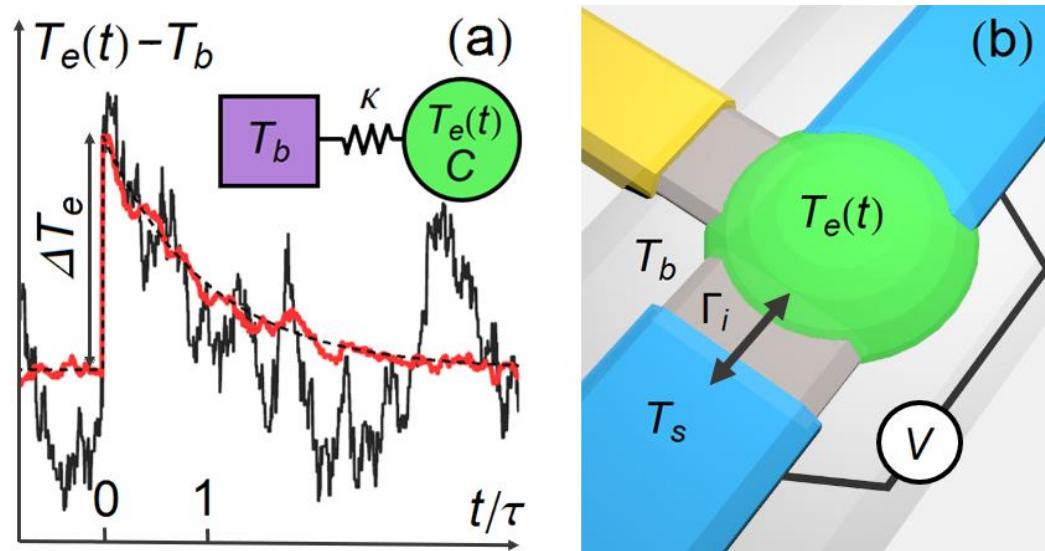


Nanoscale Quantum Calorimetry with Electronic Temperature Fluctuations

F. Brange¹, P. Samuelsson¹, B. Karimi², J.P. Pekola²

¹Lund University, ²Aalto University



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Outline

Calorimetry of heat pulses

- Calorimetry and bolometry
- Fast and sensitive temperature measurements
- Estimates for sub-meV detection

Proposed nanoscale calorimeter

- Hybrid normal-superconductor calorimeter
- Energy transfer statistics

Temperature fluctuations

- Temperature fluctuations, full statistics
- Temperature noise
- Third cumulant and back-action

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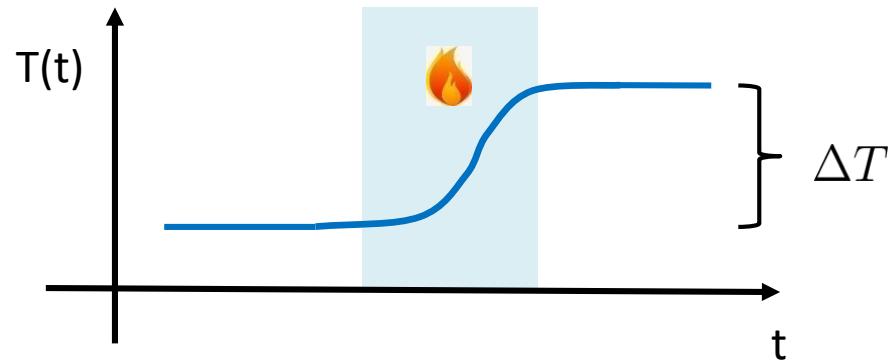
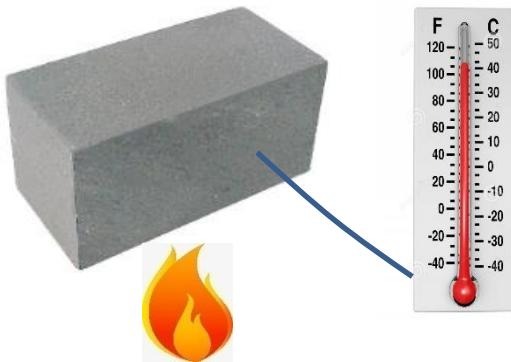
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Calorimetry

“Calorimetry is the science or act of measuring changes in state variables of a body for the purpose of deriving the heat transfer associated with changes of its state due, for example, to chemical reactions, physical changes, or phase transitions under specified constraints” - Wikipedia

- Here: measurement of energy/heat transfer via temperature change



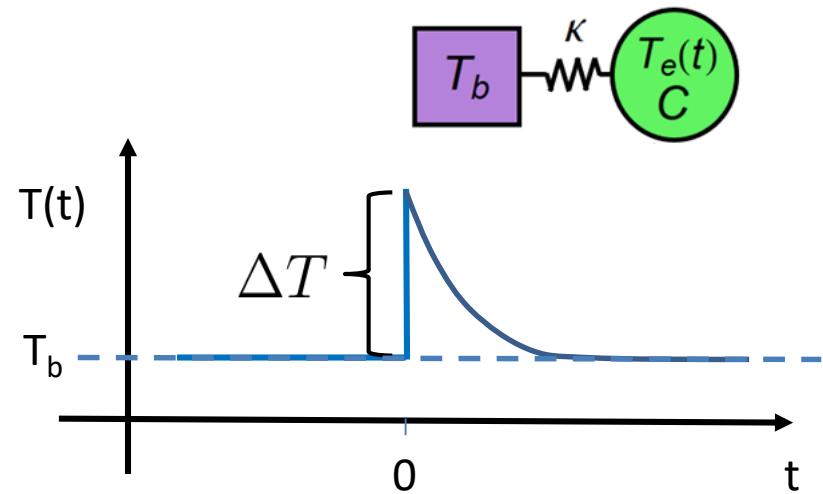
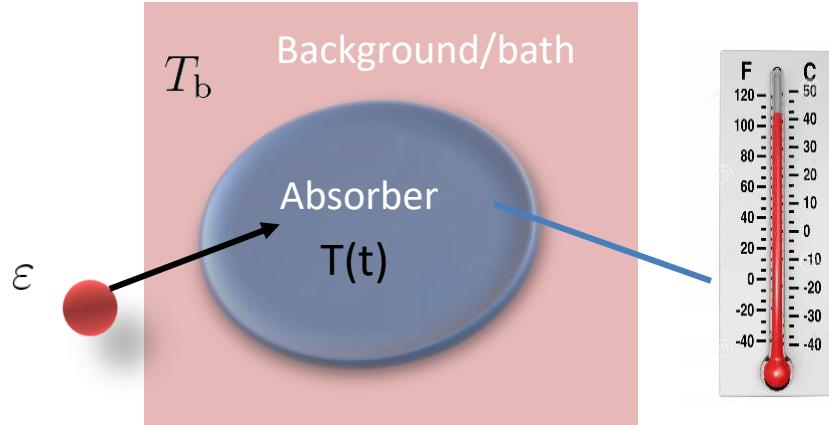
- Absorbed energy

$$E = C \Delta T$$

C - heat capacity

Quantum Calorimetry

Single particle energy detection (particle physics Kilbourne et al, Phys. Today 99)



