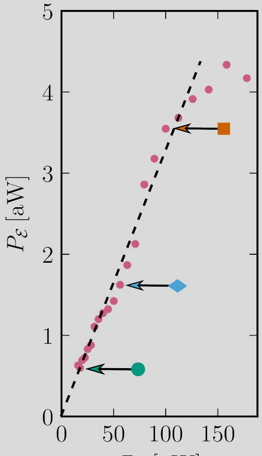
Driving a nanomechanical resonator with *"phonon wind"* in superfluid ^{4}He

- Actuating force of the phonon wind obtained from the experiment:
  $F_{\mathrm{ph}} \approx 25\,\mathrm{fN}$ at $0.5\,\mathrm{aW}$
  $F_{\mathrm{ph}} \approx 62\,\mathrm{fN}$ at $3.5\,\mathrm{aW}$ of detected power.
- From the simple arguments of the molecular kinetic theory:

$$F_{\mathrm{ph}} = \gamma n p_{\mathrm{ph}} c_{\mathrm{ph}} S,$$

  the pnonon density in a pulse

$$n_{\mathrm{ph}} \sim 2 \times 10^{21}\,\mathrm{m}^{-3}$$



Quantum Probes for Quantum Fluids