A model of calorimetric measurements in an open quantum system

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Outline

1. Calorimetric measurement on a driven qubit
2. Mathematical modeling
3. Results
4. Outlook
Calorimetric principle

Idea: measure work statistics in an Open Quantum System

- protocols bringing the system back to the initial state at the end of the horizon.
- the work $W$ done on the system under these conditions is equal to the heat $Q$ dissipated to the environment

Stylized experimental setup
Calorimetric measurement on a driven qubit

Integrated quantum circuit

- e-e interaction
- e-p interaction

Calorimeter

NIS

Resonance circuit

Qubit

Normal metal
Insulator
Superconductor


Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

Qubit driven by a monochromatic force

\[ H_q(t) = \frac{\hbar \omega_q}{2} \sigma_z + \kappa V_d(t) \]
\[ V_d(t) = \hbar \omega_q \left( e^{i\omega_L t} \sigma_+ + e^{-i\omega_L t} \sigma_- \right) \]

\[ \kappa \hbar \omega_q = \text{drive amplitude} \]
\[ \omega_L = \text{drive frequency} \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

Calorimeter: free fermion gas (effectively, more follows)

\[ H_e = \sum_k \eta_k c_k^\dagger c_k \]

\[ \eta_k = \frac{\hbar \|k\|^2}{2m} \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

Qubit calorimeter interaction

\[ H_{qe} = g \frac{\sqrt{8 \pi} \epsilon_F}{3N} \sum_{k \neq l \in S} (\sigma_+ + \sigma_-) c_k^\dagger c_l, \]

\[ N = O(10^9) \text{ fermions} \]

\[ S = \text{energy shell around } \epsilon_F \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

**Phonons**

\[ H_p = \sum_k \hbar \omega_k b_k^\dagger b_k \]

\[ \omega_k = v_s k \]

\( v_s = \text{sound speed} \quad k = \|k\| \text{ phonon wavelength norm.} \)
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

Frölich Hamiltonian

\[ H_{ep} = \lambda \sum_{k,q} \omega_q^{1/2} \left( c_k^{\dagger} c_{k-q} b_q + c_k^{\dagger} c_{k-q} b_q^{\dagger} \right) \]
Timescales

- $\tau_{ee} = O(10^0)\text{ns}$: Landau quasi-particle relaxation rate to Fermi–Dirac equilibrium in a metallic wire.
- $\tau_{ep} = O(10^4)\text{ns}$: electron-phonon interactions.
- $\tau_R = 2 - 5 \times O(10^5)\text{ns}$: transmon qubit relaxation times (Wang et al., *Applied Physics Letters*, (2015))
- $\tau_{eq} \simeq g^{-2}$: Fermi’s golden rule estimate of characteristic qubit-calorimeter time scale.

Open quantum system approach

$\tau_{ee} \ll \tau_{eq} \ll \tau_{ep} \ll \tau_R$
Phonon–fermion bath interaction

- Phonon bath temperature $T_p = O(10^{-1})K$ (cryostat)
- Fermion bath temperature $T_e$

$$T_p \simeq T_p$$

**mean energy current** $\propto T_p^5 - T_e^5$ (leading order\textsuperscript{a})

**rms energy current fluctuations** $\propto O(T_p^3)$ at $T_e = T_p$ (leading order\textsuperscript{b})


\textsuperscript{b}Pekola and Karimi, “Quantum noise of electron-phonon heat current”, 2018.
Idea of the model

Qubit: stochastic Schrödinger equation

\[ d\psi = \left( \text{deterministic dissipative drift} \right) dt + \text{Poisson jumps} \]

Calorimeter: equilibrium Fermi–Dirac ensembles at evolving \( T_e \)

\[ dT_e^2 = \frac{1}{N\gamma} dE \]

Sommerfeld expansion

\[ dE = dE_{eq} + dE_{ep} = \text{Poisson jumps} + (T_p^5 - T_e^5)dt + O(T_p^3)dw_t \]
Upshot of the modeling

"Strong drive": Floquet theory


\[ \tau_{qe} \gg \tau_m = \text{inverse separation of peaks in the radiation spectrum (RWA).} \]

- Resonant drive: \[ \tau_m / \tau_{qe} \approx g^2 / \kappa \ll 1 \]
- Temperature+population process: jump diffusion master equation

"Weak drive"

\[ g^2 / \kappa \geq 1 \]

- Temperature+state process: hybrid master equation

Short-time temperature behaviour

Initial temperature of the electron bath: $T_e = 0.1 K$

Temperature distributions after 10 periods of resonant strong drive
Short-time temperature behaviour

Initial temperature of the electron bath: $T_e = 0.1K$

(Left) Mean temperature of the calorimeter after 10 periods of driving vs driving frequency $\omega_L$ for different values of the qubit calorimeter coupling $g$. Stars= weak-drive. Lines: Floquet . (Right) Standard deviation.
Short-time temperature behaviour

Initial temperature of the electron bath: $T_e = 0.1K$
Relaxation to a steady state

The qubit-calorimeter reaches a steady state.
Effective temperature process

**Multiscale expansion:** \[ \varepsilon \propto 1/N \& s = \varepsilon t \geq O(1) \]

\[
dT_e^2 = \frac{1}{\gamma} \left( \Sigma V(T_p^5 - T_e^5) + J(T_e^2) \right) ds + \frac{1}{\gamma \sqrt{N}} \left( \sqrt{10\Sigma Vk_B T_p^3} + \sqrt{S(T_e^2)} \right) dw_s
\]

**Analytic estimates**

\[
\langle T_e \rangle \approx \left( T_p^5 + g^2 \frac{O(\hbar \omega_L^2)}{\Sigma V} \right)^{1/5}
\]

mean steady state temperature

\[
\tau \approx \left( T_p^5 + g^2 \frac{O(\hbar \omega_L^2)}{\Sigma V} \right)^{-3/5}
\]

relaxation time to steady state
Numerics vs analytic theory (Floquet)

Steady state temperature PDF

\[ \sigma = 0.004 \text{ K} \quad \xi = 0.001 \text{ K} \]

\[ \sigma = 0.005 \text{ K} \quad \xi = 0.003 \text{ K} \]
Steady state av. temperature vs drive strength

- Floquet
- Weak drive

$g^2 = 0.1$

$g^2 = 0.01$

$g^2 = 0.001$
Predictions always involve weak coupling between qubit and calorimeter.
Perturbative Markovian master equation techniques not reliable beyond the strictly weak subsystem-bath coupling limit (see e.g. Segal, *Physical Review B*, (2013)).
Strong qubit-calorimeter coupling analysis desirable.
References

Based on


- B. Donvil, P. Muratore-Ginanneschi, J. P. Pekola *Hybrid master equation for calorimetric measurements* 2018

THANKS FOR YOUR ATTENTION
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HAPPY BIRTHDAY!!!!

Enthusiasm for physics 😻