- Ideal operation (linear $\Delta T \ll T_b$, noise free)

$$T(t) = T_b + \Delta T e^{-t/\tau}, \quad t \geq 0 \quad \tau = C/\kappa$$

κ - thermal conductivity, absorber-bath

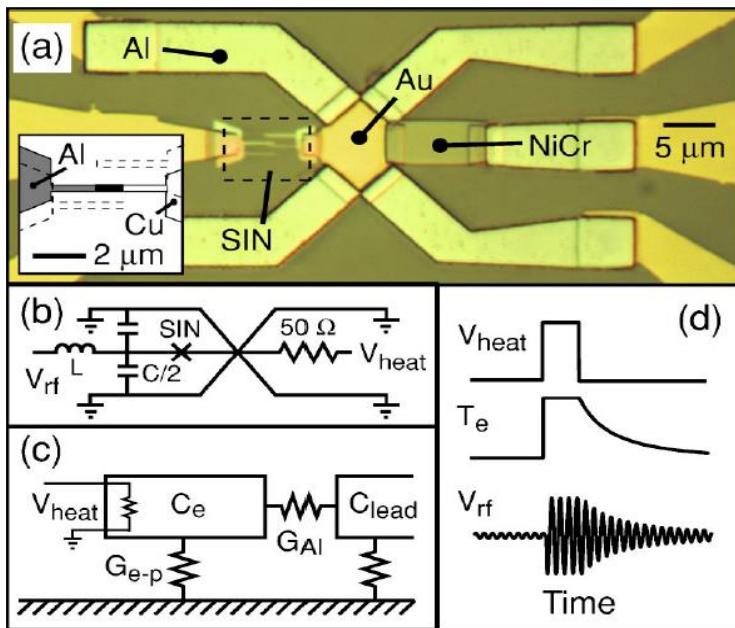
Particle energy

$$\varepsilon = C \Delta T$$

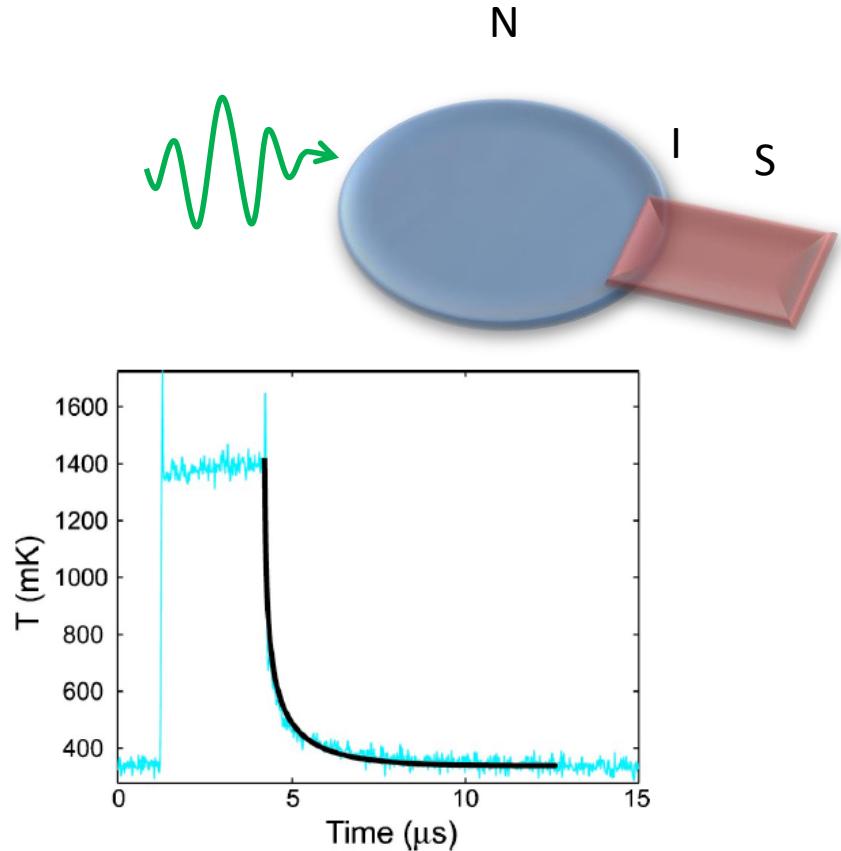
Nanoscale calorimeters and bolometers

Early experiments

- X-ray detection Nahum, Martinis, APL 95
- Fast (RF) readout Schmidt et al, PRB 04



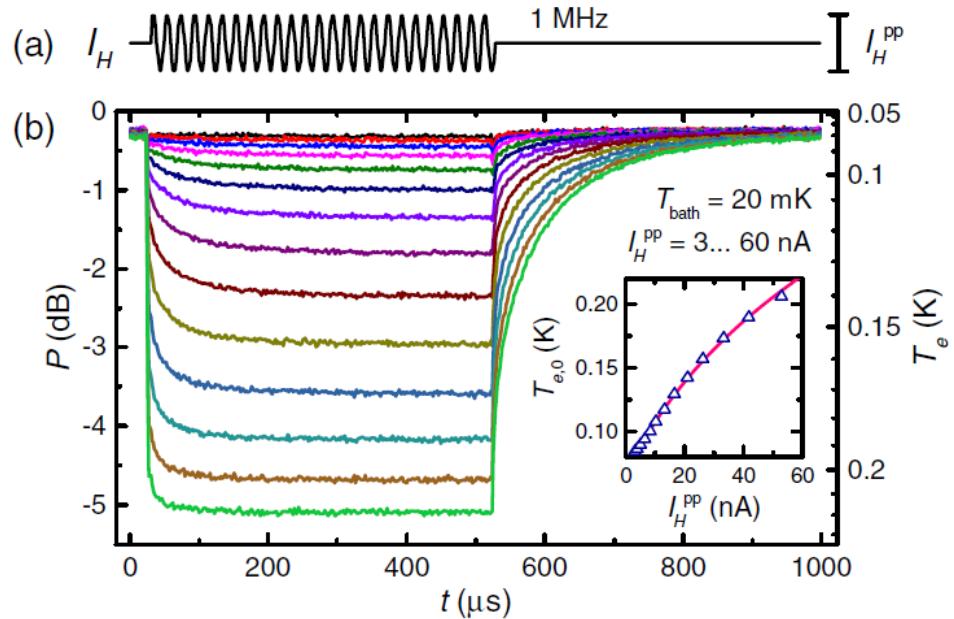
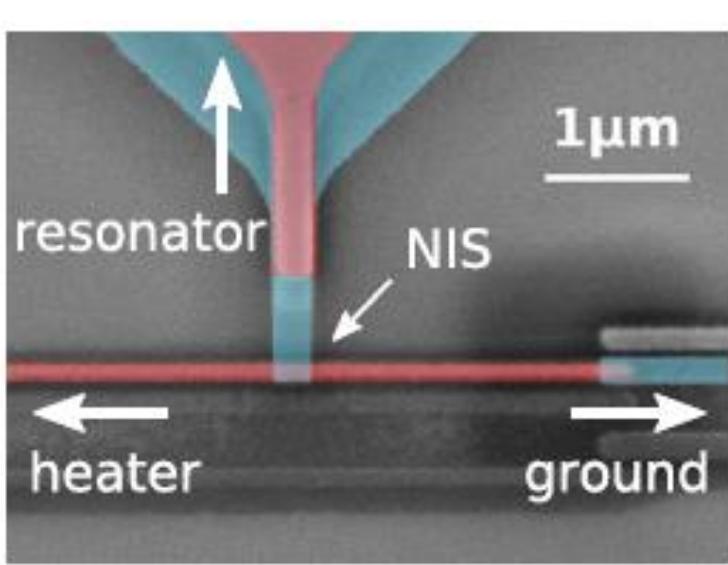
- Large sensitivity Schmidt et al, APL 05



Largely space
application driven

Recent Aalto results

Gasparinetti et al, PR App 14, Govenius et al, PRL 16, Viisanen and Pekola, PRB 2018,....



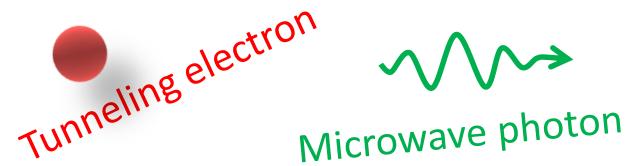
- Fast, sensitive thermometry, effectively non-invasive.
- Small absorber volume \Rightarrow small heat capacity.
- Small background noise.

Typical parameters

$$C \sim 10^3 - 10^5 k_B \quad \tau \sim 1 - 100 \mu\text{s}$$

$$T_b \sim 30 - 100 \text{ mK} \quad (\text{effective bath})$$

Can we extend to
quantum calorimetry
for sub meV-energies?



Energy quanta detection and fluctuations

Absorber-bath transfers - phonons \Rightarrow
fluctuation-dissipation like relation

$$\langle \delta T_e(t) \delta T_e(t') \rangle = \frac{k_B T_b^2}{C} e^{-|t-t'|/\tau},$$

\Rightarrow amplitude of fluctuations

$$\sqrt{\langle \delta T_e^2(t) \rangle} = T_b (k_B/C)^{1/2}$$

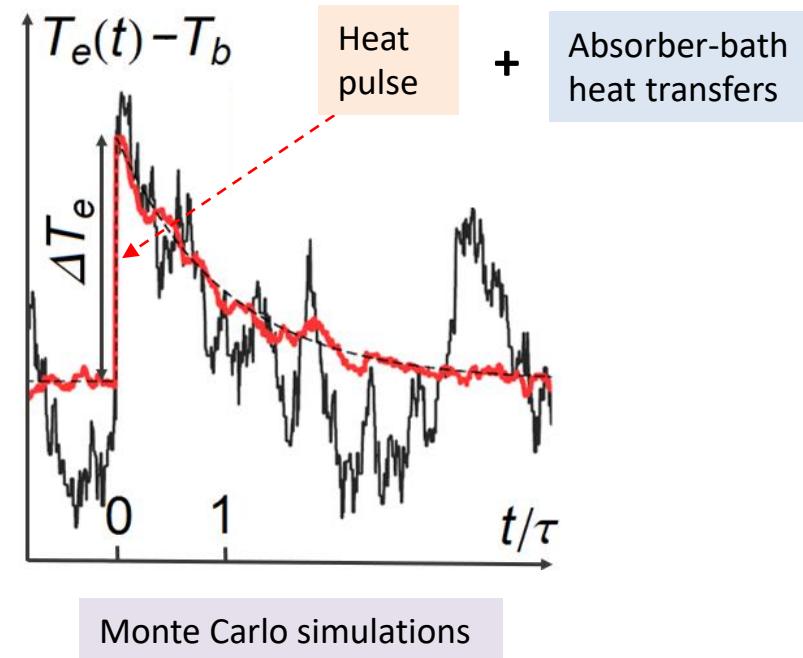
Signal-to-noise ratio (SNR), $\Delta T_e = \varepsilon / C$

$$\Delta T_e / \sqrt{\langle \delta T_e^2 \rangle} = \varepsilon / [T_b \sqrt{k_B C}]$$

Typical parameters for single energy quanta detection

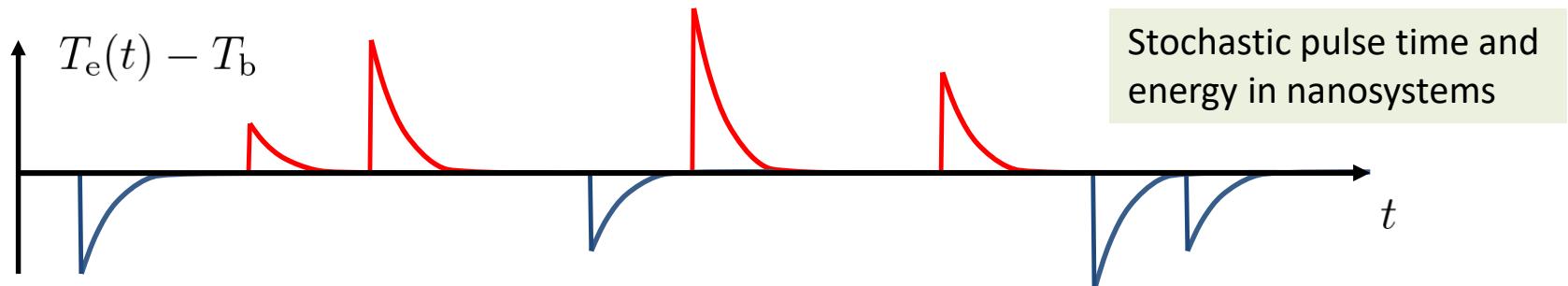
$$\varepsilon = 200 \mu eV, C = 10^3 k_B \quad \left\{ \begin{array}{l} T_b = 5 \text{ mK} \\ T_b = 30 \text{ mK} \end{array} \right. \quad \begin{array}{c} \text{---} \\ \text{—} \end{array} \quad \begin{array}{l} \text{SNR 15} \\ \text{SNR 2.4} \end{array}$$

Ex: Al gap $\Delta \approx 200 \mu eV$, 50 GHz microwave photon



Careful treatment of
fluctuations needed!

Stochastic treatment of all transfer events



Probability distribution of total energy transfer (during time t_0)

$$P_\sigma(E, T_e) = \frac{1}{2\pi} \int d\xi_\sigma e^{-iE\xi_\sigma + t_0 F_\sigma(\xi_\sigma, T_e)} \quad \sigma = i, b$$

Injection of heat pulses (i) and absorber-bath (b) transfers, T_e constant.

Poisson transfer statistics: cumulant generating function

[van den Berg et al NJP 15](#)

$$F_\sigma(\xi_\sigma, T_e) = \Gamma_\sigma(T_e) \left[\int d\varepsilon e^{i\varepsilon\xi_\sigma} P_\sigma(\varepsilon, T_e) - 1 \right],$$

Particle
transfer rate

Particle energy
distribution

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Proposed nanoscale calorimeter

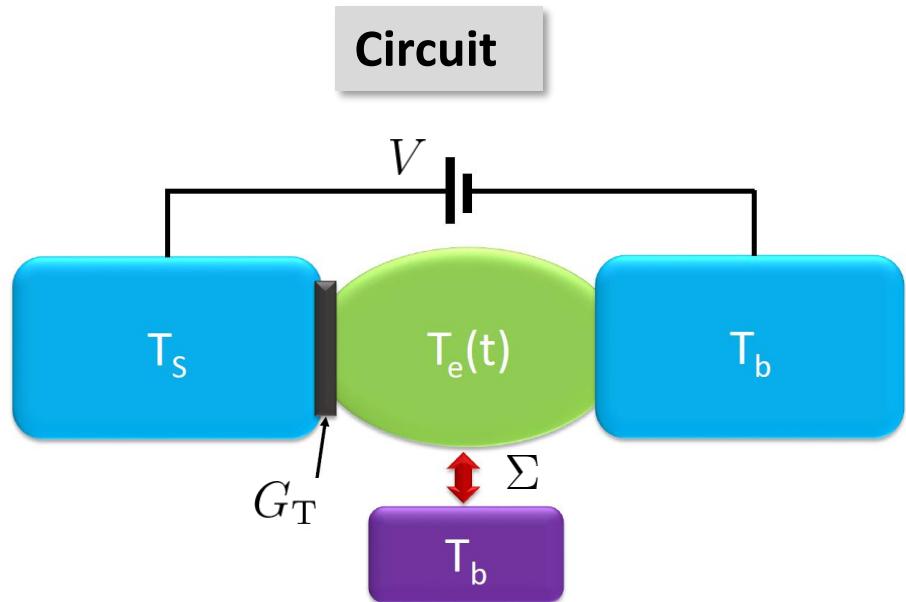
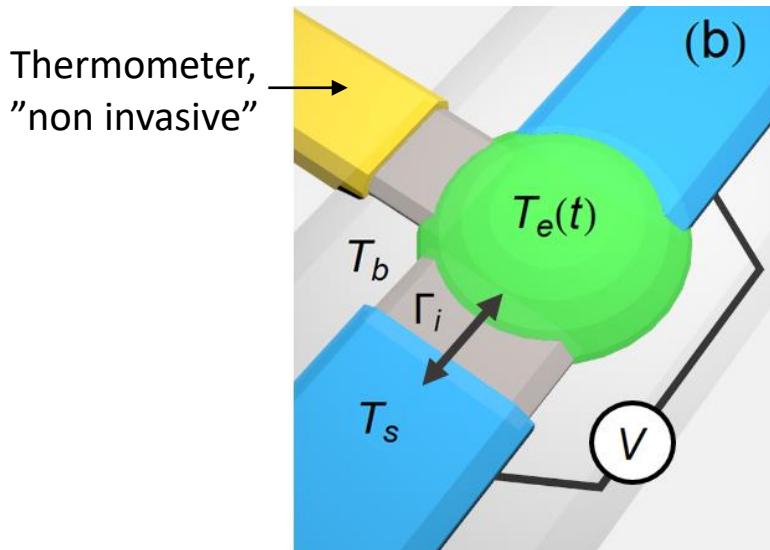
- Hybrid normal-superconductor calorimeter
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Temperature fluctuations

- Temperature fluctuations, full statistics
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Normal-superconductor set-up

Heat pulses from tunneling electrons



Parameters

- Tunneling conductance G_T
- Phonon coupling constant Σ
- Temperatures $T_s, T_b, T_e(t)$
- Applied bias V

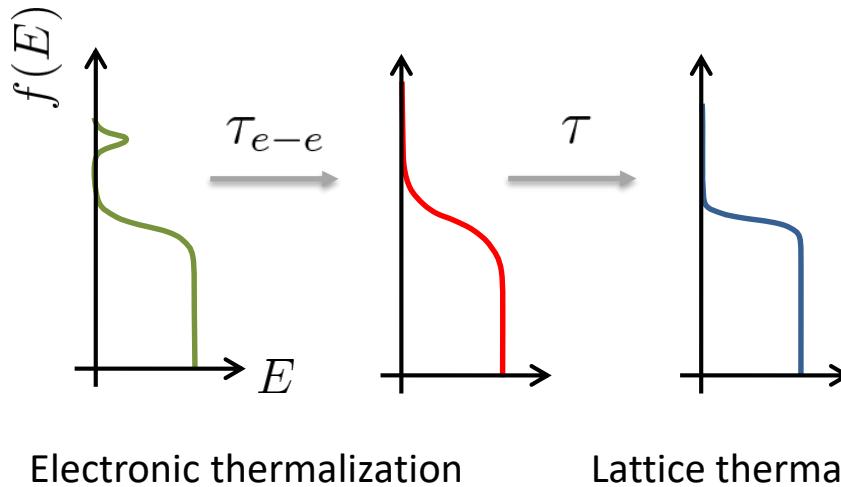
Right superconductor

- Transparent, ohmic contact
- Suppresses potential fluctuations
- Perfect heat mirror

Time scales and assumptions

Hot-electron regime

Absorber relaxation times



Additional assumptions

- No standard and inverse proximity effect
- No unwanted heating, V does not affect T_s

Quasi-equilibrium

$$\tau_{e-e} \ll \tau, 1/\Gamma_i$$

- Electronic distribution

$$f_e(E) = [1 + e^{E/k_B T_e(t)}]^{-1}$$

- Well defined $T_e(t)$

Well separated pulses

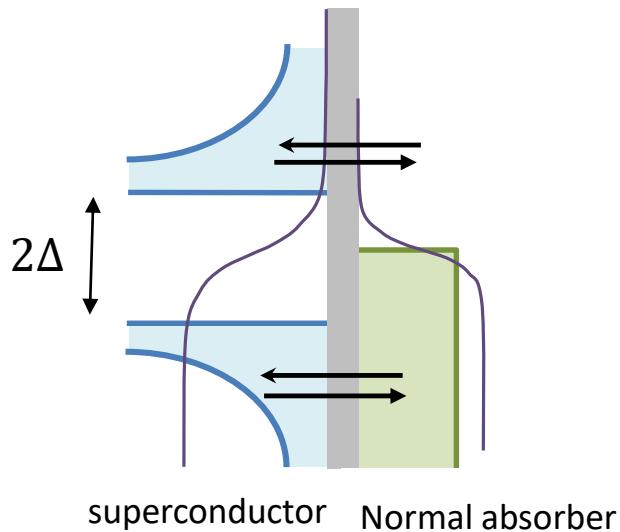
$$\Gamma_i \tau \ll 1$$



Not needed
in general

Injector – absorber, electron tunnelling

Quasiparticle picture



Spectral tunneling rates

Standard expressions

$$\Gamma_{\pm}^i(\varepsilon) = (G_T/e^2)\nu_S(\varepsilon - eV)f_{\pm}(\varepsilon - eV, T_s)f_{\mp}(\varepsilon, T_e)$$

with

$$\nu_S(\varepsilon) = |\varepsilon|/\sqrt{\varepsilon^2 - \Delta^2}\theta(|\varepsilon| - \Delta)$$

$$f_+(\varepsilon, T) = (e^{\varepsilon/[k_B T]} + 1)^{-1}, f_-(\varepsilon, T) = 1 - f_+(\varepsilon, T)$$

Cumulant generating function

$$F_i(\xi_i, T_e) = \int d\varepsilon \left[\Gamma_+^i (e^{i\xi_i \varepsilon} - 1) + \Gamma_-^i (e^{-i\xi_i \varepsilon} - 1) \right]$$

Rate into absorber

Energy counting factors

Rate out of absorber

Cumulants from $(-i)^n \partial_{\xi_i}^n F(\xi_i, T_e)|_{\xi_i=0}$, giving energy current and noise

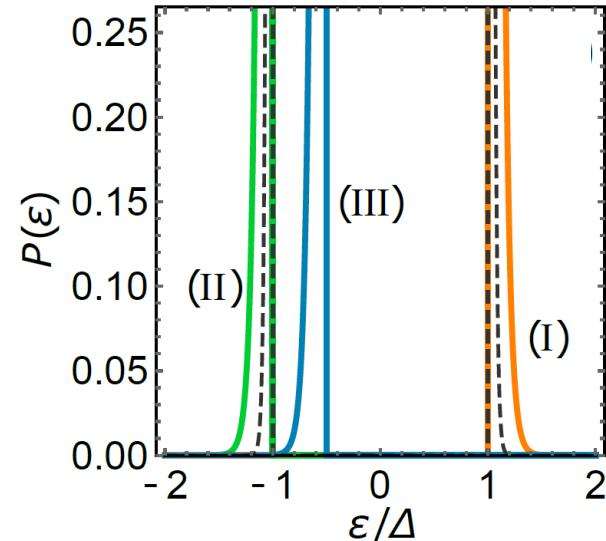
$$I_i^E = \int d\varepsilon \varepsilon [\Gamma_+^i - \Gamma_-^i] \quad S_i^E = \int d\varepsilon \varepsilon^2 [\Gamma_+^i + \Gamma_-^i]$$

Tunneling rate and energy distribution

$$\Gamma_i(T_e) = \int d\varepsilon [\Gamma_{+}^i(\varepsilon) + \Gamma_{-}^i(\varepsilon)]$$

$$P_i(\varepsilon, T_e) = [\Gamma_{+}^i(\varepsilon) + \Gamma_{-}^i(-\varepsilon)]/\Gamma_i$$

Single energy injection



Relevant regime $k_B T_s, k_B T_e \ll \Delta$, three cases with well defined energies

$$\varepsilon_I = \Delta, \varepsilon_{II} = -\Delta, \varepsilon_{III} = eV - \Delta$$

for

$$V = 0, T_s \gg T_e \quad (I)$$

$$V = 0, T_s \ll T_e \quad (II)$$

$$T_s(1 - e|V|/\Delta) \ll T_e \ll e|V|/k_B \quad (III)$$



generating function $g = \sqrt{2\pi}G_T\Delta/e^2$, c_α constant

$$F_i^{(\alpha)}(\xi_i, T_e) = g c_\alpha (e^{i\varepsilon_\alpha \xi_i} - 1), \quad \alpha = I, II, III$$

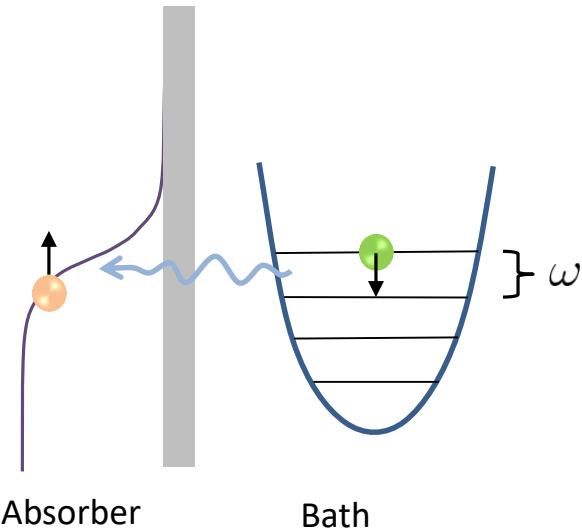
Uncorrelated/Poisson injection
of particles with energy ε_α

Bath – absorber, phonons

Bath phonon picture

Spectral rates

Standard expressions, 3D-phonons in metals



$$\Gamma_{\pm}^b(\varepsilon) = -\Sigma \mathcal{V} / [24k_B^5 \zeta(5)] \varepsilon^3 n(\pm\varepsilon, T_b) n(\mp\varepsilon, T_e)$$

with Riemann $\zeta(x)$,

$$n(\varepsilon, T) = (e^{\varepsilon/[k_B T]} - 1)^{-1}$$

and the absorber volume \mathcal{V} .

Cumulant generating function

$$F_b(\xi_b, T_e) = \int d\varepsilon [\Gamma_+^b (e^{i\xi_b \varepsilon} - 1) + \Gamma_-^b (e^{-i\xi_b \varepsilon} - 1)]$$

Cumulants $S_b^{(n)} = \partial_{\xi_b}^n F_b(\xi_b, T_e)|_{\xi_b=0}$ are ($n_{\pm} = n + (7 \pm 1)/2$, $n = 1, 2, \dots$)

$$S_b^{(n)} = \Sigma \mathcal{V} k_B^{n-1} \frac{\zeta(n_{\pm})(n+3)!}{24\zeta(5)} (T_e^{n+4} \pm T_b^{n+4}),$$

- Odd n exact
- Even n within 2%

Bath – absorber, phonons

Energy current and noise

$$S_b^{(1)} = \Sigma \mathcal{V} (T_e^5 - T_b^5) \quad \text{Wellstood et al, PRB 94}$$

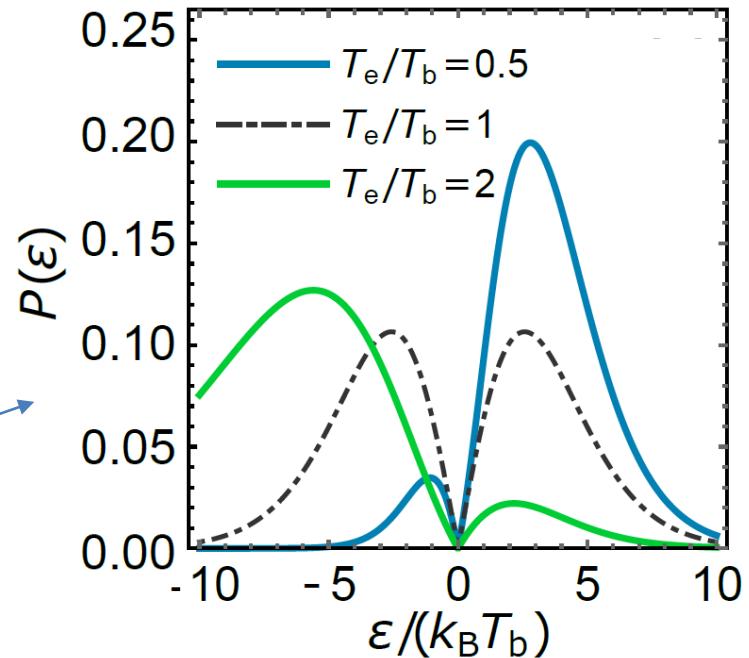
$$S_b^{(2)} \propto \Sigma \mathcal{V} (T_e^6 + T_b^6) \quad \text{Karimi, Pekola, JLTP 18}$$

Tunneling rate and energy distribution

$$\Gamma_b(T_e) = \int d\varepsilon [\Gamma_+^b(\varepsilon) + \Gamma_-^b(\varepsilon)]$$

$$P_b(\varepsilon, T_e) = [\Gamma_+^b(\varepsilon) + \Gamma_-^b(-\varepsilon)]/\Gamma_b$$

Emission and absorption of phonons



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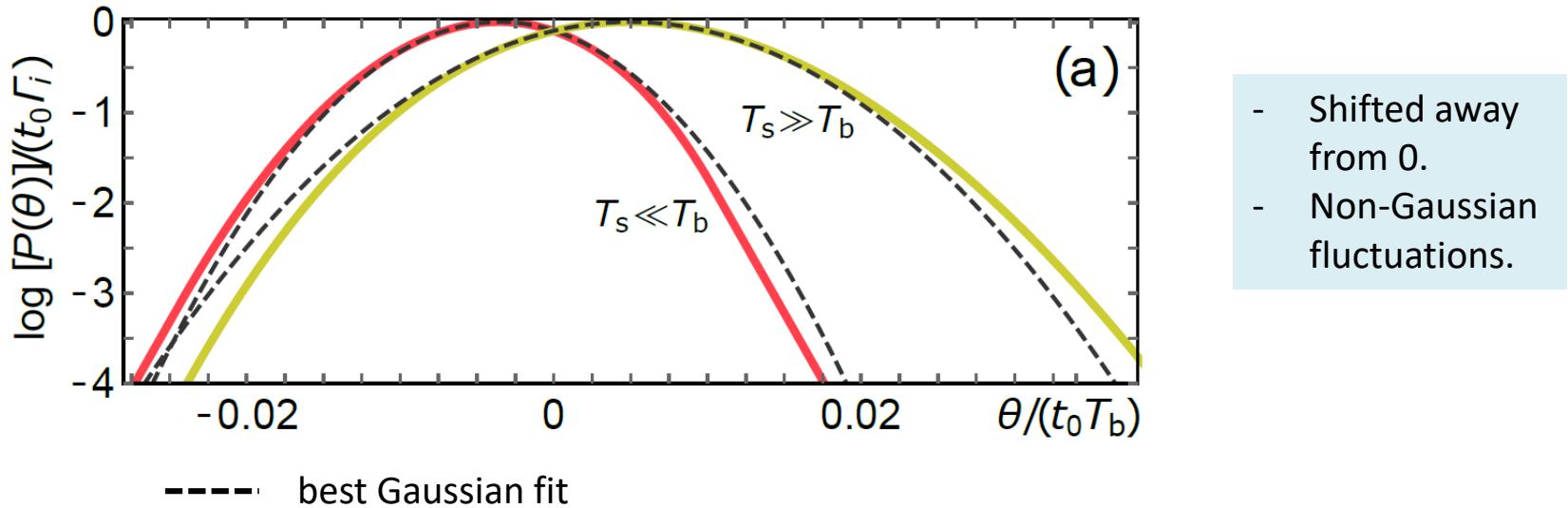
Full temperature statistics

Rates $\Gamma_i(T_e)$ and $\Gamma_b(T_e)$ depend on electron temperature \Rightarrow
Back-action of heat transfer induced T_e -fluctuations on rates.

Resulting, full, temperature fluctuations investigated via

$$P(\theta), \quad \theta = \int [T_e(t) - \bar{T}_e] dt$$

Stochastic path integral approach [Jordan et al, J. Mat. Phys 04](#), [Battista et al, PRL 13](#).
(alternative [Heikkilä and Nazarov, PRL 09](#)).



Cumulant expansion

Cumulant expansion, in terms of energy current cumulants

$$\langle\langle \mathcal{E}^n(T_e) \rangle\rangle = (-i)^n \partial_\xi^n F(\xi, T_e)|_{\xi=0} \quad F(\xi, T_e) = F_i(\xi, T_e) + F_b(\xi, T_e)$$

gives average temperature \bar{T}_e from

$$\langle \mathcal{E}(\bar{T}_e) \rangle = 0.$$

The temperature noise is, with $\kappa(T_e) = i\partial_{T_e}\partial_\xi F(\xi, T_e)|_{\xi=0}$,

$$S_{T_e}^{(2)} = \frac{1}{\kappa^2} \langle\langle \mathcal{E}^2(T_e) \rangle\rangle \quad \text{never measured!}$$

and the third cumulant

$$S_{T_e}^{(3)} = \frac{1}{\kappa^3} \left[\langle\langle \mathcal{E}^3(T_e) \rangle\rangle + 3 \langle\langle \mathcal{E}^2(T_e) \rangle\rangle \frac{d}{dT_e} \frac{\langle\langle \mathcal{E}^2(T_e) \rangle\rangle}{\kappa(T_e)} \right]$$

all evaluated at \bar{T}_e .

Low frequency
temperature
autocorrelations

back action term

Average temperature

The average temperature equation is

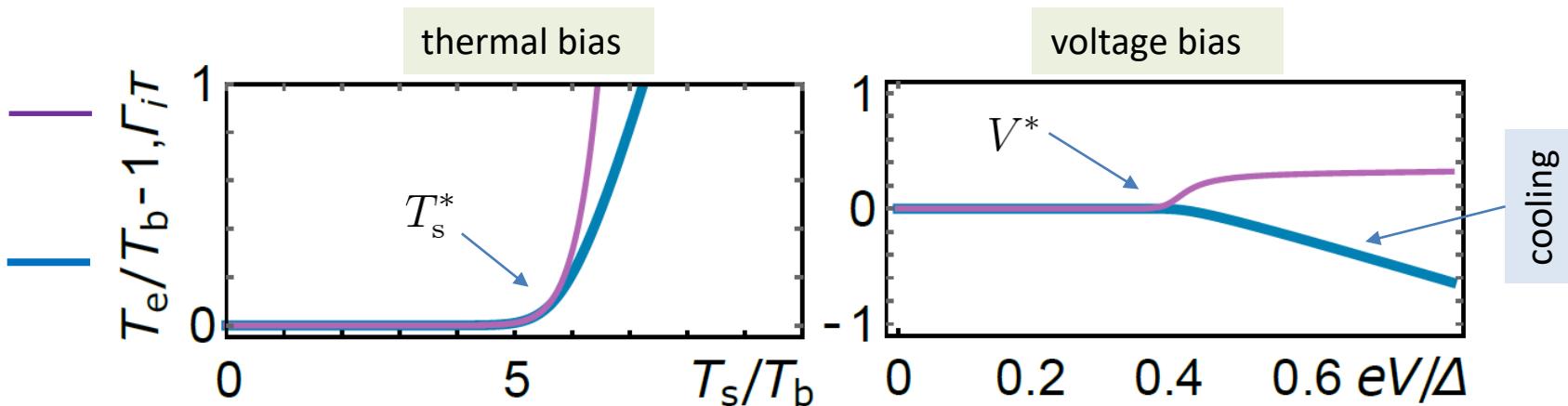
$$h(T_s) + h(\bar{T}_e) \left[-\cosh\left(\frac{eV}{k_B \bar{T}_e}\right) + \frac{eV}{\Delta} \sinh\left(\frac{eV}{k_B \bar{T}_e}\right) \right] = \frac{1}{5r} \left(\frac{\bar{T}_e^5}{T_b^5} - 1 \right)$$

where

$$h(T) = \sqrt{\frac{k_B T}{\Delta}} e^{-\frac{\Delta}{k_B T}} \quad r = \frac{\sqrt{2\pi} G_T \Delta^2}{T_b e^2 \kappa}$$

Injection tunnel rates

$$\Gamma_i = g [h(T_s) + h(T_b) \cosh(eV/k_B T_b)]$$



Well separated injection events $\Gamma_i \tau \ll 1$ gives bias limits T_s^*, V^* .

Temperature noise

Thermal bias

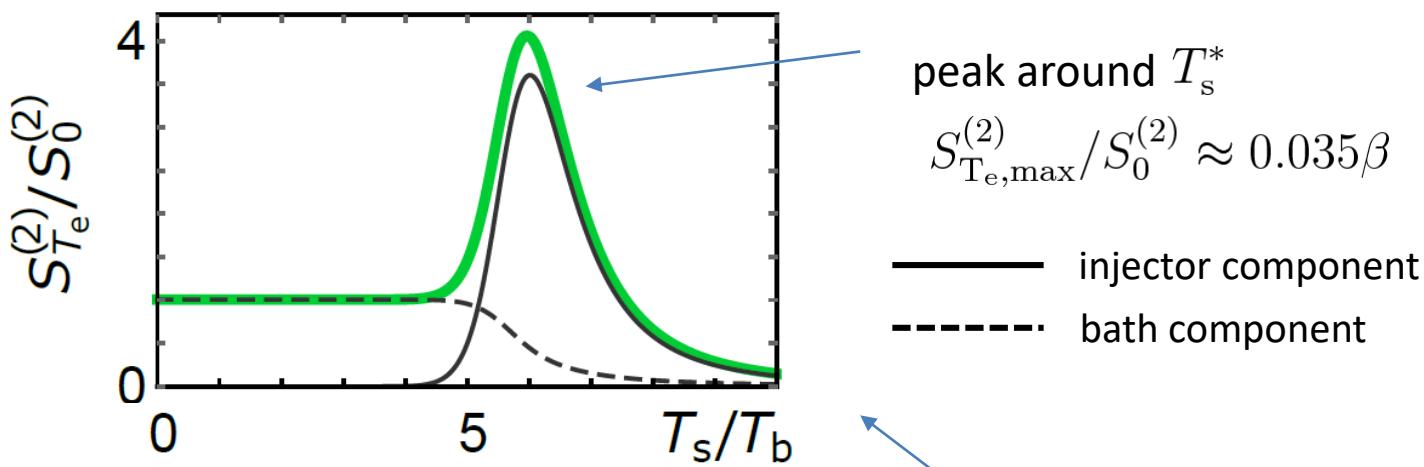
$$V = 0$$

The second cumulant, for $\beta \gg \ln(r) \gg 1$, $\beta = \Delta/(k_B T_b)$, $q = \bar{T}_e/T_b$

$$S_{T_e}^{(2)}/S_0^{(2)} = \frac{1+q^6}{2q^8} + \frac{\beta(q^5 - 1)}{10q^8}$$
$$S_0^{(2)} = 2k_B T_b^2 / \kappa$$

bath noise

injector noise



$$T_b = 0.01\Delta/k_B, C = 20\Delta/T_b$$

suppression from
increasing $\kappa(\bar{T}_e)$

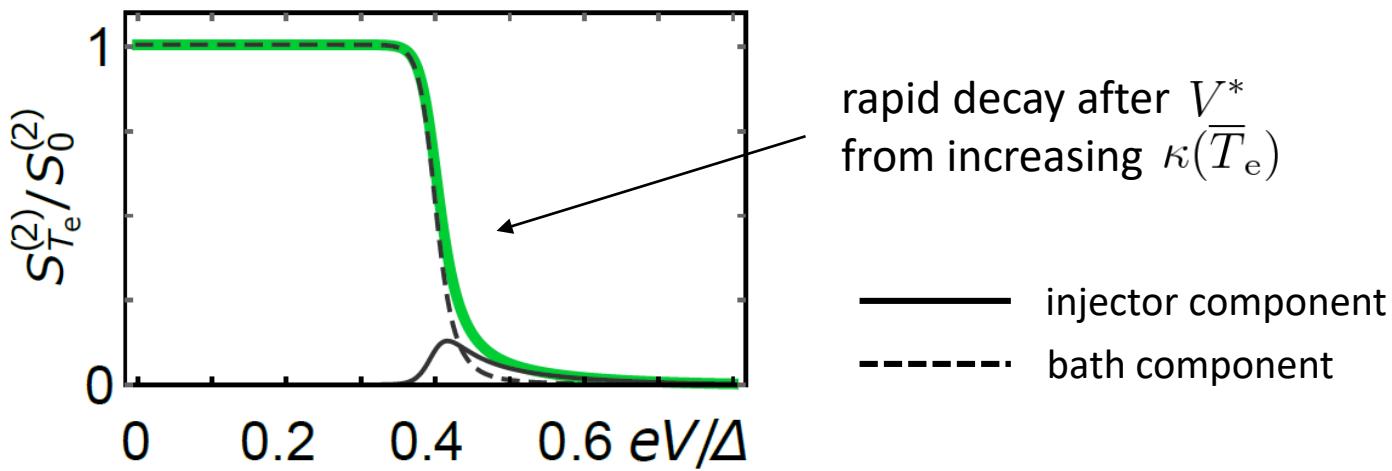
Voltage bias

$$T_s = T_b$$

The second cumulant, with $\tilde{\beta} = \beta(1 - eV/\Delta)$

$$\frac{S_{T_e}^{(2)}}{S_0^{(2)}} = \frac{q^4}{2} \frac{1 + q^6 + (\tilde{\beta}/5)(1 - q^5)}{\left(q^6 + (\tilde{\beta}/5)(1 - q^5)\right)^2}.$$

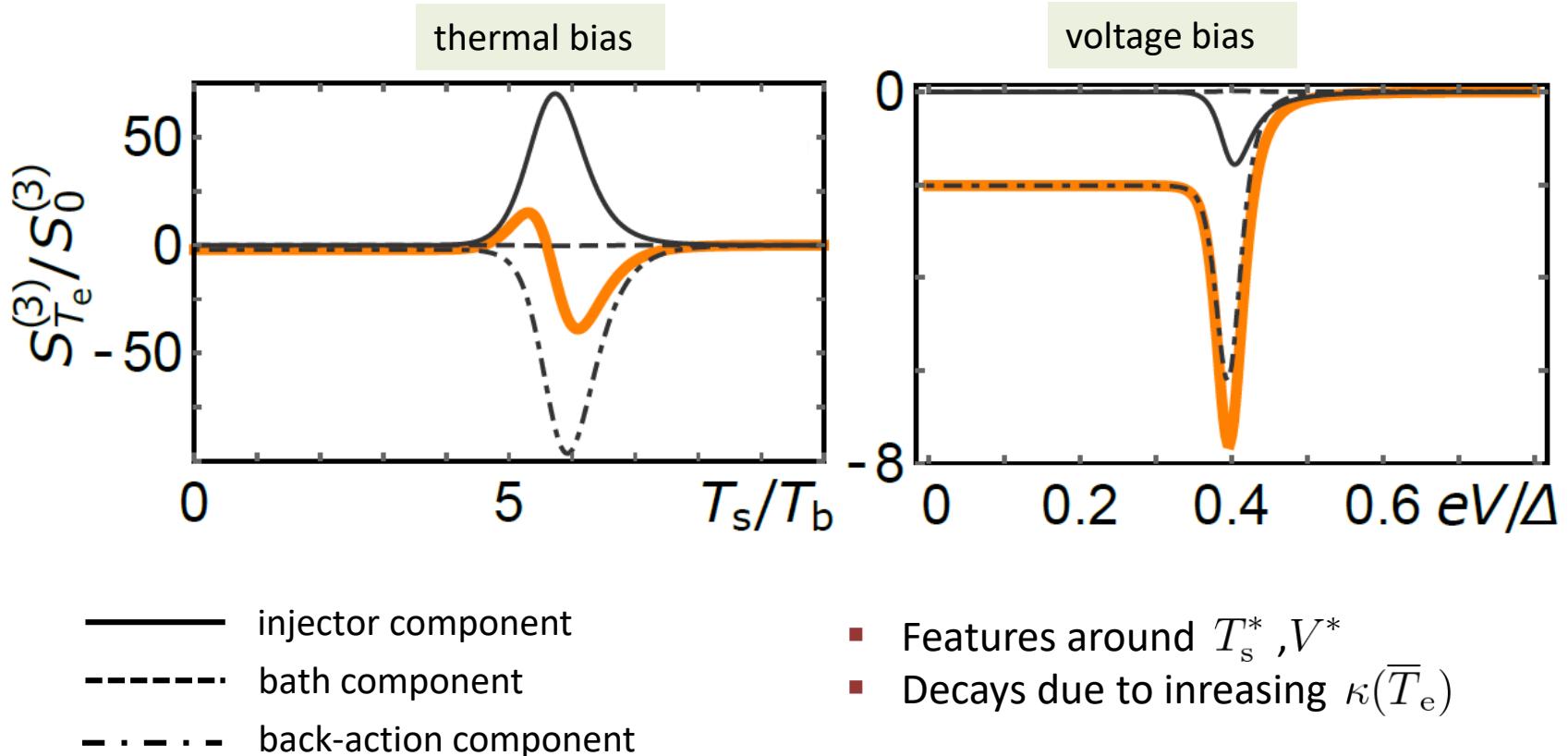
bath term injector term



Third cumulant

Three terms: bath, injector and back-action

$$S_0^{(3)} = 6k_B^2 T_b^3 / \kappa^2$$



Conclusions

- Towards single energy quantum calorimetry
- Hybrid normal-superconductor system
- Full temperature statistics
- Parameter estimates and limits

